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## Applications of AdS/CFT to strongly correlated matter: from numerics to experiments

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# **Applications of AdS/CFT to strongly correlated matter: from numerics to experiments**

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*To my wonderful wife, Martina.*



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# Foreword

This thesis will deal with the practical use of the AdS/CFT correspondence to shed some light on the properties of quantum matter. The first chapter will provide an introduction to the wider context in which this thesis is placed. This chapter will also introduce the key concepts required throughout the various later chapters. Chapters 2 to 6 consist of published papers I was an author on and are therefore rather self-contained. Due to their nature as publications, these chapters will each re-introduce the necessary notations and context although with a narrower focus.

