Spectral signatures of breaking of ensemble equivalence

Dionigi, P.

Summary

In this thesis we explore the concept of Breaking of Ensemble Equivalence (BEE) within the context of random graph models, focusing on spectral properties of adjacency matrices. Our research aims to identify spectral quantities that can distinguish between different random graph ensembles, thereby providing new insights into the structure and behavior of complex networks. We cover both theoretical aspects and practical implications, including simulations and sampling methods for random graph models.

In Chapter 1 we introduce some basic notions of random graph theory, and discuss how maximum entropy graph models are fundamental in modeling real-world networks. We explain what BEE is, what is its characterization in the context of statistical mechanics, and how it is intimately connected to differences that arise naturally between the canonical versus the microcanonical description of random graph ensembles. In order to do so, we delve into the spectral theory of random graphs and use it to investigate BEE.

In Chapter 2 we formulate a conjecture on the equivalence of measure-BEE and the presence of a gap between the largest non-centered and non-scaled largest eigenvalues of the adjacency matrix in the canonical and the microcanonical ensemble. We prove this conjecture in the setting of homogeneous graphs.

In Chapter 3 we study the same question for Chung-Lu random graphs. In particular, we prove central limit theorems for the largest eigenvalue and its associated eigenvector.

In Chapter 4 we compute the expectation of the largest eigenvalue for the configuration model, which verifies our conjecture in the setting of inhomogeneous graphs as well.

In Chapter 5 we provide numerical evidence for our findings through simulation, after a brief introduction to graph sampling. We formulate the main conclusions of our work and indicate possible further directions of research.