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FairSNA: Algorithmic Fairness in Social Network Analysis

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In recent years, designing fairness-aware methods has received much attention in various domains, including machine learning, natural language processing, and information retrieval. However, in social network analysis (SNA), designing fairness-aware methods for various research problems by considering structural bias and inequalities of large-scale social networks has not received much attention. In this work, we highlight how the structural bias of social networks impacts the fairness of different SNA methods. We further discuss fairness aspects that should be considered while proposing network structure-based solutions for different SNA problems, such as link prediction, influence maximization, centrality ranking, and community detection. This survey-cum-vision clearly highlights that very few works have considered fairness and bias while proposing solutions; even these works are mainly focused on some research topics, such as link prediction, influence maximization, and PageRank. However, fairness has not yet been addressed for other research topics, such as influence blocking and community detection. We review the state of the art for different research topics in SNA, including the considered fairness constraints, their limitations, and our vision. This survey also covers evaluation metrics, available datasets and synthetic network generating models used in such studies. Finally, we highlight various open research directions that require researchers' attention to bridge the gap between fairness and SNA.

CCS Concepts: • **Human-centered computing** → **Social network analysis**;

Additional Key Words and Phrases: Network science, social network analysis, algorithmic fairness, fairness-aware methods

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1 INTRODUCTION

People connect with each other based on different relationships, such as friendships, acquaintances, kinship, collaboration, or having the same interests and form different types of complex social networks, such as friendship networks, collaboration networks, or communication networks [67, 228]. All these social networks evolve based on day-to-day human interactions, and **social network analysis (SNA)** has been used to understand human behavior as an individual and in groups [86].

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In recent years, due to the vast amount of available data from **online social networks (OSNs)**, social network analysis has been of great interest for understanding and modelling human behavior in several dynamic processes, such as information spreading or opinion formation, at a large scale. Structural patterns of social networks have been used to answer various interesting questions, such as which users will connect in the future, which users are more important in the network given the application context, which users have high control on the network for a given dynamic phenomenon, or which users are more vulnerable.

In SNA, the network structure and the characteristics of nodes and networks are used to design methods for different problems. These methods provide state-of-the-art accuracy; however, most of them have not accounted for structural bias and inequality in social networks. For example, in health-intervention programs, such as awareness for **human immunodeficiency virus (HIV)** or prevention of suicide, we aim that the useful information should reach as many people in need as possible. However, due to the limited resources, it is not possible to personally reach every user at risk. Therefore, we target influential community leaders who can spread vital information to other members of the community or detect suspicious cases beforehand and respond appropriately. The success of these intervention programs highly relies on the underlying social networks. References [210, 222] highlighted the bias in fairness-oblivious social intervention methods [120, 138] that aim to maximize the outreach by focusing on major communities and are not fair for minor communities at risk. They might disproportionately exclude users from racial minorities or **lesbian, gay, bisexual, transgender and queer (LGBTQ)** communities from the benefits of the intervention. Therefore, it is important to ensure that the resource allocation should respect the diverse composition of communities, and each community should receive a fair proportion.

Another similar important example is link recommendation in OSNs [6]. In link recommendation, the goal is to predict future or unknown links in the network so that users will have more connections and will remain more loyal to the social networking website. The link prediction methods learn the similarity of nodes based on network structure and attribute of nodes and predict a link between a given pair of nodes if they have high similarity. However, most of the existing link prediction methods have not considered structural biases and do not efficiently predict links for small or sparse groups [147]. In OSNs, inter-community node pairs have lower similarity than intra-community links, and using the same threshold value does not predict inter-community links with high accuracy as intra-community links, as shown in Reference [187]. Link prediction methods also drive the evolution of the network and affect information accessibility of nodes [109, 213]. A biased link recommendation system might create an effect of filter bubbles [167] that segregates the network and limits the diversity of information exposed to online users. Therefore, it is important to predict all kinds of links fairly with high accuracy, irrespective of users' attributes or their communities, which will lead to diversity in the local neighborhood of the nodes. In the recent era, SNA has been used in many different applications, including studying marketing strategies [34, 72], identifying scientific leaders [35], news and rumor propagation [200], opinion and innovation diffusion [57], job hiring [218], disseminating terrorist networks [154, 181], and identifying gang leaders or predicting criminal activities using criminal networks [7, 44]. Several works, including References [80, 122, 147, 151, 177], have highlighted unfairness in different SNA algorithms, i.e., mainly occurred because of the structural bias.

In social networks, due to homophily [150], people prefer to connect with similar kinds of people. Stoica et al. [211] collected an online social networking dataset from Instagram and observed that male users on Instagram exhibit much stronger homophily among them as compared to female users. A similar phenomenon has been observed in other social networks [89, 153, 208, 217]. Another well-known network evolution phenomenon is preferential attachment, also known as rich-gets-richer, in which people prefer to connect with other people having more number of

connections and generate scale-free networks [23]. Scale-free networks have a small number of people having a high number of connections and a high number of people having a small number of connections, and therefore their degree distribution follows the power law. The homophily and preferential attachment further give rise to the glass ceiling effect, i.e., an unbreachable barrier that keeps minorities from achieving higher ranks even having comparable characteristics as majorities. They further showed that the bias of recommendation algorithms used for the network evolution in OSNs further worsens these pre-existing inequalities. Avin et al. [15] also highlighted power inequality and the glass-ceiling effect in different social networks and co-authorship networks. Besides homophilic connectivity, in real-world scale-free networks, communities' size follows power-law distribution [12, 81, 111, 162, 249]. Therefore, the algorithms might provide higher accuracy by optimizing their results for large-size communities and might not be accurate for small-size communities. All these factors should be considered while designing methods based on network structure so that the proposed methods are fair for all kinds of users and all communities, irrespective of their size or type. In algorithmic fairness, the aim is to design algorithms that consider these network inequalities and produce an outcome that is not biased for any user or any group, given a fairness constraint. The fairness constraints are defined based on the application requirements, which is further explained in Section 3.

The inequality of social networks has been an interesting topic for sociologists and economists [39, 70, 82, 149, 165, 168], though this issue is not well addressed by network scientists while designing solutions based on network structure that are applied in real life. Since the early 2020s, some researchers have proposed fairness-aware methods for some of the SNA problems, such as link prediction, influence maximization, and PageRank [147, 177, 209, 224]. However, fairness still needs to be well defined and explored for several other problems, including influence blocking, community detection, different centrality rankings, anomaly detection, and network anonymization. This survey-cum-vision discusses fairness constraints that should be considered for achieving individual or group fairness [74] for different research problems, including state-of-the-art literature and future directions. The main aim of this survey is to draw researchers' attention toward this gap.

Related Surveys. Initial surveys on fairness in **machine learning (ML)** focused on independent and identically distributed (i.i.d.) data [46, 59, 73, 152, 156, 170, 175]. These surveys did not mainly focus on relational graph datasets. There are some recent tutorials that provide a good taxonomy of fairness in ML and graph data mining and might be interesting for readers to explore more about algorithmic fairness in different applications of ML [25, 113, 229].

We would like to highlight recent surveys that have focused on graph data mining from machine learning and deep learning perspectives [58, 71, 246]. In Reference [71], the authors focused on fairness in graph data mining algorithms for knowledge graphs, recommender systems, and network embedding. Zhang et al. [246] provided a brief review on quantifying different types of fairness studied for graph-structured datasets. Choudhary et al. [58] reviewed fair-ML methods for graph-structured relational datasets. This survey considers structural bias and comes close to our work, though they have only focused on ML methods to achieve fairness. Besides this, these surveys are not focused on social networks and have considered only a few downstream tasks in graph mining, mainly node classification, link prediction, and influence maximization.

An important difference to note is that most of the works in fair graph mining have considered bipartite or heterogeneous graphs; for example, recommending movies on Netflix where nodes are users and movies, recommending items on Amazon where nodes are users and items, or hiring job candidates where nodes are employers and prospective employees. In such cases, it is easy to compute the ranking of items based on their characteristics and the given requirement context, and individual and group fairness are defined based on these rankings. However, computing the ranking of users in the case of social networks is not that straightforward, as the ranking depends

on the interconnectivity of users as well as on the application context. Another important point is that in such applications, the data are well organized, and several attributes of the nodes are known in advance, though that is not the case with OSNs. For example, sensitive attributes, such as gender or demographic location, are not available for OSNs users from their profiles. For example, most of the OSNs, such as X (formerly known as Twitter), do not collect these parameters from users for creating an account, or in some other cases, such as communication or co-authorship networks, it is difficult to infer the gender using the names.

To the best of our knowledge, this is the first survey-cum-vision that focuses on fairness in a wide range of research problems in social network analysis. One important point is that this survey considers all types of algorithms, including heuristic, probabilistic, machine learning, and deep learning-based methods, to achieve fairness. This survey also highlights many open research problems in this area and aims to bridge the gap between fairness and SNA.

Prerequisite. In this survey, we have briefly explained all required terms to understand the discussion. Still, a basic understanding of (i) network science terminologies [22] and (ii) fairness constraints [171] will be helpful in easily following the discussion.

The rest of the survey is structured as follows. In Section 2, we discuss the taxonomy of fairness in SNA. In Section 3, we discuss various research topics for which the network structure-based solutions have been proposed, and for each topic, we discuss fairness constraints, state of the art, and future directions. In Section 4, we summarize datasets used for such studies, followed by a discussion in Section 5. The survey is concluded in Section 6. All abbreviations used in this survey are summarized in Appendix A.

2 TAXONOMY OF FAIRNESS-AWARE SNA (FAIRSNA)

In **fairness-aware social network analysis (FairSNA)**, fairness has been defined by extending fairness definitions from ML [46, 87] and also proposing some novel definitions to the network context. Here, we discuss a high-level taxonomy of FairSNA that can be looked at from different perspectives, including group vs. individual fairness, feature-aware vs. feature blind fairness, and in/pre/post-processing-based fair methods.

Group vs. Individual Fairness: In recent works on algorithmic fairness, there are two basic frameworks, group fairness and individual fairness [171]. In SNA, group fairness demands that different communities, irrespective of their size or protected attribute, should be treated fairly and should receive a fair share of resources. For example, in the case of influence maximization for a social-awareness cause, the influence should be propagated equally to all the communities, and the number of influenced users in each community should be proportional to its size. In FairSNA, group fairness is defined using *equality*, *equity*, and *statistical parity* fairness constraints [80, 177]. Similarly to ML, in SNA, individual fairness aims that similar users should receive similar treatment or similar resources. In some SNA problems, such as link prediction and influence maximization, the individual fairness has been modelled using *equality at user level* [177] and *information access gap* [83], respectively, i.e., different than the individual fairness constraints in ML [46] due to the application settings. Individual fairness for other problems, such as community detection or centrality ranking, is still not defined (further discussed in Sections 3.1 and 3.2). Besides these, in SNA, fairness has also been incorporated by proposing novel fairness constraints, including *maximin*, *disparity*, and *diversity*, that ensure fairness for small and marginalized groups [80]. Maximin fairness [80] aims to improve fairness for the least benefited community. The diversity constraint [80] is inspired by the game-theoretic approach and guarantees that each community is at least as well off as if it receives the resources proportional to its size and allocates them internally.

Feature-aware vs. Feature-blind: The methods that explicitly use protected attributes to achieve fair results are called feature-aware methods, otherwise feature-blind. In FairSNA, all

proposed methods have used community membership of users or its size so that each community is treated fairly or receives equal resources. The community membership is identified using protected attributes or by applying community identification methods on the network. Therefore, FairSNA methods fall under the feature-aware category. To the best of our knowledge, no method provides feature-blind fairness in SNA. The scientific community will appreciate such methods as they will avoid the cumbersome task of collecting ground-truth community data.

There can be different approaches to design feature-blind methods (without using any protected attributes) that achieve individual or group fairness. One approach to designing an individually fair feature-blind method is to maintain individual fairness for all users based on their contextual situation or importance, irrespective of their sensitive attributes. For example, in influence blocking, an individually fair feature-blind method will make sure that the probability of saving a node from misinformation is proportional to it receiving the misinformation. These individual-fair methods will treat all nodes similarly that were similarly impacted and will not be dependent on any sensitive attribute and, therefore, will be feature-blind. Such methods for individual fairness can be extended to designing group fairness methods. Another feasible direction to design feature-blind methods is to assess that a proposed method does not discriminate against any individual or group, and then it can be used for unlabeled datasets. Some feature-blind methods in machine learning include those in References [36, 60, 76] that can be referred to for a better understanding.

Pre/In/Post-Processing: Fair algorithms can be categorized as (i) pre-processing, (ii) in-processing, and (iii) post-processing approaches [46]. In SNA, fairness is mainly achieved by using in-processing methods, in which the algorithm learns to provide fair output by considering the fairness constraints that diminish the effects of structural bias in the data. However, Laclau et al. [133] proposed a pre-processing method to repair the adjacency matrix of the graph by adding links that will obfuscate dependency on the protected attribute, and the repaired matrix can be used further to generate network embedding that provides individual as well as group fair results in downstream tasks, such as link prediction. One important point to note is that such methods can only be applied if the entire network structure is known in advance. Online social networks are highly dynamic, and their size is growing very fast with time [2, 43, 48]. Therefore, in SNA, researchers have proposed several methods that use partial network information to provide fast and efficient solutions [79, 191, 207, 221]. Therefore, the pre-processing steps, such as repairing adjacency networks, will require the entire network information and will not be suitable for large-scale dynamic networks. One can still explore the possibility of efficiently generating a repaired adjacency matrix for dynamic and partial networks.

