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Motivic invariants of character stacks

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Citation

Vogel, J. T. (2024, June 13). *Motivic invariants of character stacks*. Retrieved from <https://hdl.handle.net/1887/3762962>

Version: Publisher's Version

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Note: To cite this publication please use the final published version (if applicable).

Stellingen

behorende bij het proefschrift

“*Motivic invariants of character stacks*”

- (i) Characteristically, the Euler characteristic of the G -character stack of a surface of Euler characteristic $2 - 2g$ is determined by the characteristics of the character table of G over finite fields.
- (ii) The group of unipotent upper triangular matrices of rank 10 over the finite field of 13 elements has 609233173333356 irreducible complex representations of degree 815730721.
[Theorem 6.3.10]
- (iii) Let X and Y be quasi-projective varieties over a field k of characteristic not equal to 2, both equipped with an action of $G = \mathbb{Z}/2\mathbb{Z}$. If X can be stratified into copies of affine space on which G acts linearly, then $[X \times Y]^+ = [X]^+[Y]^+ + [X]^-[Y]^-$ in the Grothendieck ring $K_0(\mathbf{Var}_k)$, where $[X]^+ = [X // G]$ and $[X]^- = [X] - [X]^+$. [Theorem 3.6.19]
- (iv) Let G be a connected complex algebraic group, and denote by $\mathfrak{X}_G(\Sigma_g)$ the G -character stack of a closed surface Σ_g of genus g . Then the limit $\lim_{g \rightarrow \infty} e(\mathfrak{X}_G(\Sigma_g))/e(G)^{2g-2}$ is equal to the E -polynomial $e(G^{\text{ab}})$ of the abelianization of G . When G is a finite group, the limit $\lim_{g \rightarrow \infty} |\mathfrak{X}_G(\Sigma_g)|/|G|^{2g-2}$ is equal to the number of 1-dimensional irreducible representations of G .
- (v) For every $r \geq 0$, the sequences of GL_r -representation varieties $X_n = R_{\text{GL}_r}(F_n)$ of the free groups F_n , and $Y_n = R_{\text{GL}_r}(\mathbb{Z}^n)$ of the free abelian groups \mathbb{Z}^n , with the action of GL_r by conjugation, and the action of S_n by permutation, are motivically representation stable (see Definition 7.3.4).
[Theorems 7.4.1 and 7.4.2]

- (vi) There exists a closed 4-dimensional manifold with non-trivial fundamental group which has no non-trivial G -local system for any complex algebraic group G .
- (vii) Let G be the group $\mathbb{G}_m \rtimes \mathbb{Z}/2\mathbb{Z}$ over a field k of characteristic not equal to 2. The G -character stack $\mathfrak{X}_G(\Sigma_g)$ has virtual class $\frac{1}{2}((\mathbb{L}-1)^{2g-2}(2^{2g+1} + \mathbb{L} - 3) + (\mathbb{L}+1)^{2g-2}(2^{2g+1} + \mathbb{L} - 1))$, where \mathbb{L} is the class of the affine line. [Corollary 4.11.5]
- (viii) The virtual class of the SL_2 -character stack $\mathfrak{X}_{\mathrm{SL}_2}(\Sigma)$ of a closed surface Σ , in the ring $\mathrm{K}_0^{\mathbb{P}^1}(\mathbf{Stck}_{\mathbb{C}})$ (see Definition 5.1.3), depends only on the Euler characteristic of Σ . [Corollary 5.5.4]
- (ix) The morphism $\mathrm{K}_0(\mathbf{Var}_{\mathbb{A}_k^1}) \otimes \mathrm{K}_0(\mathbf{Var}_{\mathbb{A}_k^1}) \rightarrow \mathrm{K}_0(\mathbf{Var}_{\mathbb{A}_k^2})$ of rings, which sends $[X] \otimes [Y]$ to $[X \times Y]$, is not surjective for any field k . [Corollary of Example 3.2.10]
- (x) The mathematical community should, when the technology allows it, strive towards a culture where the results of their papers are formalized using proof-assistants.

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Leiden, 13 juni 2024