The parabolic Anderson model on Galton-Watson trees

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Stellingen

Behorende bij het proefschrift

The Parabolic Anderson Model on Galton-Watson Trees

1. For regular trees, the leading order asymptotics of the annealed total mass of the parabolic Anderson model is determined by the leading order asymptotics of the cumulant generating function of the potential, and a variational formula whose minimiser is non-increasing with the distance to the root.

2. A tight upper bounded for the annealed total mass on a regular tree can be obtained by computing the annealed total mass on the tree truncated at level R, adding an additional edge from the root to a vertex at level R, attaching tadpoles to all the other vertices at level R, and taking $R \to \infty$.

3. Large degrees pose problems for the derivation of the quenched total mass of the parabolic Anderson model. As such, the path expansion method is applicable only when the degrees are bounded, or when the tails of the offspring distribution are sufficiently thin.

4. The solution to the parabolic Anderson model on exponentially growing graphs with a double-exponential potential exists and admits the Feynman-Kac representation.

5. For a Galton-Watson tree with offspring distribution having all exponential moments finite, the growth rate of the volume of balls does not depend on where the ball is centred.

6. A random walk on a rooted regular tree starting from the root and conditioned to return to the root has the same distribution as an unconditioned random walk with its drift reversed. A random walk conditioned to return to the root on a Galton-Watson tree does so stochastically faster than on any embedded regular tree.

7. The inverse of a cumulant generating function λ can be written in terms of its Legendre transform $\mathcal{L}\lambda$ as

$$\lambda^{-1} = (\mathcal{L}\lambda)' \circ \mu^{-1},$$

where μ is given by

$$\mu = \lambda \circ \theta^*$$

and θ^* is the maximiser of the Legendre transform.

8. The degree-normalised random walk on a finite graph $G = (V, E)$ does not have a symmetric generator. However the large deviation rate function for the normalised local times is given by

$$I_E(p) = \sum_{\{x,y\} \in E} \left(\sqrt{\tfrac{p(x)}{\deg(x)}} - \sqrt{\tfrac{p(y)}{\deg(y)}} \right)^2,$$

which is of a similar form as the unnormalised random walk.

9. A mathematics paper that has only one 'small' lemma left to prove is often not even half completed.

10. The parabolic Anderson model on trees is a suitable model for life: we perform a random walk to seek the area of highest potential, whilst there being only one path that leads there.

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