Some works [147, 187] proposed post-processing-based link prediction methods that achieve fairness by increasing the prediction likelihood of the edges of underrepresented node-pair groups that might be less likely to be predicted using classical link prediction methods [6]. In SNA, heuristic methods have been proposed for several tasks, such as link prediction [3, 110, 140, 248] and influence maximization [52, 132], which are comparatively faster than probabilistic modelling, machine learning, or deep learning-based methods. The post-processing for such heuristic methods might provide fair and fast solutions.

3 FAIRSNA: FAIRNESS CONSTRAINTS, STATE OF THE ART, AND FUTURE DIRECTIONS

In the following subsections, we discuss various research topics in SNA, including the respective fairness constraints, state of the art, challenges, and future directions.

Notations. $G(V, E)$ denotes a network where V is the set of nodes and E is the set of edges. u, v , and w are nodes in the network, and A is the sensitive attribute. $C = \{C_1, C_2, \dots, C_i, \dots\}$ denotes communities in the network G .

3.1 Fair Link Prediction

Link prediction (LP), also known as link recommendation, has been widely used to predict future or unknown links in the network [6]. Predicting promising links in online social networks for recommending friends is important so that users will be more loyal to the website. Most of the existing link prediction methods have not considered structural biases [187].

In Figure 1, we show a small example of unfairness in link prediction using the Dutch School social network [129] that has 26 nodes (17 girls and 9 boys) and 63 edges. The homophily value of the network is 0.7 [161]. In Figure 1(a), the network is shown, and the nodes are divided into two groups based on gender; blue nodes represent girls, and pink nodes represent boys. Next, we remove around 10% of intra-community and inter-community edges uniformly at random, and the missing links are shown using dashed lines in Figure 1(b). Now, we compute the similarity scores for predicting the missing links using two heuristics methods, (i) Jaccard Coefficient [140] and (ii) Adamic Adar Index [3], and similarity scores are shown corresponding to the missing links in Figure 1(c) and (d), respectively. We can observe that the value of similarity scores for inter-community links is lower than for intra-community links. Besides this, similarity scores are lower for small and sparse communities. For example, in Figure 1(d), Adamic Adar coefficient values for the links from the pink community are smaller than the blue community. Saxena et al. [187] computed similarity scores for link prediction using nine different heuristic methods on many real-world scale-free networks and showed that inter-community links have lower scores than intra-community links. An in-depth understanding of the performance of different link prediction methods for sparse and dense communities is still an open research question. In fair link prediction, the aim is to efficiently predict all kinds of links with high accuracy, irrespective of users' attributes, their communities, or community sizes.

3.1.1 Fairness Definition. Next, we discuss fairness constraints considered in link prediction.

- (1) **Statistical Parity:** In link prediction, if the sensitive attribute is gender having two values $\{m, f\}$, then the recommended links can be divided into four groups, $G_{mm}, G_{mf}, G_{fm}, G_{ff}$; G_{mf} denotes the links recommended to group m from group f . Similarly, if a sensitive attribute A has z values $\{1, 2, \dots, i, j, \dots, z\}$, then there will be z^2 groups. Let the link recommendation probability of positive predictions for link class G_{ij} be $P(G_{ij}) = |\{p(u, v) = 1 : (u, v) \in G_{ij}\}| / |G_{ij}|$, where $p(u, v) = 1$ if the link is likely to exist (positively recommended) between nodes u and v irrespective of ground-truth label. In the case of similarity score-based methods, such as the Jaccard Coefficient and Adamic Adar index, $p(u, v) = 1$ if the computed similarity score between node pair (u, v) is higher than the given threshold [6]. The statistical parity or bias for two groups G_{ij} and G_{jk} is the difference between $P(G_{ij})$ and $P(G_{jk})$. For analyzing bias among multiple groups, the variance between the link recommendation probabilities of each group can be calculated [177]. So, the bias with respect to statistical parity, i.e., denoted by $bias^{SP}(G)$, is computed as

$$bias^{SP}(G) = Var(\{P(G_{ij})\} : (i, j) \in A).$$

- (2) **Accuracy Parity:** The accuracy parity focuses on correct predictions (for both existing and non-existing links) instead of positive predictions given a pair of nodes. The Accuracy Parity for two groups G_{ij} and G_{jk} is the difference between $P_c(G_{ij})$ and $P_c(G_{jk})$, where $P_c(G_{ij})$ is the probability of correctly predicted existing and non-existing links in group G_{ij} . Finally, the bias with respect to accuracy parity is computed as

$$bias^{AP}(G) = Var(\{P_c(G_{ij})\} : (i, j) \in A).$$

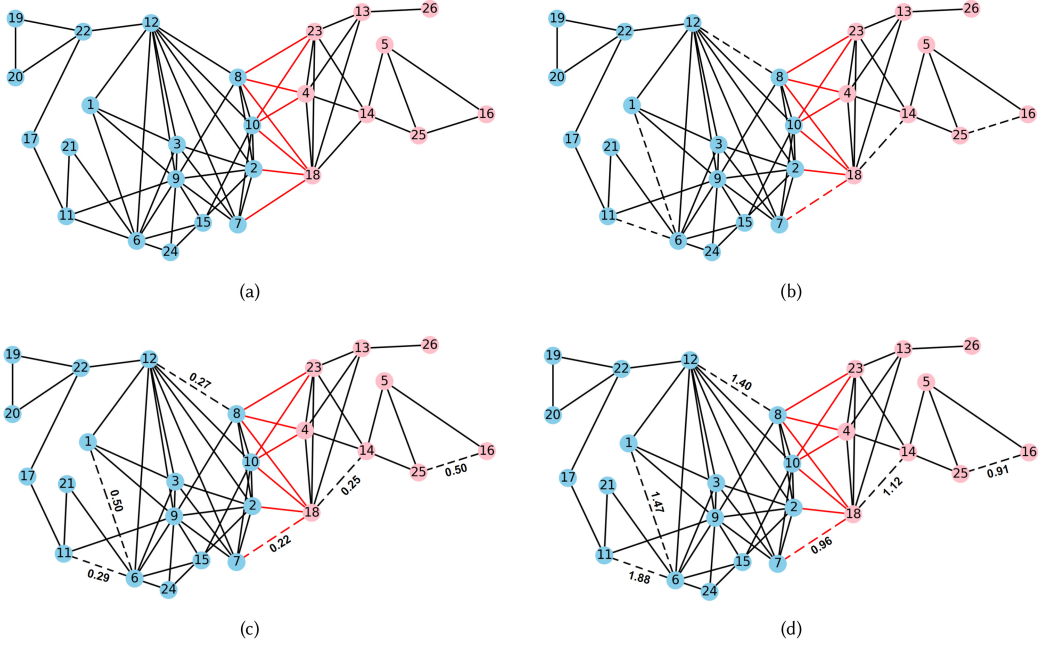


Fig. 1. (a). Dutch School Social network having two groups where blue nodes represent girls and pink nodes represent boys, (b). Dashed lines represent around 10% u.a.r. removed links from the network ((c) and (d)). Values corresponding to dashed lines show the Jaccard coefficient and Adamic Adar scores for predicting the missing links, respectively.

- (3) Equality of Recommendation at Group Level [177]: In link recommendation, the equality of recommendation at the group level (denoted by ERG) with respect to a sensitive attribute A can be computed using the total number of recommendation for a pair of groups. If $N(G_{ij})$ denotes the number of recommendation for a group G_{ij} , and $N(G_{ij}) = |\{p(u, v) = 1 : (u, v) \in G_{ij}\}|$, then the bias with respect to ERG is computed as

$$bias^{ERG}(G, A) = Var(\{N(G_{ij})\} : \{i, j\} \in A).$$

- (4) Equality of Recommendation at User Level [177]: To compute the Equality of Recommendation at user level for a user u (denoted by ERu), we compute the fraction of recommended users to u having attribute value j and denote it as $j\text{-share}(u)$. Now, the bias with respect to ERu can be computed for each sensitive attribute value j , which is the difference between a fair fraction where the recommended links from each group have the same fraction and the average $j\text{-share}$ over all users. It is computed as

$$bias^{ERu}(G, j) = \frac{1}{|A|} - \frac{\sum_{u \in V} j\text{-share}(u)}{|V|}$$

Statistical Parity is concerned with the positively predicted links, which are highly likely to exist. This approach takes into account that the link prediction model was trained on a network that has evolved with bias, leading to lower accuracy for smaller groups or inter-community links. Therefore, the methods designed based on statistical parity focus that the method should predict the likelihood of link existence for all groups proportionally equally likely. However, Accuracy Parity focuses on the correct predictions (for likelihood as well as unlikelihood of a link) given a pair of nodes rather than only positive predictions. The fairness constraints based on equality

prefer equal treatment for all groups regardless of their size and, therefore, are not desired if group size varies.

3.1.2 State of the Art. In this section, we discuss state-of-the-art fairness-aware link prediction methods.

Heuristics Methods. In heuristic or similarity-based link prediction methods, such as **Jaccard Coefficient (JC)**, **Adamic-Adar (AA)**, and **Resource Allocation (RA) Index** [6], the similarity score of a given pair of nodes is computed based on their structural similarity, which is further used for link prediction. Saxena et al. [187] studied the fairness in link prediction with respect to two classes, intra-community and inter-community links. They observed that inter-community links have lower structural similarities than intra-community links. Based on this observation, the authors proposed the **Heuristic Method-Extended using Intra and Inter Community Thresholds (HM-EIICT)** framework, where a different threshold value is considered for intra-community and inter-community node pairs for link prediction. The proposed method was verified on several real-world networks, and even the simple heuristic methods extended using the given approach, such as JC-EIICT and RA-EIICT, provide better total accuracy as compared to all nine considered baselines. The proposed method specifically highly improves the accuracy of inter-community link prediction. The work can be further extended where different thresholds can be decided for different communities, as some communities that are sparse and small might have lower similarity scores. However, this is not explored in the work, as it only focuses on inter vs. intra-community links.

Network Embedding-based Methods. Besides heuristic methods, network embedding-based methods have also been proposed for fair link prediction. Rahman et al. [177] observed a high group-level bias in link recommendation using the Node2Vec embedding [98], i.e., a well-known network embedding method. They proposed a fairness-aware network embedding method, called Fairwalk, in which a biased random walk is proposed that explores diverse neighborhoods. In the proposed Fairwalk, the neighbors are partitioned based on the sensitive attribute, and each group gets the same probability of being chosen by the random walker, irrespective of its size. The analysis showed that the presentation of each group is fair in the explored fairwalk traces, and Fairwalk reduced biases under the statistical parity and equality of representation fairness constraints. The proposed method can be further extended to choose the random walker probabilities based on group size or tune the parameters based on the application requirement.

Khajehnejad et al. [122] also proposed a reweighting method that can be applied to any random walk-based network embedding method, such as deepwalk and node2vec, to generate fair network embedding. The proposed method, called CrossWalk, updates the transition probabilities such that the random walker is more likely to cross group boundaries by up-weighting edges closer to the periphery of the groups or inter-group edges. The crosswalk method first computes the proximity of each node with respect to other groups, i.e., the expected number of times the nodes from other groups will be visited in random walks starting from the given node, and then the proximity is used to reweight the transition probabilities of the edges. Therefore, it will embed the periphery nodes from different groups closer if they are more similar while preserving the network structure information. The method is evaluated for influence maximization, node classification, and link prediction using total accuracy (non-fairness-based metric) and disparity (fairness-based metric), and the results on real-world as well as synthetic networks show that the method achieves a high disparity on the cost of a very small decrease in the accuracy. One limitation of this work is that the method has been verified on small datasets having two to three groups based on attributes such as age or political inclination, and the performance of the model for large-size networks having multiple communities is not discussed.

In a recent work, Saxena et al. [189] proposed a network embedding-based method, called NodeSim, for fair link prediction, where the links are categorized into two classes: (i) intra-community links and (ii) inter-community links. The authors proposed a NodeSim random walk that explores the intra- as well as inter-community neighborhood of a node based on nodes' structural similarities. In the NodeSim random walk, the probability of moving to the next node depends on both the community label and its similarity with the current node. The authors train a logistic regression model for link prediction using vector representation of nodes and their community information. The proposed method outperforms baseline methods for both intra- and inter-community link prediction. To the best of our knowledge, this is the first work that uses community information while training the link prediction model to improve both inter- and intra-community link prediction accuracy that was measured using ROC-AUC value. One limitation of this work is that the proposed method has not been evaluated for its performance in link prediction in all communities, which might be interesting to understand better the performance of the proposed embedding method for different types of communities.

In network embedding methods, an adversarial approach has also been used to generate a fair embedding that is invariant to protected attributes [38]. The adversarial learning framework has two important components: (i) generator and (ii) discriminator. The generator generates network embedding, while the discriminator aims to predict the sensitive attribute value based on the generated output. When the discriminator fails to predict the sensitive attribute, then it is considered that the generated embedding is decoupled from the sensitive attributes and used for further downstream tasks. Masrour et al. [147] proposed an adversarial learning-based network embedding method, called **Fairness-aware Link Prediction (FLIP)**, that provides fairness while recommending links between people belonging to the same and different genders. The method was verified on small networks having a limited number of groups; therefore, the scalability of the proposed method is questionable for large-size networks having multiple protected attributes with multiple values.

