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## Spectral signatures of breaking of ensemble equivalence

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# Spectral Signatures of Breaking of Ensemble Equivalence

Pierfrancesco Dionigi

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# Spectral Signatures of Breaking of Ensemble Equivalence

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