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## The parabolic Anderson model on Galton-Watson trees

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### Citation

Wang, D. (2024, May 28). *The parabolic Anderson model on Galton-Watson trees*. Retrieved from <https://hdl.handle.net/1887/3754826>

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**Note:** To cite this publication please use the final published version (if applicable).

# The Parabolic Anderson Model on Galton-Watson Trees

Daoyi Wang



# The Parabolic Anderson Model on Galton-Watson Trees

Proefschrift

ter verkrijging van  
de graad van doctor aan de Universiteit Leiden,  
op gezag van rector magnificus prof.dr.ir. H. Bijl,  
volgens besluit van het college voor promoties  
te verdedigen op dinsdag 28 mei 2024  
klokke 13:45 uur

door

Daoyi Wang  
geboren te Beijing, China  
in 1996

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The research in this thesis was supported by the Netherlands Organisation for Scientific Research through NWO Gravitation Grant NETWORKS-024.002.003.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
§1.1	Introduction . . . . .	2
§1.1.1	Definitions, intermittency and problems . . . . .	2
§1.1.2	The parabolic Anderson model on a lattice . . . . .	4
§1.1.3	The parabolic Anderson model on random graphs . . . . .	6
§1.1.4	Overview of the results . . . . .	9
§1.1.5	Open problems . . . . .	12
<b>I</b>	<b>Parabolic Anderson model: Annealed</b>	<b>15</b>
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<b>2</b>	<b>Annealed parabolic Anderson model on a regular tree</b>	<b>17</b>
§2.1	Introduction and main results . . . . .	18
§2.1.1	Background and motivation . . . . .	18
§2.1.2	The PAM on a graph . . . . .	19
§2.1.3	The PAM on an unrooted regular tree: annealed total mass for large times and key variational formula . . . . .	21
§2.1.4	Discussion . . . . .	23
§2.2	Proof of the main theorem: lower bound . . . . .	24
§2.2.1	Killing and lower variational formula . . . . .	25
§2.2.2	Limit of the lower variational formula . . . . .	25
§2.3	Proof of the main theorem: upper bound . . . . .	26
§2.3.1	Backbone, projection, periodisation and upper variational formula . . . . .	26
§2.3.2	Limit of the upper variational formula . . . . .	32
§2.4	Analysis of the upper variational formula . . . . .	32
§2.4.1	Identification of the rate function for the local times on the truncated tree . . . . .	33
§2.4.2	Limit of the upper variational formula . . . . .	39
§2.5	Analysis of the variational problem on the infinite regular tree . . . . .	39
§2.5.1	Two properties . . . . .	40
§2.5.2	Proof of the two properties . . . . .	41
<b>3</b>	<b>The parabolic Anderson model on a periodic Galton-Watson tree</b>	<b>47</b>

§3.1	Introduction and main results . . . . .	48
§3.1.1	Definition of the periodic Galton-Watson tree . . . . .	48
§3.1.2	The PAM on a periodic Galton-Watson tree . . . . .	48
§3.1.3	Assumptions on the potential . . . . .	49
§3.2	Main theorem: annealed total mass for large times . . . . .	49
§3.3	Proof of main theorem: lower bound . . . . .	50
§3.4	Proof of main theorem: upper bound . . . . .	50
§3.5	Backbone, projection, periodisation and upper variational formula . . . . .	51
§3.5.1	Backbone . . . . .	51
§3.5.2	Projection . . . . .	52
§3.5.3	Periodisation . . . . .	52
§3.5.4	Large deviation rate function . . . . .	53
§3.6	Limit of the upper variational formula . . . . .	61
<b>A</b>	<b>Appendix: Part I</b>	<b>63</b>
§A.1	Large deviation principle for the local times of Markov renewal processes	63
§A.2	Sojourn times: cumulant generating functions and Legendre transforms	64
§A.2.1	General observations . . . . .	64
§A.2.2	Exponential sojourn time . . . . .	65
§A.2.3	Non-exponential sojourn time . . . . .	65
§A.3	Large deviation estimate for the local time away from the backbone . . . . .	68
<b>II</b>	<b>Parabolic Anderson model: Quenched</b>	<b>75</b>
<b>4</b>	<b>Parabolic Anderson model on a Galton-Watson tree revisited</b>	<b>77</b>
§4.1	Introduction and main results . . . . .	78
§4.1.1	The PAM and intermittency . . . . .	78
§4.1.2	The PAM on a graph . . . . .	79
§4.1.3	The PAM on a Galton-Watson tree . . . . .	80
§4.2	Structural properties of the Galton-Watson tree . . . . .	82
§4.2.1	Volumes . . . . .	82
§4.2.2	Degrees . . . . .	85
§4.2.3	Tree animals . . . . .	86
§4.3	Preliminaries . . . . .	87
§4.3.1	Maximum size of the islands . . . . .	87
§4.3.2	Mass up to an exit time . . . . .	88
§4.3.3	Principal eigenvalue of the islands . . . . .	89
§4.3.4	Maximum of the potential . . . . .	90
§4.3.5	Number of intermediate peaks of the potential . . . . .	91
§4.4	Path expansions . . . . .	92
§4.4.1	Mass of the solution along excursions . . . . .	93
§4.4.2	Key proposition . . . . .	95

§4.5	Proof of the main theorem . . . . .	98
§4.5.1	Upper bound . . . . .	98
§4.5.2	Lower bound . . . . .	101
§4.6	Existence and uniqueness of the Feynman-Kac formula . . . . .	104
<b>5</b>	<b>The parabolic Anderson model on a Galton-Watson tree with normalised Laplacian</b>	<b>109</b>
§5.1	Introduction and main results . . . . .	110
§5.1.1	The PAM and intermittency . . . . .	110
§5.1.2	The PAM on a Galton-Watson tree . . . . .	111
§5.1.3	Main results and discussion . . . . .	112
§5.2	Preliminaries . . . . .	113
§5.2.1	Related Spectral Problems . . . . .	114
§5.2.2	Structural properties of the Galton-Watson tree . . . . .	117
§5.2.3	Estimates on the potential . . . . .	117
§5.3	Path expansions . . . . .	119
§5.3.1	Mass of the solution along excursions . . . . .	120
§5.3.2	Key proposition . . . . .	122
§5.4	Proof of the main theorem . . . . .	125
§5.4.1	Upper bound . . . . .	125
§5.4.2	Lower bound . . . . .	128
§5.5	Analysis of the variational formula . . . . .	131
§5.5.1	Alternative representations for $\chi$ . . . . .	131
§5.5.2	Identification of the minimiser . . . . .	133
<b>B</b>	<b>Appendix: Chapter 4</b>	<b>139</b>
§B.1	Largest eigenvalue . . . . .	139
	<b>Bibliography</b>	<b>141</b>
	<b>Samenvatting</b>	<b>145</b>
	<b>Summary</b>	<b>147</b>
	<b>Acknowledgements</b>	<b>149</b>
	<b>Curriculum Vitae</b>	<b>150</b>
	<b>Publications</b>	<b>151</b>