All the above-discussed methods provide fairness using in-processing or post-processing techniques to provide a fair output. However, pre-processing steps can also be taken to remove structural bias from a network dataset. Laclau et al. [133] proposed an embedding-agnostic repairing method that repairs the adjacency matrix by adding edges to obfuscate the dependency on the sensitive attribute, and the repaired adjacency matrix can be further used for the network embedding. The group-fairness-aware repair will add edges that obfuscate the original network structure both within and across the sensitive groups; however, the individual-fairness-aware repair will keep the within-group structure almost intact. The impact on the graph structure is measured using the assortativity coefficient [160]. The authors tested the proposed method for binary as well as multi-class settings and showed that it could control the tradeoff between individual and group fairness. As we discussed, the pre-processing methods to repair adjacency matrices are yet to be extended for dynamic streaming networks. Otherwise, the computational complexity will be very high if we compute the repaired adjacency matrix for every change in a dynamic network.

In Table 1, we summarize fairness-aware link prediction methods, where the first and second columns provide the reference and name of the proposed method, followed by the method type, i.e., the underlying designing structure of the method, fairness constraints and fairness evaluation metrics, and baselines used in the work.

3.1.3 Future Directions. The fairness-aware link prediction methods are helpful in increasing the diversity in a network. However, there are some limitations that might be hurdles in designing and testing fair solutions and should be considered for improving the research in this direction. Lichtnwalter and Chawla [142] discussed various limitations of fair link prediction methods. One

Table 1. Fairness-aware Link Prediction Methods

Ref	Proposed Method	Method Type	Fairness Constraints and Evaluation	Baselines
[177]	Fairwalk	Network Embedding-based method	Statistical Parity and Equality of representation	Node2Vec [98]
[38]	Adversarial learning	Fairness-aware embedding	AUC	Non-compositional adversary embedding [75]
[147]	FLIP	Adversarial learning-based network embedding	Intra- vs. Inter Community Link Prediction Fairness, AUC	JC [140], AA [3], PA [159], DeepWalk [169]
[187]	HM-EIICT	Heuristic Method	Intra- vs. Inter Community Link Prediction Fairness, Accuracy	JC [140], AA [3], RA [248], CACN and CARA [110], CRS-RA [137], CMS-RA [19], ICRA [231]
[122]	CrossWalk	Network Embedding-based method	Disparity	DeepWalk [169], Fairwalk [177]
[189]	NodeSim	Network Embedding-based method	Intra- vs. Inter Community Link Prediction Fairness, AUC	JC [140], AA [3], RA [248], DeepWalk [169], Node2Vec [98], Splitter [77], NECS [139]
[133]	Graph-repair	Embedding-agnostic repairing method for the adjacency matrix	Representation Bias, Statistical Parity, Disparate Impact, Consistency, AUC	Node2Vec [98], Fairwalk [177], CNE [112], DEBAYES [42]

of the main problems is the imbalance of both the classes (i) existing links and (ii) non-existing links. The empirical analysis showed that the considered link prediction methods, including preferential attachment [159], Adamic-Adar index [3], and PropFlow predictor [141], might not provide practical conclusions if evaluated (using ROC area and precision-recall curve area [64]) on imbalanced datasets. It implies that we can get more robust results by considering the imbalance of classes in training and testing datasets. Another important point to note is that the currently available data of OSNs is based on the evolution of networks, where the users formed connections based on their personal choice plus based on the existing friend recommendation system used by the platform. Therefore, there might be inherited bias in the data due to the algorithmic unfairness of link recommendation methods, and the network might have less diverse links that might impact the training and testing of the proposed methods. However, this has not been looked at in the past and might affect the training and testing of the model. A promising solution in this direction, which might be the formation of a fair dataset or better testing methods, will be well appreciated by the scientific community.

We also observed that there is minimal work on fairness-aware link prediction methods for other types of networks, such as directed networks [31], weighted networks [186], temporal networks [105], multilayered networks [126], and so on. The methods proposed for undirected networks might not work well for other types of networks. For example, in directed networks, the direction of an edge is an important driving factor in their evolution; on X, if user a follows user b , then user b may or may not follow user a . Therefore, it is essential to consider the structural bias of different

types of networks and then propose specific methods for specific types of networks. Apart from these, the fairness of the proposed methods has not been verified with respect to different types of communities, such as weak and sparse communities, varying size communities, and multiple protected attributes. What if the methods are unfair for some specific communities or specific types of users, and if yes, how can the accuracy for them be further improved? A theoretical and empirical analysis of the bias of different link prediction methods on diverse types of groups will be interesting to study.

3.2 Fair Centrality Ranking

In a social network, each user has some unique characteristics that define its importance in accordance with the given application context. There exist several centrality measures, such as degree centrality, closeness centrality, betweenness centrality, and pagerank, that assign a centrality value to each user based on its characteristics, its position in the network, and the network structure [197]. The assigned centrality value is further used to compute the centrality ranking of the nodes. The inequalities and biases among individuals in a society with respect to different parameters, such as race, gender, or ethnicity [201], have not been considered while computing the centrality value or centrality ranking of users in a given social network.

In Figure 2, we plot the Dutch school social network using two fairness-oblivious and two fair centrality rankings, where the node's size corresponds to its centrality rank. Figure 2(a) and (b) show the degree and pagerank centrality of the nodes, respectively, and one can observe that large-size communities have more highly ranked nodes that negatively affect the social capital of nodes belonging to smaller communities. In Figure 2(c) and (d), centrality ranking is computed using locally fair pagerank and fairness sensitive pagerank, respectively, which were proposed by Tsioutsoulakis et al. [224]. These fair methods allocate pagerank fairly to both communities, and one can observe that the distribution of pagerank is more balanced using fair methods. The locally fair pagerank method assigns higher pagerank values to the nodes that are local to the group, and the fairness sensitive pagerank considers both local and global connectivity of the nodes.

In ML, fairness has been explored in ranking to generate an ordered list of items, and several fairness definitions have been proposed to assess and quantify fairness in the ranking. These methods focus on group or individual fairness-aware ranking [47, 92], pairwise orderings [30], and exposure bias [204]. However, the case is a bit different in social networks, as each user's importance depends on other users as all users are interconnected to form a complex network. Therefore, it is necessary to define fairness constraints for nodes' ranking in complex social networks so that the ranking is invariant to structural bias. Next, we discuss how group fairness proposed for ranking in ML [92] can be extended to apply to OSNs where users are grouped into communities.

3.2.1 Fairness Definitions. Let us assume that a given network $G(V, E)$ has $C = \{C_1, C_2, \dots, C_l\}$ communities, and τ^k shows top- k ranked users based on a given centrality measure.

- (1) Demographic Parity: A group fairness ranking, with respect to communities C , should satisfy the following constraint to achieve the demographic parity [92].

$$|\{u \in \tau^k \text{ and } u \in C_i\}| \leq \left\lceil \frac{|C_i|}{|V|} \cdot k \right\rceil, \forall k \text{ \& \& } \forall C_i \in C$$

and

$$|\{u \in \tau^k \text{ and } u \in C_i\}| \geq \left\lfloor \frac{|C_i|}{|V|} \cdot k \right\rfloor, \forall k \text{ \& \& } \forall C_i \in C.$$

This fairness constraint assures that the representation of candidates from different communities is balanced in the top- k list.

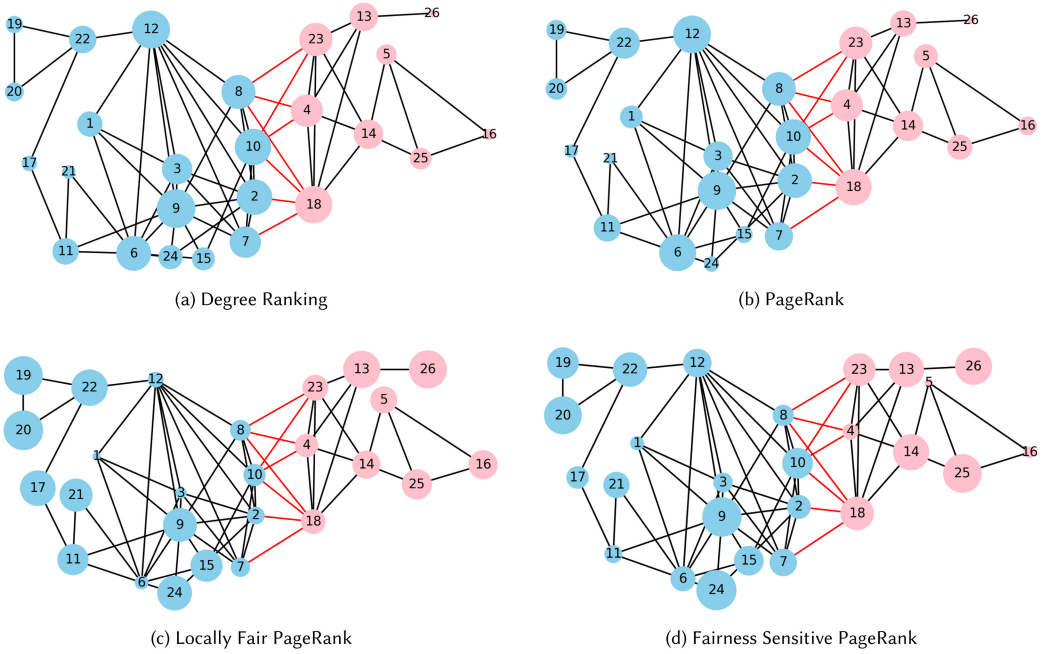


Fig. 2. Nodes are ranked based on four different centrality rankings, and nodes' size corresponds to their centrality rank, where (a) degree ranking and (b) PageRank are fairness-oblivious centrality ranking and (c) locally fair PageRank and (d) fairness sensitive PageRank are fairness-aware PageRank from Reference [224].

- (2) ϕ -fairness: Tsioutsoulouklis et al. [224] defined ϕ -fairness for pagerank, where a generated ranking is fair if the fraction of the total mass allocated to the members of the protected group is ϕ . It is defined as follows: Given a graph $G = (V, E)$ having a protected group $R \subset V$, a generated pagerank PR is ϕ -fair if $PR(R) = \phi$, where $PR(R)$ denotes the total pagerank mass allocated to the nodes of group R . ϕ -fairness is similar to the statistical parity when ϕ is equal to the fraction of protected nodes in the network.

Initially, Reference [224] defined ϕ -fairness for the network having two groups, though it can be further extended for networks having multiple groups, where a ϕ vector can be defined to denote the mass allocation of pagerank to all groups in the network.

3.2.2 State of the Art and Future Directions. Fairness constraints for centrality ranking are not yet well defined. Given that there exist several centrality measures suitable for different applications, the respective fairness constraints for centrality rankings are yet to be discussed. How a biased ranked list of nodes might affect minor communities should be looked at in depth. Karimi et al. [115] showed that the degree ranking of users from the minority community depends on the relative community sizes and the homophilic index. In a homophilic network, the nodes from minor communities suffer and are not able to achieve a higher degree rank as compared to the nodes from major communities. Therefore, it is important to define methods that can generate a fair centrality ranking of nodes irrespective of their community sizes.

In one recent work, Tsioutsoulouklis et al. [224] studied the bias in the PageRank and observed that the pagerank is not equally distributed among groups based on different sensitive attributes, such as demographics or gender. They proposed a parity-based definition of fairness (ϕ -fairness) that focuses on maintaining the proportion of pagerank allocated to the candidates of each group.

Fairness can be achieved using two methods: (i) fairness-sensitive pagerank and (ii) locally fair pagerank. The fairness-sensitive pagerank method modifies the jump factor of the pagerank method so that the random walker will reach the nodes of the underrepresented group with a higher probability and will achieve the fair pagerank. Locally fair pagerank aims to achieve fairness by imposing the fairness constraint on each node so that each node acts fairly in the process of computing pagerank. In the locally fair pagerank method, each node will assign its pagerank to both groups fairly equally; thus, the process is fair at each step and not only on the convergence. Overall, the gist is that the proposed methods manipulate jump vector or transition probabilities for the nodes so that nodes from both groups are fairly represented in the ranking, and, finally, each node gets a fair rank. The proposed approach can be further extended to get rank-aware fairness or define other notions of fairness. The authors further proposed a link recommendation method that aims to recommend links that will maximize the fairness gain in pagerank [223]. The proposed method will reduce network bias and lead toward a diverse and fair network evolution.

One simple solution to achieve group fairness might be to use representative ranking that has been applied in various recommendation methods [92]. The representative ranking maintains the representation ratio of each protected group in the final ranking. In recommendation, the quality of an item/member is quantified using its characteristics; however, in social networks, the quality needs to be quantified using local as well as global influence of the users and their characteristics. The local and global influence of a user depends on its local and global connectivity. These influence values can be further used to generate a final representative ranking. However, centrality measures to generate a meaningful local ranking of the nodes in their communities that can be further used to generate a fair global ranking are yet to be studied, especially with respect to different applications. Reference [188] shows the bias in fairness-aware representative ranking if a group is sub-active on a platform, and this should also be considered while generating a fair ranking.

Given the dynamic nature of large-scale real-world networks, researchers have proposed several methods to update centrality value in dynamic networks [117, 244], approximate centrality value [18, 196, 237], or estimate centrality ranking using local neighborhood information [190, 192, 195] for fairness-oblivious centrality measures (that has been defined without using any fairness constraints) [197]. One should further consider extending these methods to estimate, update, or approximate fair centrality ranking. As we already discussed, the pre-processing step to repair the adjacency matrix of a network seems like an infeasible option for large-scale dynamic networks; one can further explore efficient methods to update fair-centrality ranking in dynamic networks when the fair ranking was computed by incorporating fairness constraints during pre-processing of the data.

