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## The parabolic Anderson model on Galton-Watson trees

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# The Parabolic Anderson Model on Galton-Watson Trees

Daoyi Wang



# The Parabolic Anderson Model on Galton-Watson Trees

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