



Universiteit
Leiden
The Netherlands

Computational speedups and learning separations in quantum machine learning

Gyurik, C.

Citation

Gyurik, C. (2024, April 4). *Computational speedups and learning separations in quantum machine learning*. Retrieved from <https://hdl.handle.net/1887/3731364>

Version: Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/3731364>

Note: To cite this publication please use the final published version (if applicable).

Bibliography

- [8] Scott Aaronson. The learnability of quantum states. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 463, 2007.
- [9] Scott Aaronson, Jan 2013. <https://csttheory.stackexchange.com/questions/15066/consequences-of-bqp-subseteq-p-poly>.
- [10] Scott Aaronson and Alex Arkhipov. The computational complexity of linear optics. In *Proceedings of the forty-third annual ACM symposium on Theory of computing*, 2011.
- [11] Jayadev Acharya, Ibrahim Issa, Nirmal V Shende, and Aaron B Wagner. Measuring quantum entropy. *2019 IEEE International Symposium on Information Theory (ISIT)*, 2019.
- [12] Jayadev Acharya, Alon Orlitsky, Ananda Theertha Suresh, and Himanshu Tyagi. Estimating rényi entropy of discrete distributions. *IEEE Transactions on Information Theory*, 63:38–56, 2016.
- [13] Michał Adamaszek. Extremal problems related to Betti numbers of flag complexes. *Discrete Applied Mathematics*, 173:8–15, 2014.
- [14] Michał Adamaszek and Juraj Stacho. Complexity of simplicial homology and independence complexes of chordal graphs. *Computational Geometry*, 57:8–18, 2016.
- [15] Leonard Adleman. Two theorems on random polynomial time. In *19th Annual Symposium on Foundations of Computer Science (SFCS 1978)*, pages 75–83. IEEE Computer Society, 1978.
- [16] Hamed Ahmadi and Pawel Wocjan. On the quantum complexity of evaluating the Tutte polynomial. *Journal of Knot Theory and its Ramifications*, 19:727–737, 2010.
- [17] Werner Alexi, Benny Chor, Oded Goldreich, and Claus P Schnorr. Rsa and rabin functions: Certain parts are as hard as the whole. *SIAM Journal on Computing*, 17:194–209, 1988.
- [18] Zeyuan Allen-Zhu, Yuanzhi Li, and Yingyu Liang. Learning and generalization in overparameterized neural networks, going beyond two layers. *Advances in neural information processing systems*, 32, 2019.
- [19] Dana Angluin. Learning regular sets from queries and counterexamples. *Information and computation*, 75(2):87–106, 1987.
- [20] Anurag Anshu, Srinivasan Arunachalam, Tomotaka Kuwahara, and Mehdi Soleimanifar. Sample-efficient learning of quantum many-body systems. In *2020 IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS)*, pages 685–691. IEEE, 2020.

- [21] Martin Anthony and Peter L Bartlett. Function learning from interpolation. *Combinatorics, Probability and Computing*, 9, 2000.
- [22] Simon Apers, Sayantan Sen, and Dániel Szabó. A (simple) classical algorithm for estimating betti numbers. *arXiv:2211.09618*, 2022.
- [23] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.
- [24] Sanjeev Arora and Boaz Barak. *Computational complexity: a modern approach*. Cambridge University Press, 2009.
- [25] Pablo Arrighi and Louis Salvail. Blind quantum computation. *International Journal of Quantum Information*, 4, 2006.
- [26] Srinivasan Arunachalam and Ronald de Wolf. Guest column: A survey of quantum learning theory. *ACM SIGACT News*, 48, 2017.
- [27] Ryan Babbush, Jarrod McClean, Craig Gidney, Sergio Boixo, and Hartmut Neven. Focus beyond quadratic speedups for error-corrected quantum advantage. *Physical review X Quantum*, 2021.
- [28] Peter L Bartlett. The sample complexity of pattern classification with neural networks: the size of the weights is more important than the size of the network. *IEEE transactions on Information Theory*, 44, 1998.
- [29] Peter L Bartlett and Philip M Long. Prediction, learning, uniform convergence, and scale-sensitive dimensions. *Journal of Computer and System Sciences*, 56, 1998.
- [30] Bela Bauer, Sergey Bravyi, Mario Motta, and Garnet Kin Chan. Quantum algorithms for quantum chemistry and quantum materials science. *Chemical Reviews*, 120:12685–12717, 2020.
- [31] Marcello Benedetti, Erika Lloyd, Stefan Sack, and Mattia Fiorentini. Parameterized quantum circuits as machine learning models. *Quantum Science and Technology*, 4, 2019.
- [32] Dominic W Berry, Andrew M Childs, and Robin Kothari. Hamiltonian simulation with nearly optimal dependence on all parameters. *Proceedings of 56th Annual Symposium on Foundations of Computer Science*, 2015.
- [33] Dominic W Berry, Yuan Su, Casper Gyurik, Robbie King, Joao Basso, Alexander Del Toro Barba, Abhishek Rajput, Nathan Wiebe, Vedran Dunjko, and Ryan Babbush. Quantifying quantum advantage in topological data analysis. *arXiv preprint arXiv:2209.13581*, 2022.
- [34] Jacob Biamonte, Mauro Faccin, and Manlio De Domenico. Complex networks from classical to quantum. *Communications Physics*, 2:1–10, 2019.
- [35] Jacob Biamonte, Peter Wittek, Nicola Pancotti, Patrick Rebentrost, Nathan Wiebe, and Seth Lloyd. Quantum machine learning. *Nature*, 549, 2017.