3.3 Fair Influence Maximization

Influence maximization (IM) is a well-known problem in network science in which a group of users, called “influencers” (also “seed nodes”), is chosen to spread the information who can influence a maximum number of people in the network [16, 21, 138]. The main approaches to solve IM include greedy methods, approximation algorithms, heuristic methods, metaheuristic methods, gossip algorithms, multiagent approaches, and community-based approaches [21]. IM methods have been applied in many applications, including marketing, news spread, awareness spread, and online trend-setting [16, 106, 242]. Initially proposed fairness-oblivious methods for social influence maximization only focused on maximizing the total outreach of information in a given social network and did not consider the fairness in outreach [138]. Therefore, the achieved outreach might be biased toward large size communities.

For example, in Figure 3, we show an example of influence maximization on the Dutch school social network. In this example, influence propagation is modelled using the **Independent**

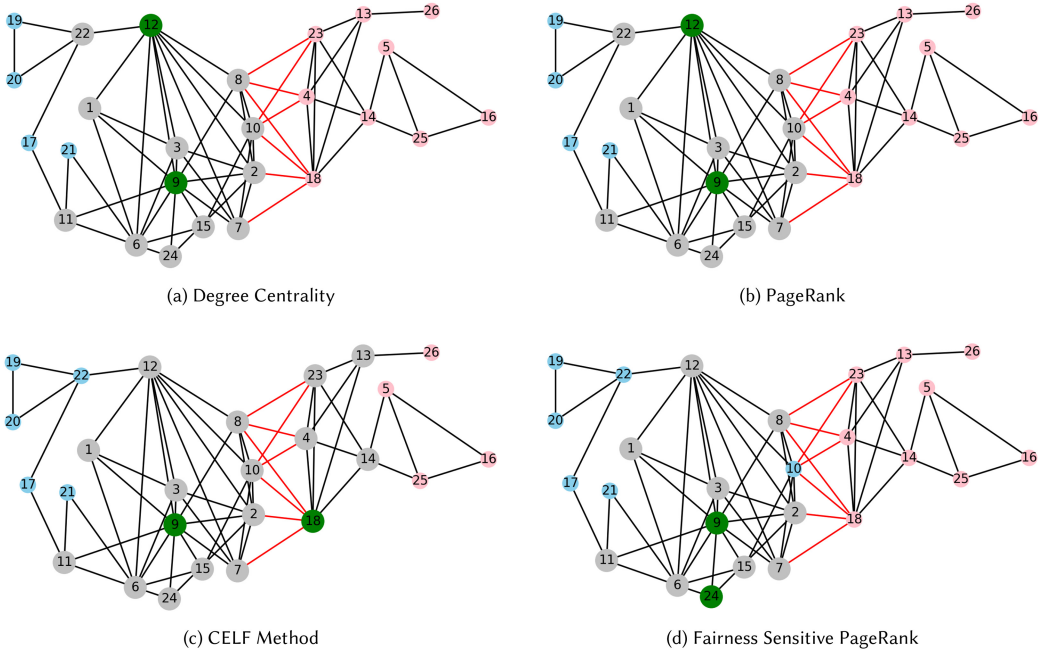


Fig. 3. Influence propagation spread using Independent Cascade model where the top-2 seed nodes are chosen using four different methods, including degree, pagerank, CELF, and fairness sensitive pagerank. Seed nodes are shown in green color and influenced nodes are shown in grey color. A node's size is proportional to the probability of it getting infected.

Cascade (IC) model, and the probability of information propagation for intra-community edges is 0.1 and for inter-community edges is 0.05. We choose top-2 nodes for influence propagation using four methods, (i) degree centrality, (ii) pagerank, (iii) **Cost-Effective Lazy Forward selection (CELF)** method [135], and (iv) fairness sensitive pagerank [224], and the influence spread is shown in Figure 3(a)–(d), respectively. Initially, the nodes are coloured pink and blue based on the groups they belong to, as shown in earlier figures. In this figure, the top-2 chosen seed nodes are shown in green color, and the IC model is simulated 1,000 times to compute the probability of a node getting influenced. Finally, the nodes with a significant probability of getting influenced (higher than 0.1) are shown in grey, and a node's size is proportional to its probability of getting influenced. One can observe that in some cases, influence has not reached the small community, and it shows the bias in outreach. Therefore, it is important to consider information access equality while selecting the top- k seed nodes. A fairness-aware influence maximization method considers both the total outreach of information and fairness in outreach while defining the optimization function.

Wang et al. [232] defined information access equality as follows: “for a given process and seeds, the majority and minority nodes should receive information at similar rates across various stages of the spreading processes.” They analyzed simple vs. complex information spreading processes on the networks evolved with different evolution mechanisms, including majority/minority dichotomy, homophily, preferential attachment, and diversity. The authors observed that equality in information access depends on both the network structure as well as on the spreading process. They further observed that homophily and preferential attachment-based network growth highly affect the information access quality. However, introducing diversity in the network can help in gaining information access equality. Teng et al. [219] also studied the bias and glass ceiling effects

for the minor community (mainly the community having females) in influence propagation on Instagram and Facebook networks where each user belongs to males or females group. They also observed that the fairness in influence propagation is affected by both factors: (i) structure homophily (people of the same kind are more likely to be connected) and (ii) influence diffusion homophily (people are more likely to be influenced by other people of the same kind or having same attributes) [11].

Recently, researchers have focused on designing fairness-aware methods for influence maximization, as they can help achieve a diverse outreach [11, 209, 210, 222]. The proposed methods show that the feature-aware methods can achieve more diverse outcomes in outreach and seed selection than being feature-blind [209].

3.3.1 Fairness Definitions. Next, we discuss different group fairness constraints concerning the influence maximization problem.

- (1) Equality [80]: It considers that a proposed IM method is fair in choosing the influencers if the number of influencers from each community is proportional to its size. If $\{C_1, C_2, \dots, C_l\}$ are communities, and S is the initial chosen set of influencers, then it is defined as

$$\frac{|\{u \in S | u \in C_i\}|}{|C_i|} = \frac{|\{u \in S | u \in C_j\}|}{|C_j|}, \forall i, j.$$

Equality is also referred to as “fairness for early-adopters.”

- (2) Equity [80]: The equity constraint suggests that the proposed method is fair in outreach if the expected number of influenced nodes for all communities are proportional to their size,

$$\frac{E|I(S, C_i)|}{|C_i|} = \frac{E|I(S, C_j)|}{|C_j|}, \forall i, j,$$

where $I(S, C_i)$ is the influence outreach achieved by seed nodes S in community C_i . This type of fair methods ensure that the information or product awareness reaches the population in a calibrated manner and each community has an equal fraction of aware people. This is important in the news or health-awareness spread, where unequal distribution can manipulate one type of population or can lead to amplifying the echo-chamber effect. This type of fairness is also referred to as “fairness in outreach.”

- (3) Maximin [80]: The maximin fairness aims to maximize the influence of a minimally influenced group as compared to others with respect to their size in the network. Given a set of communities, the maximin fairness will maximize $\min_{C_i \in C} E|I(S, C_i)|/|C_i|$.

The maximin fairness focuses on the least influenced community, and therefore, the overall outreach is impacted and is lower than the achieved by other fairness-aware methods.

- (4) Disparity [8]: The disparity of a method is computed by the maximum disparity in the normalized outreach across all communities. It is defined as

$$\max_{i, j \in \{1, 2, \dots, |C|\}} \left| \frac{E|I(S, C_i)|}{|C_i|} - \frac{E|I(S, C_j)|}{|C_j|} \right|.$$

- (5) Diversity [80]: The diversity measure allocates resources according to the network structure of each community. It follows two steps. In the first step, a fraction of initial influencers are chosen proportional to the size of the community, and the influence is propagated in the induced sub-graph of that community. In the second step, the outreach of influence in each community is used as a lower bound on the influence received by the nodes of that community. In simple words, it ensures that each community receives outreach at least equal to the internal outreach of that community. The steps are summarized below.

- (a) Let G_{C_i} denotes the induced subgraph for a community C_i , and $k_{C_i} = \lceil |S| \cdot |C_i| / |V| \rceil$ and $Outreach_{C_i} = \max_{S \subset C_i: |S|=k_{C_i}} |I(S, G_{C_i})|$.
- (b) For each community C_i , the diversity constraint ensures that it holds $E|I(S, C_i)| \geq Outreach_{C_i}$.
- (6) Welfare Functions [178]: The welfare principles represent the preferences of a fair decision-maker when choosing between two solutions. A welfare function that adheres to these principles would consistently rank the most desirable solution, in terms of both fairness and efficiency, as the highest. Therefore, maximizing the welfare function would lead to the most desirable solution. Rahmattalabi et al. [178] proposed two classes of welfare functions: (i) $W_\alpha(U) = \sum_{i=1}^n U_i^\alpha / \alpha$ for $\alpha < 1, \alpha \neq 0$, and (ii) $W_\alpha(U) = \sum_{i=1}^n \log(U_i)$ for $\alpha = 0$, where n is the total number of nodes in the network, U is the utility value, and α is a constant that controls the inequality. These parameters measure the benefit or goodness of a utility distribution. Typically, a user's utility for an item(set) is determined by the difference between the user's valuation for that item(set) and the corresponding cost paid. For example, in influence maximization, the utility value will measure the expected fraction of influenced nodes in a community (benefit) from a given seed set (associated cost). These two proposed functions satisfy the following five welfare functions: Monotonicity, Symmetry, Independence of Unconcerned Individuals, Affine Invariance, and Influence Transfer. In group fairness-aware IM, the utility of each user can be defined as the average utility of users of its community. Using this group-fairness, a welfare function can be defined in terms of the average utilities over communities as $W_\alpha(U) = \sum_{i=1}^n U_i^\alpha / \alpha = \sum_{C_i \in C} |C_i| \cdot U_{C_i}^\alpha / \alpha$, where U_{C_i} is the average utility of community C_i .

In the IM problem, we can define a fair solution in which the selection of seed nodes S maximizes welfare according to the optimization problem described as $\text{maximize}_{\{S: S \subseteq V \& |S| \leq k\}} W_\alpha(U(S))$. Now, given fairness constraints (written as a feasible set F), an objective function that can be used as a welfare function for IM can be defined as

$$\max_S \sum_{C_i \in C} |C_i| \cdot U_{C_i}(S) + I_F(U(S)) := \max_S W_F(U(S)),$$

where $I_F(U)$ is equal to 0 if $U \in F$ and $-\infty$ otherwise, and $U_{C_i}(S)$ denotes the utility for community C_i , i.e., the expected fraction of the influenced nodes in that community.

Equality fairness centers around early adopters, or seed nodes, for influence spread. However, equity focuses on fairness for all groups regarding influence spread, also known as total outreach, by ensuring that outreach propagates in each group proportionally to their size. Maximin, Disparity, and Diversity fairness measures aim to minimize the outreach gap between different groups with respect to their size. In group fairness, Maximin fairness ensures a fair distribution and aims to maximize the influence of a minimally influenced group. Maximin fairness can be similarly defined for individual fairness, in which the minimum probability of getting influenced for each individual should be as high as possible. In contrast, the Diversity fairness constraint ensures that while maintaining fairness, any group should not receive less influence than they would have received in a fairness-oblivious algorithm. Finally, we discussed the welfare function for group fairness that is based on social welfare theory and principles of social welfare. It provides a spectrum of fairness as we vary parameter α and $\alpha = -\infty$ refers to the leximin fairness, i.e., a subclass of Maximin fairness. A utility-based welfare function can similarly be defined for individual fairness.

3.3.2 State of the Art. Most of the fairness-aware IM methods have been proposed in the past three years.

Group Fairness. Stoica and Chaintreau [209] considered two types of fairness in IM: (i) fairness in early adopters (called seed nodes) and (ii) fairness in outreach. Early adopters play an important role in the diffusion process, and having early access to the information or products may help establish their role as influencers and gain people's trust. These days, influencers earn money, which can help them gain social capital or financial benefits [119]. For example, influencers on Instagram are paid by companies to promote products. However, a calibrated outreach is more useful in spreading awareness information, where we aim that all communities should be equally aware of the given information. Both kinds of fairness are important, and which one should be chosen depends on the application. In this work, the authors compared two techniques for choosing early adopters: (i) greedy approach, i.e., a strategic method to choose the next node that maximizes the outreach, and (ii) highest degree node, i.e., a heuristic method. They assessed the performance of both methods using the IC model [121] for influence propagation and showed that the strategic method achieves fair outreach as compared to the heuristic method on the Instagram dataset. The authors also theoretically proved that degree heuristics in some conditions having differentiated thresholds for choosing top- k seed nodes in each community provides higher outreach with more diversity. In simple words, the authors suggest that an equal number of chosen nodes from each community, irrespective of their degree, provides a better diverse outreach.

