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The parabolic Anderson model on Galton-Watson trees

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The Parabolic Anderson Model on Galton-Watson Trees

Daoyi Wang

The Parabolic Anderson Model on Galton-Watson Trees

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Contents

1	Introduction	1
§1.1	Introduction	2
§1.1.1	Definitions, intermittency and problems	2
§1.1.2	The parabolic Anderson model on a lattice	4
§1.1.3	The parabolic Anderson model on random graphs	6
§1.1.4	Overview of the results	9
§1.1.5	Open problems	12
I	Parabolic Anderson model: Annealed	15
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2	Annealed parabolic Anderson model on a regular tree	17
§2.1	Introduction and main results	18
§2.1.1	Background and motivation	18
§2.1.2	The PAM on a graph	19
§2.1.3	The PAM on an unrooted regular tree: annealed total mass for large times and key variational formula	21
§2.1.4	Discussion	23
§2.2	Proof of the main theorem: lower bound	24
§2.2.1	Killing and lower variational formula	25
§2.2.2	Limit of the lower variational formula	25
§2.3	Proof of the main theorem: upper bound	26
§2.3.1	Backbone, projection, periodisation and upper variational formula	26
§2.3.2	Limit of the upper variational formula	32
§2.4	Analysis of the upper variational formula	32
§2.4.1	Identification of the rate function for the local times on the truncated tree	33
§2.4.2	Limit of the upper variational formula	39
§2.5	Analysis of the variational problem on the infinite regular tree	39
§2.5.1	Two properties	40
§2.5.2	Proof of the two properties	41
3	The parabolic Anderson model on a periodic Galton-Watson tree	47

§3.1	Introduction and main results	48
§3.1.1	Definition of the periodic Galton-Watson tree	48
§3.1.2	The PAM on a periodic Galton-Watson tree	48
§3.1.3	Assumptions on the potential	49
§3.2	Main theorem: annealed total mass for large times	49
§3.3	Proof of main theorem: lower bound	50
§3.4	Proof of main theorem: upper bound	50
§3.5	Backbone, projection, periodisation and upper variational formula	51
§3.5.1	Backbone	51
§3.5.2	Projection	52
§3.5.3	Periodisation	52
§3.5.4	Large deviation rate function	53
§3.6	Limit of the upper variational formula	61
A	Appendix: Part I	63
§A.1	Large deviation principle for the local times of Markov renewal processes	63
§A.2	Sojourn times: cumulant generating functions and Legendre transforms	64
§A.2.1	General observations	64
§A.2.2	Exponential sojourn time	65
§A.2.3	Non-exponential sojourn time	65
§A.3	Large deviation estimate for the local time away from the backbone	68
II	Parabolic Anderson model: Quenched	75
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4	Parabolic Anderson model on a Galton-Watson tree revisited	77
§4.1	Introduction and main results	78
§4.1.1	The PAM and intermittency	78
§4.1.2	The PAM on a graph	79
§4.1.3	The PAM on a Galton-Watson tree	80
§4.2	Structural properties of the Galton-Watson tree	82
§4.2.1	Volumes	82
§4.2.2	Degrees	85
§4.2.3	Tree animals	86
§4.3	Preliminaries	87
§4.3.1	Maximum size of the islands	87
§4.3.2	Mass up to an exit time	88
§4.3.3	Principal eigenvalue of the islands	89
§4.3.4	Maximum of the potential	90
§4.3.5	Number of intermediate peaks of the potential	91
§4.4	Path expansions	92
§4.4.1	Mass of the solution along excursions	93
§4.4.2	Key proposition	95

§4.5	Proof of the main theorem	98
§4.5.1	Upper bound	98
§4.5.2	Lower bound	101
§4.6	Existence and uniqueness of the Feynman-Kac formula	104
5	The parabolic Anderson model on a Galton-Watson tree with normalised Laplacian	109
§5.1	Introduction and main results	110
§5.1.1	The PAM and intermittency	110
§5.1.2	The PAM on a Galton-Watson tree	111
§5.1.3	Main results and discussion	112
§5.2	Preliminaries	113
§5.2.1	Related Spectral Problems	114
§5.2.2	Structural properties of the Galton-Watson tree	117
§5.2.3	Estimates on the potential	117
§5.3	Path expansions	119
§5.3.1	Mass of the solution along excursions	120
§5.3.2	Key proposition	122
§5.4	Proof of the main theorem	125
§5.4.1	Upper bound	125
§5.4.2	Lower bound	128
§5.5	Analysis of the variational formula	131
§5.5.1	Alternative representations for χ	131
§5.5.2	Identification of the minimiser	133
B	Appendix: Chapter 4	139
§B.1	Largest eigenvalue	139
	Bibliography	141
	Samenvatting	145
	Summary	147
	Acknowledgements	149
	Curriculum Vitae	150
	Publications	151