Motivic invariants of character stacks

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Stellingen

behorende bij het proefschrift
"*Motivic invariants of character stacks*"

(i) Characteristically, the Euler characteristic of the G-character stack of a surface of Euler characteristic $2-2g$ is determined by the characteristics of the character table of G over finite fields.

(ii) The group of unipotent upper triangular matrices of rank 10 over the finite field of 13 elements has 609233173333356 irreducible complex representations of degree 815730721.
[Theorem 6.3.10]

(iii) Let X and Y be quasi-projective varieties over a field k of characteristic not equal to 2, both equipped with an action of $G = \mathbb{Z}/2\mathbb{Z}$. If X can be stratified into copies of affine space on which G acts linearly, then $[X \times Y]^+ = [X]^+[Y]^+ + [X]^-[Y]^-$ in the Grothendieck ring $\mathrm{K}_0(\mathbf{Var}_k)$, where $[X]^+ = [X /\!\!/ G]$ and $[X]^- = [X] - [X]^+$. [Theorem 3.6.19]

(iv) Let G be a connected complex algebraic group, and denote by $\mathfrak{X}_G(\Sigma_g)$ the G-character stack of a closed surface Σ_g of genus g. Then the limit $\lim_{g\to\infty} e(\mathfrak{X}_G(\Sigma_g))/e(G)^{2g-2}$ is equal to the E-polynomial $e(G^{\mathrm{ab}})$ of the abelianization of G. When G is a finite group, the limit $\lim_{g\to\infty} |\mathfrak{X}_G(\Sigma_g)|/|G|^{2g-2}$ is equal to the number of 1-dimensional irreducible representations of G.

(v) For every $r \geq 0$, the sequences of GL_r-representation varieties $X_n = R_{\mathrm{GL}_r}(F_n)$ of the free groups F_n, and $Y_n = R_{\mathrm{GL}_r}(\mathbb{Z}^n)$ of the free abelian groups $\mathbb{Z}_n$, with the action of GL_r by conjugation, and the action of S_n by permutation, are motivically representation stable (see Definition 7.3.4).
[Theorems 7.4.1 and 7.4.2]

(vi) There exists a closed 4-dimensional manifold with non-trivial fundamental group which has no non-trivial G-local system for any complex algebraic group G.

(vii) Let G be the group $\mathbb{G}_m \rtimes \mathbb{Z}/2\mathbb{Z}$ over a field k of characteristic not equal to 2. The G-character stack $\mathfrak{X}_G(\Sigma_g)$ has virtual class $\frac{1}{2}((\mathbb{L}-1)^{2g-2}(2^{2g+1}+\mathbb{L}-3)+(\mathbb{L}+1)^{2g-2}(2^{2g+1}+\mathbb{L}-1))$, where $\mathbb{L}$ is the class of the affine line. [Corollary 4.11.5]

(viii) The virtual class of the SL_2-character stack $\mathfrak{X}_{\mathrm{SL}_2}(\Sigma)$ of a closed surface Σ, in the ring $\mathrm{K}_0^{\mathbb{P}^1}(\mathbf{Stck}_{\mathbb{C}})$ (see Definition 5.1.3), depends only on the Euler characteristic of Σ. [Corollary 5.5.4]

(ix) The morphism $\mathrm{K}_0(\mathbf{Var}_{\mathbb{A}^1_k}) \otimes \mathrm{K}_0(\mathbf{Var}_{\mathbb{A}^1_k}) \to \mathrm{K}_0(\mathbf{Var}_{\mathbb{A}^2_k})$ of rings, which sends $[X] \otimes [Y]$ to $[X \times Y]$, is not surjective for any field k. [Corollary of Example 3.2.10]

(x) The mathematical community should, when the technology allows it, strive towards a culture where the results of their papers are formalized using proof-assistants.

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