Stoica et al. [210] further compared three different methods for selecting early adopters: (i) agnostic seeding, i.e., feature blind and chooses all nodes above the given threshold irrespective of their community membership; (ii) parity seeding, which maintains the ratio of early adopters with respect to their community size, or we can say it uses differentiated degree thresholds for both groups; and (iii) diversity seeding that uses differentiated thresholds based on the influence outreach. The authors theoretically showed that if the budget is less than a specific threshold (depending on the network), then the diversity seeding achieves a larger expected outreach than the agnostic seeding and gets close to the parity outreach. The authors verified the theoretical results on the DBLP dataset, having 81% males and 19% females with a high homophilic structure. The authors used the degree heuristic for all three kinds of seeding. For diversity seeding, the authors used a relaxation parameter that selects the seed nodes in between agnostic and parity seeding, and the best results were considered. The diversity seeding showed a slight improvement over other methods to get an efficient and diverse outreach using the independent cascade model, especially for the minor community. However, an increase in the outreach of minor community is at the cost of a decrease in the outreach of major community. Teng et al. [219] further proposed disparity seeding-based methods for a different IM problem setting, where top- k seed nodes are selected such that a given target gender ratio ζ should be maintained in the influenced nodes with an error margin e .

Tsang et al. [222] studied fair resource allocation for IM from a group perspective using Maximin and Diversity constraints. The authors showed that fairness-aware IM is non-submodular as opposed to the classic IM problem (refer to Section 3 in Reference [222] for the detailed proof). The authors proposed a method using multi-objective submodular optimization that can satisfy either fairness constraints and provides an asymptotic approximation guarantee of $1 - 1/e$. The authors theoretically proved that the price of group fairness can be very high under a range of network structures for both fairness constraints. One notable finding is that the price of fairness gets worse if nodes join multiple communities (overlapping community structures). The proposed solution with diversity constraint performs 55–65% better as compared to the greedy approach and also gives competitive performance for maximin constraint. However, when the algorithm was applied with the maximin constraint, it provided the best maximin value, as expected. In the experiments, the price of fairness ranges from 1.05 to 1.15, which is comparatively very small, as provided by the theoretical bounds. It suggests that applying fairness-aware methods in practice is less costly than observed in the theoretical worst-case scenario.

Becker et al. [29] proposed to use probabilistic strategies instead of deterministic strategies to achieve group fairness using maximin. The results are promising and show that randomization can be further used to achieve efficient and fair solutions for other fairness constraints in IM. In this work, fairness is ensured for the minimally influenced group, and there can still be huge variation in the influence outreach of different groups. Therefore, they further proposed a deterministic solution where the optimization function ensures fair outreach for all groups by enforcing demographic parity (equity) while maximizing the total outreach [28]. The main benefit of using such methods is that the fairness violation in the resulting outreach is strictly bounded.

Farnadi et al. [80] formulated fairness in IM using **Mixed Integer Programming (MIP)**, where the fairness requirements were enforced using linear constraints or updating the objective function. The proposed framework is a generalized framework that can achieve four types of group fairness (equity, equality, maximin, and diversity) with competitive results. The authors also mentioned that each kind of fairness has certain properties, and a proposed method might not be able to satisfy all these properties using a single fairness metric. One main limitation of this work is scalability, and the method has only been tested on small networks having 500 nodes. This work mainly focused on group fairness, though one can further extend the proposed framework for studying individual fairness using welfare theory.

Khajehnejad et al. [123] proposed the first work that used adversarial learning-based network embedding to identify seed nodes for achieving fair IM. The adversarial graph embedding is generated by co-training an auto-encoder for network embedding and a discriminator to discern sensitive attributes. Therefore, in the generated embedding, the nodes belonging to different sensitive attribute groups are similarly distributed. Once the embedding is generated, a clustering algorithm is applied, and then the seed nodes are chosen fairly from the identified clusters. A simple way is to choose the centroids of k -clusters as k -seed nodes, though it might not be fair for different groups. For achieving fairness, the identified k -clusters are again divided into subgroups based on the sensitive attribute; for example, each cluster will be divided into two groups (males and females) if the sensitive attribute is gender having two values. Then, the number of seed nodes selected from each subgroup is proportional to its size, according to the given budget k . The proposed method reduces disparity, given that it is competitive in total outreach obtained. This is the first work that uses network embedding to choose seed nodes for fair IM. Similar network embeddings can also be explored for other SNA downstream tasks, such as fair node classification or fair link prediction.

Balance Tradeoff. A series of works have focused on designing IM methods that aim to balance the tradeoff between maximum outreach and fairness; next, we will discuss them. Teng et al. [219] proposed disparity seeding, which focuses on both factors, maximizing the outreach and influencing the required fraction of the target community. In disparity seeding, the influential users are ranked using PageRank, Target HI-index, and Embedding index, and the seed nodes are selected using simulation-based learning. The proposed method outperformed [210] on the considered datasets; however, the running time can be a concern on large-scale datasets as the seed nodes are identified using a simulation-based approach.

Anwar et al. [11] also proposed a method that greedily chooses seed nodes by maximizing an objective function depending on both the total outreach and outreach balance. The algorithm outperforms [210] in achieving a better total outreach for the given seed nodes size. Rahmattalabi et al. [178] proposed a family of welfare functions with an inequity aversion parameter that can be varied to get the required tradeoff between fairness and maximum outreach. They also observed that high welfare could be obtained without a significant reduction in the total outreach, as observed in Reference [222].

Time Critical Fair IM. All above-discussed works did not consider the deadline or time constraint while maximizing the influence. However, in real-life applications, receiving the

information before the deadline is crucial [51]. For example, spreading the information about any marketing discount that is valid for a short period of time is time critical. In **Time Critical IM (TCIM)**, the aim is that within the given time deadline, the fraction of influenced nodes across different groups should follow the fairness constraint. Ali et al. [8] studied group-fairness in TCIM, where the fraction of influenced nodes should be equal across all groups. The authors mainly worked on two variants of the TCIM problem: (i) FAIRTCIM-BUDGET, where the budget (i.e., the number of seed nodes) is fixed, and the aim is to choose seed nodes that maximize the time-critical influence, and (ii) FAIRTCIM-COVER, where the aim is to find a minimal size set of seed nodes such that the given fraction of the population is influenced within the given time deadline. As the problem is NP-hard, the authors proposed a greedy approximation solution with provable guarantees. The empirical results showed that fairness comes with the price of reduced total outreach, though it is bounded as guaranteed by the proposed solution. The proposed methods evaluated fairness in outreach at the deadline of influence propagation (given time limit t), though it will also be interesting to evaluate its fairness as well as the performance at each timestep during influence propagation.

Individual Fairness. In individual fairness, the aim is to balance the probability of receiving the information for each individual. Simply put, the probability of receiving the information for each individual should be the same irrespective of their connectivity and community. Fish et al. [83] investigated individual fairness using the notion of the information access gap, where they aim to maximize the minimum probability of receiving the information by an individual to reduce this gap. The authors showed that maximizing the minimum probability of information access is NP-hard. The authors used the maximin social welfare function as an objective function and proved that it reduces the access gap. They further proposed various greedy-based algorithms that provide good results. In *greedy* solution, the probabilities are computed for each node, and the node that maximizes the objective function is added to the solution. However, the proposed solution is slower as the probabilities will be computed for each node. Therefore, the authors further proposed a faster solution, called *Myopic*, in which, in each round, the node with the currently smallest probability will be added to the seed nodes without evaluating the objective function. Another faster variation is *Naive Myopic*, in which the probabilities are estimated initially, and then top- k nodes with the smallest probabilities are chosen as the seed nodes. An alternative fast solution, called *Gonzalez*, avoids estimating the probabilities and chooses seed nodes that are distant from each other. This method will pick the next seed node that is furthest from the current seed nodes. The empirical results on real-world networks showed that both the *Myopic* and *Naive Myopic* methods provide better total outreach with improved information access for each node in the network. These greedy methods have high time complexity, and a faster heuristic method will always be well received.

Jalali et al. [109] observed that the information unfairness increases with increasing the number of seed nodes k and influence probability p in real-world networks. The reason is that as k will increase, information cascades will reach nodes of the small-size community, though the number of cascades passing through the local neighborhood in a large-size community will increase exponentially. Therefore, the difference in the distribution of information access between both groups will increase. However, if p is small, even if k is high, then the cascades will reach a very small number of nodes and will die out. The authors proposed a link recommendation method, called *MaxFair*, that predicts the given number of links, which will minimize the information unfairness in the network. The edges receive a positive score for increasing information flow between group pairs with below-average flow and a negative score for increasing flow between above-average group pairs. Then, a final score is computed for all pairs of unconnected nodes, and the pair having the highest score is selected in each iteration. The proposed method shows a decrease in information unfairness and can be adopted for link recommendation in OSNs for fair information flow. The

diverse link recommendation methods will reduce structural bias and will also impact other network phenomena, such as the opinion formation of users and forming echo chambers in OSNs. Swift et al. [213] also studied a similar problem of recommending potential probable edges that maximizes outreach while achieving group fairness and proved that it is NP-hard. They further proposed a scalable incremental solution that reduces the unfairness to around zero (~ 0.0004) and increases the outreach by 43%.

In Table 2, we summarize works on fairness-aware IM. In the table, columns are as follows: (i) Ref is reference of the publication; (ii) name of the proposed methods; (iii) complexity of the problem statement and if there is any other remark to mention; (iv) considered fairness constraints; (v) top- k denotes if the proposed method aims to choose top- k seed nodes based on the given budget; (vi) Target Nodes denotes if the method aims to save the given target nodes; (vii) α denotes decontamination ratio that includes constraints, such as maintain the ratio of influenced nodes in different communities or maintain the influence ratio for the minimally influenced community; (viii) t denotes if any time constraint is considered, such as the deadline for influence outreach, (ix) diffusion model denotes the used influence propagation model; and (x) baselines considered in the work.

3.3.3 Future Directions. Fairness-aware IM methods for various constraints applicable in practical applications still have not been proposed. For example, in **IM with Priority (IMP)**, the aim is to find a set of k seed nodes that obtains the maximum outreach, given that the influence should spread to a given target set of nodes, called priority set, with at least the given threshold T [172]. The fairness-aware solution for IMP is still an open question. The solutions will have to consider various cases, such as whether the priority set is equally distributed over all communities or not, and if not, then what is the best achievable solution. Other interesting constraints are time constraints for time-critical information, outreach threshold that should be achieved, and rigid users that might not be propagating the information. The generic solutions that might achieve fairness given any combination of constraints will also be very appreciated.

Individual fairness in IM has not been explored much and still requires researchers' attention. Fish et al. [83] proposed a method to reduce the information access gap to improve individual fairness. However, the applicability of the method for time-constrained information is not explored. In the case of social interventions or awareness programs, the information accessibility must be fair for each individual. Therefore, it is essential to propose methods that can maintain individual fairness for time-critical information propagation. Another important point to note is that all the proposed fairness-aware methods have used the IC model for information propagation. Therefore, the achieved fairness with respect to other influence propagation models, such as the linear threshold model or SIR model, is still not explored [16, 212]. Besides these, there are several other extensions of ICM to model influence propagation, such as shortest path model [124], penta-level spreading model [99, 198], trust-based latency-aware independent cascade [157], conformity-aware cascade model [136], continuous-time Markov Chain-independent cascade model [250], and dynamic independent cascade model [220]. An in-depth comparative analysis of different IM methods for different influence propagation models from a fairness perspective will help get a deeper understanding of the problem and further design promising solutions. For example, in the case of the linear threshold model [120], the existing IM methods [41, 52, 96] focus on identifying a critical group of nodes that maximize the influence propagation locally, and therefore, the information might be constrained within the major community and may not reach to the minor community. Therefore, the bias in such critical mass models, including the linear threshold model [120], tripping model [240], and multiple adoption linear threshold model [174], should be further studied.

Apart from these, in recent years, researchers have focused on identifying topic-based influential users [166, 226]. At the same time, some studies have shown gender-based biases in online

Table 2. Fairness-aware Influence Maximization Methods

Ref	Proposed Methods	Complexity & Remarks	Fairness Constraints	Top- k	Target Nodes	α	t	Diffusion Model	Baselines
[209]	Differentiated Seeding	–	Equity, Equality, Maximin, Diversity	✓	–	–	–	IC	Greedy
[83]	Greedy, Myopic, Naive Myopic, Gonzalez	NP-hard	Maximin	✓	–	–	–	IC	TIM+ [216], Random
[80]	MIP	Used Mixed Integer Programming encodings of fairness measures in influence maximization	Equity, Equality, Maximin, Diversity	✓	–	–	–	IC	CELF [135], Simpath [96], TIM [216], IMM [215], FairIM [222]
[222]	Tsang Method	fairness-aware IM is non-submodular; approximation ratio $\rightarrow (1 - 1/e)$ as $k \rightarrow \infty$	Maximin, Diversity	✓	–	–	–	IC	Greedy [120]
[178]	DC and Maximin Fair IM	Prove monotonicity and submodularity of the resulting optimization problem; Greedy algorithm for fair IM through welfare maximization	Maximin, Diversity	✓	–	–	–	IC	Fairness-oblivious IM
[210]	Parity & Diversity Seeding	–	Equality	✓	–	–	–	IC	Agnostic seeding
[123]	Fair-Emb	–	Equality	✓	–	–	–	IC	Greedy [120], Tsang et al. [222]
[8]	FAIRTCIM-COVER	$k \leq \ln(1 + V)(\sum_{i=1}^k S_i^*)$, where S_i^* is an optimal solution	Disparity	–	–	✓	✓	IC	–
[8]	FAIRTCIM-BUDGET	Lower bound of total Influence $\geq (1 - 1/e) \cdot \frac{H(z)}{z}$ where z is the optimal solution and H is a monotone concave function	Disparity	✓	–	–	✓	IC	–
[29]	Set-based and Node-based solution	approximate within a factor of $(1 - 1/e)$ plus an additive small error ϵ	Maximin	✓	–	–	–	IC	Myopic [83], Greedy, Tsang et al. [222]

