Computational speedups and learning separations in quantum machine learning

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Summary

In this thesis, the contribution of quantum computers to machine learning has been explored, falling under the domain of Quantum Machine Learning. This domain promises novel perspectives and methods for addressing complex issues in machine learning by leveraging the unique capabilities of quantum computers. Quantum computers differ fundamentally from classical computers in their use of quantum mechanics, resulting in unique computational abilities. Unlike classical bits, which can be either 0 or 1, quantum computers employ qubits that can exist as both 0 and 1 simultaneously due to a phenomena called "superposition". Moreover, "entanglement" enables quantum computers to bring qubits into a mutually dependent state, allowing for complex parallel computations. Within the research domain of quantum machine learning, this thesis has explored various proposals regarding how quantum computers can enhance certain components of machine learning.

The first proposal examined in this thesis is the applications of quantum computers in topological data analysis. Topological data analysis is an innovative approach that extracts robust properties from datasets by understanding their inherent "shape". The focus was specifically on quantum algorithms for linear algebra, aiming to determine if they could offer superpolynomial speedups compared to classical methods. The results of this thesis demonstrated that existing quantum algorithms, along with algorithms developed in this thesis (with applications in numerical linear algebra, machine learning, and complex network analysis), solve problems that are classically deemed intractable according to widespread assumptions in complexity theory. Specifically, these results showed that the speedup provided by quantum algorithm methods for topological data analysis is resilient against the development of faster classical algorithms. These findings shed light on the potential power of quantum computers in addressing complex problems in topological data analysis, machine learning, and network analysis.

Another aspect of this thesis is the investigation of structural risk minimization in the context of quantum machine learning models. Structural Risk Minimization (SRM) is a principle in machine learning that seeks a balance between model complexity and performance on new data. It involves selecting a model from a given family by striking a balance between training error (how well the model fits the training data) and a complexity term (penalizing overly complex models). The focus on this thesis was on understanding the impact of certain design choices in machine learning models based on parameterized quantum circuits. In essence, a parameterized quantum circuit can be seen as a quantum variant of a neural network, manipulating a set of

qubits depending on parameters and seeking the right parameters for the problem at hand. The research in this thesis explored whether important settings within new quantum machine learning models based on parameterized quantum circuits can be identified, influencing both complexity measures and training error, which is crucial for the successful implementation of SRM. In particular, this thesis demonstrated how to construct new quantum machine learning models with favorable performance guarantees based on the SRM principle. These insights contribute to optimizing quantum machine learning models, enhancing their performance.

The subsequent topic explored in this thesis was how quantum computers can improve reinforcement learning. Reinforcement learning revolves around learning through interaction to achieve a specific goal, typically modeled by the interaction between an "agent" (the learner) and an "environment". The agent takes continuous actions, and after each action, the environment responds by providing the agent with a "reward". The goal of the agent is to maximize these rewards over time. In this thesis, quantum models based on parameterized quantum circuits were introduced within reinforcement learning. Notably, these models demonstrated comparable performance to traditional classical models (such as deep neural networks), while showing superior performance in certain scenarios. These results suggest that quantum models can be a powerful tool for solving complex problems in reinforcement learning.

The final part of the research in this thesis focused on identifying learning tasks within computational learning theory for which quantum learning algorithms have exponential advantages over classical algorithms. Computational learning theory is a mathematical framework introduced in the 1990s with the aim of providing formal arguments about why and how machine learning can be successful in practice. This thesis delved deep into the details to precisely define what it means for a quantum learner to have an exponential advantage over its classical counterparts. It then explored previous cases of exponential advantages, identifying the exact source of classical complexity and the advantage of quantum models. Finally, it investigated the general belief that quantum machine learning performs best in scenarios with data generated by quantum processes. This involved establishing a framework in which any problem with data generated by quantum processes can lead to an exponential quantum advantage. This opens doors to the application of quantum computing in specific scenarios where classical algorithms fall short.

In summary, this thesis provides an exploration of quantum machine learning, applying the unique capabilities of quantum computers to diverse domains within machine learning. The proposals researched ranged from the application of quantum algorithms in topological data analysis, understanding the influence of design choices on structural risk minimization and the introduction of quantum models in reinforcement learning to examinations of learning tasks in computational learning theory where quantum learning algorithms can offer exponential advantages. As a whole, this thesis contributes to the understanding of the promising role of quantum computers in addressing complex problems within machine learning.