(Continued)

Table 2. Continued

Ref	Proposed Methods	Complexity & Remarks	Fairness Constraints	Top- k	Target Nodes	α	t	Diffusion Model	Baselines
[11]	Balanced IM	$(1 - 1/e)$ approximation guarantee. problem is monotone and submodular	Categorical Balance in outreach	✓	–	–	–	IC	Diversity Seeding [210]
[219]	Disparity Seeding	NP-Hard	Disparity (ratio of influenced users is ζ within an error margin e)	✓	–	✓	–	IC	Diversity seeding [210], IM-balanced [91]
[179]	SetMOGWO	–	Maximin, Diversity, Group Activation Speed	✓	–	–	✓	IC	Tsang et al. [222]
[28]	ind_lp, grdy_grp+lp, and maxmin+lp	$(1 - 1/e, 1 - 1/e)$ -approximation guarantee for ind_lp	Equity	✓	–	–	–	IC	Greedy, Greedy maxmin, Greedy Prop, Uniform, MIP [80], Tsang et al. [222], Mult-weight [29], Myopic [83]
[144]	CEA-FIM	$O(\delta \cdot n \cdot (k \cdot n \cdot w + pop \cdot \log(pop) \cdot g_{max}))$, where δ : number of live-edge graphs, n : number of nodes, k : seed set size, w : number of edges, pop : size of population, and g_{max} : maximum number of iterations	Maximin, Diversity	✓	–	–	–	IC	Greedy [120], Tsang et al. [222], Fair-Emb [123], CrossWalk [122], SetSOGWO [179]
[184]	FIMM	$11/e$ approximation guarantee	Welfare function	✓	–	–	–	IC	–
[95]	FIMMOGA	NP-Hard	Gini coefficient-based fairness	✓	–	–	✓	IC	Degree, Closeness, Betweenness, PageRank, Eigenvector, DegreeDiscountIC [53], K-core [125], IMM [215], RIS [37], NSGA-II [66], MOEA/D [245], awGA [90]

communication on social media [146, 153, 164]. However, these two topics have been studied separately and should be analyzed together to highlight the impact of communication biases and following patterns on the (perceived) influential ranking of different types of users in a range of topics from home decor to STEM. Another practical topic has been to compute influence probabilities in real-world networks, and most of these works have yet to consider nodes' characteristics based on sensitive attributes [227], and one can look further to understand it better.

Link recommendation methods have also been designed to improve the social capital (based on information access) of a node [26, 27]. These methods can consider different types of advantages for a node based on its location in the network for achieving fairness as well as maximum outreach, such as (i) broadcast advantage that shows the reachability to the least advantage nodes, (ii) influence advantage that considers the reachability to most of the nodes, and (iii) control advantage that shows the node's ability to act as an intermediary or broker in propagating the information [26]. However, there are several open directions that should be looked at further, such as augmenting the information access fairness in directed networks using link recommendation, as in directed networks, the influence propagation for a pair of nodes may not be symmetric, and optimizing for one might lead to tradeoff for another. An additional crucial direction involves analyzing the probability of accepting suggested links and examining the tradeoff between various fairness constraints and recommended links, considering different influence propagation models.

3.4 Fair Influence Blocking Maximization

Social Media is beneficial in spreading valuable information and awareness among people, though at the same time, it has also been misused to propagate rumors, fake news, propaganda, or misinformation. The adverse impact of fake information propagation has been seen in several major events, such as the USA and Indonesian presidential elections [40, 97, 185]. It motivated researchers to study **influence blocking maximization (IBM)**, also known as influence minimization, on social media. In IBM, the aim is to identify a minimal set of users whose blocking or immunization will minimize the spread of misinformation in the network [78, 101, 173, 241]. In the case where only misinformation propagates in the network (also known as a single cascade of misinformation), the immunized or blocked nodes do not spread the misinformation further [173]. However, in the case of competitive information propagation (also known as truth-campaigning), we aim to choose seed set nodes that propagate the counter-true information to mitigate the negative influence of misinformation [101, 194].

3.4.1 Fairness Definitions. Fairness in IBM is yet to be defined than being solved. Here, we define some fairness constraints that can be considered in IBM. Let us assume that $I(R, G)$ denotes the rumor outreach (set of influenced nodes) from rumor starters R on the network G and $I(S, R, G)$ denotes the set of influenced nodes by rumor outreach in the network G , given R is the set of rumor starters and S is the set of immunized nodes. A node u is called *saved node* by immunized nodes S if $u \in I(R, G)$ and $u \notin I(S, R, G)$.

- (1) **Equality:** It ensures that the chosen immunized nodes in each community should be proportional to the number of nodes that might be affected by misinformation in the absence of the immunized nodes. This definition is based on the fact that the communities that might be more affected should have more immunized nodes to save them. It is defined as

$$\frac{|\{u \in S | u \in C_i\}|}{E|\{u \in I(R, G) | u \in C_i\}|} = \frac{|\{u \in S | u \in C_j\}|}{E|\{u \in I(R, G) | u \in C_j\}|}, \forall i, j.$$

- (2) **Equity**: The equity constraint suggests that the proposed solution is fair in saving nodes if the expected number of saved nodes for all communities are proportional to their sizes,

$$\frac{E|\{u \in I(R, G) \& u \notin I(S, R, G) \mid u \in C_i\}|}{|C_i|} = \frac{E|\{u \in I(R, G) \& u \notin I(S, R, G) \mid u \in C_j\}|}{|C_j|}, \forall i, j.$$

The equity constraint can also be defined as the expected number of saved nodes for all communities should be proportional to the nodes influenced by the rumor in respective communities. This will ensure that each community gets the resources compared to the number of people who would have been affected by the rumor spread.

- (3) **Maximin** [193]: The maximin fairness aims to maximize the fraction of saved nodes for a community that has minimum proportion of saved nodes. It will maximize the maximin value that is computed as

$$\min_{C_i \in C} \frac{E|\{u \in I(R, G) \& u \notin I(S, R, G) \mid u \in C_i\}|}{E|\{u \in I(R, G) \mid u \in C_i\}|}.$$

Other group and individual fairness definitions discussed for IM can also be similarly extended for IBM concerning the saved nodes.

3.4.2 State of the Art and Future Directions. First, we highlight that the solutions for fairness-aware influence maximization can not directly be applied to achieve fair IBM, as the solution in IBM is also dependent on the set of rumor starters. Therefore, given the same network and the same budget for immunizing nodes, the solution might differ for different sets of rumor starters; that is not the case in IM. A practical solution to minimize the misinformation spread is by propagating its counter-true information, i.e., also known as “truth-campaigning” [200]. The psychology-based studies have shown that users are more probed to find the correct information and believe in this once they have been exposed to both fake and true information [200]. Most of the existing truth-campaigning methods have focused on maximizing the total number of saved nodes [194]. The saved nodes refer to the nodes who believe in true information if the true information is propagated and would have been believed in misinformation in the absence of true information. Fair truth-campaigning methods should ensure that a community can not be manipulated, given its small size or connectivity with the network. Saxena et al. [193] proposed a fairness-aware influence blocking method, called **Fairness-aware Weighted Reversible Reachable System (FWRRS)**, that aims to maximize maximin fairness. The proposed method uses weighted reversible reachable trees to compute the blocking power of each node with respect to different communities and then chooses top- k truth campaigning nodes using six selection steps that maximize the fairness-aware optimization function. According to the experimental results, the proposed FWRRS method outperforms both fairness-oblivious and fairness-aware baselines in terms of both fairness and saved nodes. These results also highlighted that fairness does not always have a cost in terms of efficiency, and in many cases, it serves as a catalyst for improving overall effectiveness in the future. The main limitations of this work are that the whole network structure and the community information should be known in advance to achieve group fairness.

Given an abundance of literature on IBM [200], it is surprising to see that no other work has concerned fairness and structural bias, though there are many interesting questions to be explored. A comparative study of state-of-the-art theoretical and simulation-based methods from a fairness perspective will provide in-depth insights into the impact of structural biases on these methods. These insights can help further propose fair methods for IBM, truth-campaigning-based misinformation mitigation, and competitive information propagation maximization. Fair solutions for competitive information propagation will also be interesting for several applications, such as marketing or political parties’ agenda sharing.

3.5 Fair Community Detection

In social networks, the nodes are organized into communities. As per the definition proposed by Barabási in his book, “in network science, we call a community a group of nodes that have a higher likelihood of connecting to each other than to nodes from other communities” [22]. The understanding of community structure has played an essential role in understanding the network evolution as nodes join communities, and these communities are further connected with each other to form a large-scale complex network. As we already discussed, communities play a crucial role in defining fairness for other discussed problems, and therefore, their fairness is dependent on fair communities. For example, if the ground-truth community labels are not known, and communities are not well identified using community detection methods (or small communities are ignored), then feature-aware fair link prediction methods might not be fair for the nodes of minor communities. Ghasemian et al. [94] compared 16 community detection methods and showed that the number of identified communities varies a lot across different methods. They further studied the impact of community detection methods on link prediction and link description tasks, and observed that no method is always the best for such downstream tasks across all networks. The authors did not study the impact of different community detection methods on different types of nodes or communities having varying sizes and densities. Fairness is not yet well defined and studied for **community detection (CD)** methods, given that it has a vast literature [85].

3.5.1 Fairness Definitions. Here, we discuss some fairness aspects that should be considered while identifying communities. One important aspect is that small as well as large size communities should be identified equally well. Similarly, sparse as well as dense communities should be identified well if they co-exist in a network. Let us assume that the ground-truth communities in a network G are $C = \{C_1, C_2, \dots, C_i, \dots\}$, and the communities identified using a community detection method are $C' = \{C'_1, C'_2, \dots, C'_i, \dots\}$.

A fairness constraint for community detection based on *demographic parity* can be defined as

$$P(C'(u, v) = 1 | u, v \in C_i) = P(C'(u, v) = 1 | u, v \in C_j), \forall i, j,$$

where $C'(u, v)$ is 1 if both nodes u and v belong to the same community in C' ; otherwise, 0. This constraint ensures that the nodes belonging to the same community in the ground truth should be identified in the same community by the given community detection method, irrespective of their community sizes. However, there is one issue with this definition: If the community detection method returns the entire network as one community, then the fairness constraint is satisfied. The reason is that it does not consider if the nodes belong to two different communities. Therefore, the quality of the identified communities should also be considered while maximizing fairness.

Another important fairness constraint based on *equity* can be defined as the ratio of misclassified nodes for all communities should be maintained. First, the ground-truth communities should be mapped to the identified communities. An identified community C'_j is mapped to C_i if $|C'_j \cap C_i| \geq |C'_j \cap C_k| \forall k$. All ties while mapping can be resolved using a heuristic or uniformly at random. If a ground-truth community is not mapped with any identified community, then map it to an empty set. Now, the fairness constraint can be defined as

$$\frac{|C'_i \cap C_i|}{|C_i|} = \frac{|C'_j \cap C_j|}{|C_j|}, \forall i, j.$$

For simplicity, in the formula, we assumed that C_i is mapped to C'_i community. The **Kullback-Leibler (K-L)** divergence and **Earth Mover Distance (EMD)** (refer to the supplementary materials Section B) can be used to compute the fairness of a proposed method by comparing achieved

and desired distribution. Other methods, such as Jaccard similarity, can also be used for community mapping.

Khajehnejad et al. [122] used *disparity* to compare the fairness of node classification methods. In general, disparity fairness focuses on reducing the disparity or differences in outcomes between different groups. In Reference [122], disparity is computed as a variance of the fraction of correctly labeled nodes in each group.

3.5.2 State of the art and Future Directions. Mehrabi et al. [151] highlighted the bias of community detection methods for small-size communities. They showed that well-known methods, such as Louvain [32] and CESNA [243], fail to assign lowly connected nodes to proper communities or assign them to various small size communities and exclude them from being included in different analyses. They proposed a method, called CLAN, for attributed networks that assigns lowly connected nodes to large size communities rather than assigning them to several small, meaningless communities. The method works in two steps. In the first step, an unsupervised community detection method that only uses network structure, such as the Louvain method [32], can be applied. The next step is supervised, in which the nodes assigned to insignificant communities are re-assigned to major communities using nodes' attributes that were not used in the first step. This work does not define fairness constraints as it mainly focuses on the bias aspect. Another important point to note is that this is not the only way to reduce bias toward lowly connected nodes. One should aim to identify small communities correctly, and then they should be treated equally well as large communities in other downstream tasks.

Since fairness has almost never been considered in the community detection context, a simple in-depth analysis of community detection methods from a fairness perspective will be very interesting. In literature, plenty of community detection methods of different types, including modularity optimization-based methods, label propagation-based methods, betweenness centrality-based methods, representation learning-based methods, spectral properties-based methods, and genetic algorithm-based methods, have been defined [13, 85, 176]. An exhaustive comparative analysis of different types of methods is required to understand which kind of methods are fairer for which kind of networks or which method is fairer. This analysis will help understand which kind of methods are more respectful to small-size communities and identify them well. A better understanding will provide insights for designing fair community detection methods. Fair community detection methods will be useful in better interpreting results for other research problems, as they will ensure that all kinds of communities are well identified irrespective of their size, density, or connectivity. Apart from crisp communities, where a node only belongs to one community, a fairness-aware analysis of overlapping communities [9] will also be of great interest, as this will highlight if the lowly connected nodes are well assigned to all communities they belong to, and if not then what is the reason and how it can be further improved. Fair community detection in other types of networks, such as directed networks, weighted networks, multilayer networks, and hypergraphs, should also be looked at given their applications in practice [55, 107, 143, 145]. Another important point is that several evaluation metrics exist to compare the quality of communities identified using different community detection methods; however, none has considered fairness [49]. How do different evaluation metrics perform compared to each other from fairness perspectives? Defining evaluation metrics for fair community detection is still an open research direction.

Fairness has been explored in machine learning-based clustering methods and is mainly defined from two different perspectives. From the first perspective, a clustering of nodes is called fair if each protected group is equally present in each cluster [56, 128]. The principle behind this definition is that each identified cluster should be a good representation of the data and should reflect its diversity. Another well-known fairness constraint is that the cluster center should be a good

representative of the members of the cluster by being “close” to the points assigned to it [1, 93]. In this fairness constraint, the centers should represent different (protected) groups very well. This definition is much closer to the definition of communities in network science and can be applied to identify fair communities using a low-dimensional vector representation of the nodes learned using a network embedding technique.

3.6 Other Research Topics

There are several other topics in SNA for which the network structure-based methods have been designed. Next, we briefly discuss some of these topics and highlight why fairness is an essential factor to be considered.

3.6.1 Fairness in Opinion Formation. In social networks, a user’s opinion is dependent on the opinion of other neighboring users and the user’s personal bias. Therefore, opinion formation models take the network structure into consideration while modelling the opinion dynamics [239]. Nguyen et al. [163] studied opinion dynamics on complex social networks where each individual holds a binary opinion on a certain subject and may change her/his opinion to match the opinion of the majority of its neighbors. The dynamics of opinion change is governed by two distinct rules: classic majority and influence majority. The classic majority rule involves individuals adopting the majority opinion of their neighbors, while the influence majority rule allows individuals to evaluate the influence of each opinion and choose the one that is more influential. To determine influence, they calculate the sum of the influence of each opinion holder in the individual’s neighborhood area, where the influence of each individual is estimated based on their number of social connections. The authors theoretically showed that the minor community nodes converge to the opinion of the majority in a dense network; however, it has not been verified on real-world data. The authors also did not consider the impact of community structures, users’ bias, and several other factors that might impact the opinion of users. Reference [239] also observed that highly connected nodes have a stronger impact on opinion formation dynamics. It is important to understand the opinion formation for minor communities given that we know that minor communities are not able to achieve a higher rank in the network due to homophily [115]. To the best of our knowledge, the fairness of opinion formation models for minor communities has not yet been studied, and one main limitation might be the lack of real-world datasets.

3.6.2 Fairness in Anomaly Detection. In anomaly detection, we aim to identify unusual instances in different applications, including malicious users detection in OSNs, fraud detection, and suspicious bank transaction detection [10, 131]. Most of the proposed anomaly detection methods are dependent on network structure as some specific structural pattern can convey abnormal behavior [5, 54]. Unfairness in such systems might affect some particular communities, for example, targeting users from a specific community while identifying suspicious users. Davidson and Ravi [63] compared five classic anomaly detection methods and showed that their outputs are unfair; however, these works might mislead someone to conclude that their results are fair, especially when the number of outliers and the number of protected status variables are small. Such analysis raises the question of whether anomalous nodes and links detection methods [225, 233] are fair or not for different protected groups in complex networks. If not, then anomaly detection methods for complex network data should address these issues and focus on the fairness of all protected groups.

3.6.3 Fairness in Network Anonymization. Since the mid-2000s, a massive amount of social networking data has been released publicly and analyzed to better understand complex networks and their different applications. However, ensuring the privacy of the released data has been a primary

Table 3. FairSNA State of the Art Summary

Research Topic	Individual fairness	Group Fairness	Feature-Aware	Feature-Blind	Pre-Processing	In-Processing	Post-Processing
LP	[133, 177]	[122, 133, 147, 177, 187, 189]	[122, 133, 147, 177, 187, 189]	–	[133]	[122, 147, 177, 189]	[147, 187]
CR	–	[224]	[224]	–	–	[224]	–
IM	[83]	[8, 11, 28, 29, 80, 95, 122, 123, 144, 178, 179, 184, 209, 210, 219, 222]	[8, 11, 28, 29, 80, 83, 95, 122, 123, 144, 178, 179, 184, 209, 210, 219, 222]	–	–	[8, 11, 28, 29, 80, 83, 95, 122, 123, 144, 178, 179, 184, 209, 210, 219, 222]	–
IBM	–	[193]	[193]	–	–	[193]	–
CD	–	[122]	[122, 151]	–	–	[122]	[151]

concern. Most of the graph anonymization techniques can be categorized as (i) graph modification methods and (ii) clustering-based methods [45, 247]. Briefly, we highlight whether the graph anonymization affects the analysis for different protected groups using anonymized data is not yet studied. Besides, linkability is used to obtain useful information by mapping the data collected from different sources. This is a privacy threat, and the extent of its impact on different types of user groups in anonymized data should be analyzed to propose better fair methods.

3.6.4 Graph Coverage Problem. Fairness has also been explored in graph theory problems. Habibi et al. [100] proposed approximation algorithms for submodular maximization under fairness constraints for both monotone and non-monotone functions. The authors verified the proposed solutions on various real-life problems, including the maximum coverage problem that is of our interest. The results showed a huge improvement in the bias (around 15% difference) given that the objective value obtained by the fair solution is similar to the unfair baseline methods. The fair solution of maximum coverage can be used further in influence maximization to obtain a fair seed set for starting the spread. However, the application of such solutions for influence maximization is still an open question. The theoretical as well as empirical results in this direction will be interesting for the scientific community.

Recently, network structure has also been used to design solutions for role analytics, fake news detection, echo chamber detection, hate spreading user detection, and mental health prediction [84, 118, 180, 199, 230], and the proposed methods might also inherit the bias of network structure that should be addressed.

In Table 3, we summarize state-of-the-art FairSNA methods that include LP, **centrality ranking (CR)**, IM, IBM, and CD. It shows that most fair methods are feature-aware and used in-processing, and individual fairness is not well studied in SNA. This table highlights the gap in research on fairness in SNA. In this work, we have discussed ample open research directions, and hopefully, in the future, it will help in bridging this research gap.

In this section, we also discussed metrics used to evaluate fairness for different problems; they are further summarized in the Appendix B for the sake of completeness.

4 DATASETS

One main limitation in designing fairness-aware methods in SNA is the limited availability of datasets, as the labeling of the dataset is a difficult task. In Table 4, we summarize real-world datasets used in different works, i.e., mentioned in the last column. Some methods were also verified on other complex networks than social networks, mentioned in the last three rows of Table 4. In most research works, researchers have used synthetic network generating models to create benchmark datasets for an in-depth understanding of the impact of different parameters, such as

Table 4. Real-world Network Datasets

Dataset	#Nodes	#Edges	Protected Attributes	Remarks	Works
Instagram 1 [209]	539,023	640,211	Gender	45.57% males & 54.43% females	[209]
Instagram-London [177]	53,902	165,184	Gender, Race	Races: African, Caucasian, Asian	[177]
Instagram-Los Angeles [177]	82,607	482,305	Gender, Race	African, Caucasian or Asian	[177]
DBLP [209]	53,307	288,864	Gender	81% males and 19% females	[109, 209, 224, 232]
DBLP-Data Mining and Database [223]	16,501	66,613	Gender	25.7% females; Considered publications from 2011 to 2020	[223]
DBLP-continent [209]	3,980	6,965	Continent	labels have 5 continent values	[133, 206]
Enron [203]	144	1,344	Gender	76% men and 24% women	[109]
Norwegian Interlocking Directorate Network [202]	1,421	3,855	Gender	63% men and 37% women	[109]
Github Follower network [116]	89,630	167,359	Gender	5.7% females	[232]
Brazilian network [182]	16,730	39,044	Gender	10,106 sex workers & 6,624 sex buyers; 40% minority nodes	[134]
POK Swedish Dating Network [104]	29,341	–	Gender	minority group has 44% nodes	[134]
Dutch School Friendship Network [205]	26	221	Gender	17 Girls & 9 Boys	[147]
Google+ Network [148]	4,938	547,923	Gender	anonymized gender data	[147]
Facebook Network [148]	4,039	88,234	Gender	anonymized gender data	[133, 147, 206]
Rice University Facebook [155]	1,205	42,443	Gender, Age	Nodes' Attributes: student id, age (18 to 22), and major	[8, 123]
Pokec social network [214]	1,632,803	30,622,564	Gender, Age	Nodes' attributes: gender, age, hobbies, interest, education	[61, 69, 100]
X network [183]	18,470	61,157	Political Inclination	61% nodes in Political (left)	[224]
X network-2 [17]	3,560	6,677	Political Inclination	2598 nodes in neutrals, 782 nodes in liberals, and 180 nodes in conservatives group	[122]
HIV Prevention for homeless youth [238]	60–70	–	birth sex, gender identity, race, and sexual orientation	4 datasets each having 60-70 nodes	[222]
Homeless Youth Social Networks [24]	124–296	111–326	Race	six social networks of homeless youths from US	[178]
APS citation network [134]	1853	3,627	Research Fields	37% Classical Statistical Mechanics, and 63% Quantum Statistical Mechanics	[232]
US Politics Blogs network [4]	1,222	19,089	Political (left)	52% nodes in Political (left)	[133, 224]
US Politics Books	92	748	Political (left)	47% nodes in Political (left)	[224]
Co-purchasing network [62]					

homophily, size of the minor community, or network density. The synthetic models to generate homophilic networks having minor major communities are (i) Homophily BA Model [115, 134], (ii) Diversified Homophily BA [232], (iii) Directed Homophily model [11], (iv) Organic Growth Model [211], and (v) **High Clustering Homophily Barabási-Albert (HICH-BA)** model [193]; refer the Appendix C for detailed synthetic models.

5 DISCUSSION

Since the early 2000s, social network analysis has emerged as a widely employed approach to understand different complex phenomena, encompassing both individual and group-oriented aspects of human behavior. Structural properties of nodes and networks provide insights into understanding their roles in the system. Therefore, SNA methods, such as influence maximization, recommendation systems, and centrality ranking, use network structure and nodes' attributes. As we discussed earlier, these networks have structural inequalities due to homophily and different sizes

of protected groups that might impact the outcome for some specific groups based on their size, density, or protected attributes. It is, therefore, important to account for these inequalities and consider fairness while designing network structure-based solutions for such problems as they might unfairly impact some specific groups of society.

In SNA, the biases can be handled by modifying/processing the data, by improving the process, or by modifying/processing the output of the existing methods to achieve fairness in the final outcome. If the bias is handled by modifying the input data, then these methods will be classified as pre-processing methods. If we design the methods that handle the bias during the process of generating the outcome, it is called in-processing, and the approaches to modify the output to make it fair are classified as post-processing. In network data, in-processing steps mainly update the adjacency matrix of the network so that the dependency on the protected attributes is obfuscated [133]. The updated matrix can be used further to design methods for different downstream tasks, such as link prediction or node classification. In-processing methods aim to maximize the optimization function subject to the fairness constraint based on the given application problem. However, the post-processing methods use fairness-oblivious methods and then process their outcome to be fair based on the fairness criteria. For example, in link prediction, the post-processing step will increase the prediction probabilities for the types of node-pairs that are less likely to be predicted correctly by a given algorithm.

In FairSNA algorithms, the choice of fairness constraint is dependent on the application requirement, and a designed method might not perform well for multiple fairness constraints [80]. How to determine the most suitable fairness metric for a particular use case is a challenging question and an ongoing research topic [88, 102, 103, 234]. This becomes further challenging as many fairness constraints cannot be satisfied simultaneously [127], as each fairness metric is associated with a distinct set of empirical and normative assumptions [235]. Other interesting questions to explore include *What types of bias and unfairness does the law address when it prohibits discrimination?*, *What role can fairness metrics play in establishing legal compliance?* [236], and *Can we morally justify the ways how fairness metrics are optimized?* [235]. Within the wider Algorithmic Fairness research community, the interdisciplinary nature of tackled problems has been widely acknowledged [65, 158]. There are attempts to bridge the understanding gaps in terminology, formulation of research questions, and research methods used in computing sciences, social sciences, and humanities at large.

One another important factor to consider while designing fair methods for complex networks is the sparsity of real-world networks [68]. This further creates a data imbalance problem, and the imbalance is too huge in some cases. For example, in link prediction, the number of existing links is much lower than the number of non-existing links [142]. Most of the machine learning models for fairness-aware link prediction have been trained by taking an equal number of node pairs that will form links as positive cases and node pairs that will not form a link as negative cases, and have been verified similarly to prove their fair outcome for all groups or individuals given a fairness constraint [122, 147, 189]. However, this is not the case in real-world scenarios, and therefore, fairness-aware methods should consider these imbalances while training and testing, as it has been shown that data imbalance might affect the performance of methods if they are designed without considering it [142].

Other important factors to consider are the price of fairness and the computational cost of fairness-aware methods vs. fairness-oblivious methods. In FairSNA, many works, including References [20, 123, 222], have studied the price of fairness theoretically as well as experimentally. Reference [222] showed that the price of fairness in practice is not as high as provided by theoretical bounds for influence maximization. The reason is that the theoretical limits are based on worst-case scenarios, and such cases rarely happen in real-world networks. In some cases, the price

of fairness might be high for making the initial impact fair, though it might pay later by being more efficient in terms of both fairness and accuracy. For example, in influence maximization, when we choose a small number of seed nodes given a fairness constraint, then initially, the total accuracy might be lower as compared to baselines; the reason is that if you propagate the influence in all communities (majorities and minorities), then the total outreach might be lower as the number of influenced nodes in majority groups might be lower as compared to fairness-oblivious methods. However, as we keep choosing seed nodes, the method might become more efficient as compared to fairness-oblivious baselines as the influence will propagate to all groups and might also attain higher total outreach [20, 123].

Besides the price of fairness, if we talk about the computational cost, then intuitively it might seem that the fairness-aware SNA methods might have a high computational cost, though this is not always the case. In the case of heuristic methods, such as References [187, 210], the computational cost is similar to fairness-oblivious methods. In the case of approximation and greedy methods, the fairness-aware methods optimize the given function, and it does not affect the computational cost in all cases. In the case of network-embedding-based methods, the fairness is embedded by updating the probability of random-walker based on network connectivity. The computational complexity to compute the probability in fairness-aware network embedding methods, such as NodeSim [189], FairWalk [177], CrossWalk [122], is the same as in some fairness-oblivious methods, such as Node2Vec [98]. Once the random walker generates an ordered pair of sequences of nodes, the embedding is generated using the word2vec model, and therefore the computational cost is not affected in such cases. Therefore, the computational cost of fair methods might not always be high as compared to fairness-oblivious methods. However, as we highlighted in Section 3, most of the fairness-aware methods consider the whole structure of the network and use community information, and therefore require that either the label of nodes be known or should be identified using structure-based community detection methods. These pre-processing steps might increase the computational cost in some cases. We refrain to comment on the computational cost for dynamic networks, as there is still a research gap on designing efficient fairness-aware methods for dynamic networks.

Fairness-aware methods are slowly getting attention in other types of complex networks, such as heterogeneous networks, bipartite networks, attributed networks, hypergraphs, and multiplex networks [114], though there is still a big research gap with many open questions. For example, in attributed networks, it will be interesting to understand how the fairness of different algorithms is impacted as we have fewer and fewer metadata or labels for the nodes. How to generate fair embedding when attribute distribution is skewed? Throughout this survey, we discussed network inequalities in social networks and highlighted the research gap in FairSNA. However, algorithmic fairness should also be considered for other types of complex networks, such as information networks, chemical networks, or biological networks. For example, in citation networks, some clusters or groups based on research areas might be bigger than some other research groups [50], and therefore this should be considered while designing recommendation models for research citation. The methods designed for social networks will be insightful for other complex networks.

6 CONCLUSION

In this survey-cum-vision, we discussed the taxonomy of FairSNA, followed by various SNA-related research topics, including why fairness is an important aspect to be considered, how fairness can be defined, state of the art, and our vision as future directions. We highlighted in Section 3 that most of the fair methods had been proposed in the past few years, and still, the research is in its infancy state. Fairness has not yet been explored for some problems, and we defined fairness constraints that can be considered for such problems. Most of the proposed methods in FairSNA

are feature-aware and use in-processing techniques. However, feature-blind fair methods will be much more useful in SNA due to privacy concerns as well as the availability of limited features. Another important point to note is that individual fairness is mostly unexplored in SNA, and the price of fairness for achieving individual fairness is unknown. We observed that different network embedding techniques have been used to achieve fairness in downstream tasks, such as link prediction and influence maximization, and provide promising results. This direction can also be further explored to achieve fairness for other downstream tasks.

Through this survey, we would like to get the attention of researchers, including network scientists, data scientists, and machine learning experts, toward this gap in research. We have discussed several open research directions that one can look at. In the future, this survey should motivate researchers to consider fairness while proposing solutions in social network analysis and computational social science.

APPENDICES

A ABBREVIATIONS

In Table 5, we list all abbreviations used in this survey.

Table 5. Abbreviations used in This Survey

Abbreviation	Explanation
CD	Community Detection
CR	Centrality Ranking
FairSNA	Fairness-aware Social Network Analysis
IBM	Influence Blocking Maximization
IC	Independent Cascade
IM	Influence Maximization
LP	Link Prediction
ML	Machine Learning
NLP	Natural Language Processing
OSNs	Online Social Networks
SNA	Social Network Analysis

B FAIRNESS EVALUATION METRICS

Here we summarize metrics used to evaluate fairness with their applications.

- (1) **Price of Fairness (PoF)**: This is the ratio of the optimal solution without a fairness constraint to the best achievable solution under the fairness constraint. For example, in influence maximization, the PoF is the ratio of maximum outreach for any choice of k seed nodes to the outreach achieved under a given fairness constraint. It can be computed as

$$PoF = \frac{I_{opt}}{I_{fairness}}.$$

- (2) **Utility Gap**: This computes the gap in utilities of communities or, mainly, the gap between the communities with the highest and lowest utilities. In influence maximization, it can measure the utility gap between the communities having the highest and lowest outreach.
- (3) **K-L Divergence [130]**: This is used to compare two distributions. In influence maximization, it can be used to compare the achieved outreach with the expected outreach ratio of different communities.

- (4) EMD: The EMD is used to compare two distributions, and it also considers the distance between the values that is not considered in the K-L divergence [232]. For example, the divergence between $[1, 1, 1, \dots]$ and $[2, 2, 2, \dots]$ is equal to the divergence between $[1, 1, 1, \dots]$ and $[10, 10, 10, \dots]$, though they have different EMD values.

The other evaluation metrics include Power inequality [14, 232], Moment Glass Ceiling [14, 108, 232], and Modularity Reduction [147].

C SYNTHETIC NETWORK GENERATING MODELS

The synthetic models to generate homophilic networks with minor and major communities that have been used in studying FairSNA are as follows.

- (1) *Homophily BA Model* [115, 134]: In this model, the links are formed using two rules: preferential attachment and homophily. The model uses the following parameters: (i) m , i.e., the fraction of minority nodes; (ii) l , i.e., the number of edges formed by each new node; and (iii) H , i.e., the homophily matrix where the entry $H_{g_u g_v}$ is the probability to connect two nodes u and v belonging to group g_u and g_v , respectively. The network evolves as follows at each timestep:
- (a) A new node v is added to the network. It belongs to the minority group with probability m and to the majority with $1 - m$. The group of node v is denoted as g_v .
 - (b) Node v makes l connections, and the probability to connect with a node u is defined as

$$\Pi_u = \frac{H_{g_v g_u} d(u)^\alpha}{\sum_w H_{g_v g_w} d(w)^\alpha},$$

where $d(u)$ is the degree of node u .

If $\alpha = 0$, then the network has no preferential attachment and is referred to as *Random Homophily*.

- (2) *Diversified Homophily BA*: Wang et al. [232] proposed the Diversified Homophily BA model to promote inter-community edges while maintaining some degree of homophily. The model requires the following parameters: (i) m , the fraction of minority nodes; (ii) l , the total number of edges for each new node; (iii) H , the homophily matrix that contains the probability of inter and intra-group connections; (iv) α , preferential attachment strength; (v) l_d , the number of diversified edges for each node; and (vi) p_d , the diversification probability. The network evolves using the following steps at each timestep:
- (a) A new node v is added to the network. It belongs to the minority group with probability m and to the majority with $1 - m$. The group of node v is denoted as g_v .
 - (b) The node v is connected with $l - l_d$ nodes using Homophily BA attachment rule where the probability of connecting with a node u is defined as $\Pi_u = \frac{H_{g_v g_u} d(u)^\alpha}{\sum_u H_{g_v g_u} d(u)^\alpha}$. The nodes connected at this step are referred to as S_v .
 - (c) In this step, the node v will make l_d diversified connections. The probability of connecting two nodes v and w is computed as

$$p_{vw} = \begin{cases} p_d, & g_v \neq g_w \\ 1 - p_d, & g_v = g_w. \end{cases}$$

For node $u \in S_v$, create a set of their neighbors denoted by N_{S_v} . Next, the node will make l_d diversified connections by connecting to $k \in N_{S_v}$ with the probability $\Pi_{vw} \propto p_{vw} \frac{1}{|d(w) - d(u)|}$, where $d(u)$ is the degree of node u and $u \in S_v$. The main idea behind this is that the nodes belonging to opposite groups should be connected by maintaining a similar

degree as of existing neighbors. When $\alpha = 0$, it will remove the preferential attachment, and the generated network will be called *Diversified Homophily*.

- (3) *Directed Homophily Network*: Anwar et al. [11] proposed a model to generate directed homophilic networks based on the network evolution models proposed by Bollobás et al. [33] and Karimi et al. [115]. In the evolution process, at each timestep, a directed edge is added to the network as follows:
 - (a) With probability α , a new node v is added, and it is connected to an existing node w . The node v is assigned to the majority category with probability p_M and to the minority category with probability $(1 - p_M)$. The node w is chosen with the probability proportional to $h(v, w)d_{in}(w) + \delta_{in}$, and $h(v, w) = h$ if v and w belong to the same category, and otherwise $h(v, w) = (1 - h)$, where h is a variable and its value depends on the required homophily in the network, $d_{in}(w)$ is the in-degree of node w , and δ_{in} is a constant.
 - (b) With probability β , an edge is added from an existing node v to an existing node w . Node v is chosen from all existing nodes with the probability proportional to $d_{out}(v) + \delta_{out}$, where $d_{out}(v)$ is the out-degree of node v and δ_{out} is a constant, and node w is chosen from all existing nodes with probability proportional to $h(v, w)d_{in}(w) + \delta_{in}$.
 - (c) With probability γ , a new node w is added and is connected with an edge from an existing node v . The node w is assigned to the majority category with probability p_M and otherwise to the minority category. Node v is chosen from existing nodes with probability proportional to $h(v, w)d_{out}(v) + \delta_{out}$.
- (4) *Organic Growth Model*: Stoica et al. [211] proposed this model to study the glass ceiling effect in the network. The proposed model assumes that each node belongs to one of the two communities, called minor or major communities. As the network grows, one node and one directed edge are added at each timestep using the following steps.
 - (a) Minority-majority partition: A new node u joins the network and belongs to the minor community with probability r and the major community with probability $1 - r$ (for $0 \leq r \leq 1/2$).
 - (b) Randomness: With probability η , the new node u connects with a randomly chosen existing node v .
 - (c) Preferential attachment: With probability $1 - \eta$, the node u chooses an existing node uniformly at random and copies one of its edges. In simple words, the probability of choosing another endpoint is directly proportional to its degree, as in preferential attachment law.
 - (d) Homophily: If the newly added node belongs to a different group than the chosen node for the connection, then the connection is created with probability ρ , and this process will be repeated until an edge is formed. ρ is the homophily factor ($0 \leq \rho \leq 1$) and controls that a node that is less similar is less probable to be chosen for making the connection.

The model is referred to as the “organic growth” of the network, as the newly added nodes form new connections without the influence of any external forces.

- (5) *HICH-BA Model*: Saxena et al. [193] proposed the HICH-BA model that can create synthetic networks having multiple communities with a given size distribution and regulated homophily and clustering coefficient. The proposed model have following six parameters, (i) n , i.e., the total number of nodes; (ii) p_N , i.e., the probability of adding a new node to the network (with probability $1 - p_N$, the network self-grows); (iii) r , i.e., an array where the i_{th} entry is the probability of a new node joining the community i ; (iv) h , i.e., the homophily factor; (v) p_t , i.e., the probability of making a triad connection; and (vi) p_{PA} , i.e., the probability of adding an edge using preferential attachment.

The network starts with a seed graph having n_0 nodes, and at each timestep, the following steps are executed:

- (a) With probability p_N , a new node v is added to the network and it joins a community $c \in C$ according to the probabilities in r . Next, the node v connects to a node $u \in c$. With probability p_{PA} , the connection is based on preferential attachment and otherwise at random. The probability of choosing the node u is computed as

$$p_u = \begin{cases} \frac{d_u}{\sum_{w \in c \setminus \{v\}} d_w} & \text{with probability } p_{PA} \\ \frac{1}{|c|} & \text{otherwise} \end{cases}.$$

- (b) With probability $1-p_N$, the network self-grows by adding a new edge using triad formation or random connection.

- (i) Triad Connection: With probability p_t , a triad connection will be made from a u.a.r. chosen node u and $u \in c$. Let us assume S is the set of second distance neighbors that are not directly connected with u . Now, the triad connection will be formed with a node $v \in S$ using homophily and preferential attachment as follows:

$$p_v = \begin{cases} \frac{d_v}{\sum_{w \in c \cap S} d_w} & \text{with probability } h * p_{PA} \\ \frac{1}{|c \cap S|} & \text{with probability } h * (1 - p_{PA}) \\ \frac{d_v}{\sum_{w \in S \setminus c} d_w} & \text{with probability } (1 - h) * p_{PA} \\ \frac{1}{|S \setminus c|} & \text{otherwise} \end{cases}.$$

- (ii) Random Connection: With probability $1 - p_t$, an edge is created between the node u ($u \in c$) that is selected u.a.r., and the node v that is selected from $G \setminus \{u\}$ using the probabilities as follows:

$$p_v = \begin{cases} \frac{d_v}{\sum_{w \in c} d_w} & \text{with probability } h * p_{PA} \\ \frac{1}{|c|-1} & \text{with probability } h * (1 - p_{PA}) \\ \frac{d_v}{\sum_{w \in G \setminus c} d_w} & \text{with probability } (1 - h) * p_{PA} \\ \frac{1}{|G \setminus c|} & \text{otherwise} \end{cases}.$$

The proposed model can generate both scale-free and random homophilic networks having a desired community size distribution and regulated clustering coefficient.

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