- [36] Manuel Blum and Silvio Micali. How to generate cryptographically strong sequences of pseudorandom bits. *SIAM journal on Computing*, 13, 1984.
- [37] Andrej Bogdanov and Luca Trevisan. Average-case complexity. *Theoretical Computer Science*, 2006.
- [38] Xavi Bonet-Monroig, Ramiro Sagastizabal, M Singh, and TE O’Brien. Low-cost error mitigation by symmetry verification. *Physical Review A*, 2018.
- [39] Adam D Bookatz. Qma-complete problems. *Quantum Information & Computation*, 14:361–383, 2014.
- [40] Fernando GSL Brandão. *Entanglement theory and the quantum simulation of many-body physics*. PhD thesis, University of London, 2008.
- [41] Michael J Bremner, Richard Jozsa, and Dan J Shepherd. Classical simulation of commuting quantum computations implies collapse of the polynomial hierarchy. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 467, 2011.
- [42] Michael J Bremner, Ashley Montanaro, and Dan J Shepherd. Average-case complexity versus approximate simulation of commuting quantum computations. *Physical review letters*, 117, 2016.
- [43] Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. Openai gym. *arXiv preprint arXiv:1606.01540*, 2016.
- [44] Michael Broughton, Guillaume Verdon, Trevor McCourt, Antonio J Martinez, Jae Hyeon Yoo, Sergei V Isakov, Philip Massey, Murphy Yuezhen Niu, Ramin Halavati, Evan Peters, et al. Tensorflow quantum: A software framework for quantum machine learning. *arXiv preprint arXiv:2003.02989*, 2020.
- [45] Brielin Brown, Steven T Flammia, and Norbert Schuch. Computational difficulty of computing the density of states. *Physical review letters*, 2011.
- [46] Kaifeng Bu, Dax Enshan Koh, Lu Li, Qingxian Luo, and Yaobo Zhang. On the statistical complexity of quantum circuits. *arXiv preprint*, 2021.
- [47] Kaifeng Bu, Dax Enshan Koh, Lu Li, Qingxian Luo, and Yaobo Zhang. Rademacher complexity of noisy quantum circuits. *arXiv preprint*, 2021.
- [48] Chris Cade and P Marcos Crichigno. Complexity of supersymmetric systems and the cohomology problem. *arXiv preprint*, 2021.
- [49] Chris Cade and P Marcos Crichigno. Complexity of supersymmetric systems and the cohomology problem. *arXiv 2107.00011*, 2021.
- [50] Chris Cade, Marten Folkertsma, and Jordi Weggemans. Complexity of the guided local hamiltonian problem: improved parameters and extension to excited states. *arXiv 2207.10097*, 2022.

- [51] Chris Cade and Ashley Montanaro. The quantum complexity of computing Schatten p -norms. *13th Conference on the Theory of Quantum Computation, Communication and Cryptography*, 2018.
- [52] Matthias C Caro and Ishaun Datta. Pseudo-dimension of quantum circuits. *Quantum Machine Intelligence*, 2, 2020.
- [53] Matthias C Caro, Elies Gil-Fuster, Johannes Jakob Meyer, Jens Eisert, and Ryan Sweke. Encoding-dependent generalization bounds for parametrized quantum circuits. *Quantum*, 5, 2021.
- [54] Berta Casas and Alba Cervera-Lierta. Multi-dimensional fourier series with quantum circuits. *arXiv 2302.03389*, 2023.
- [55] Shantanav Chakraborty, András Gilyén, and Stacey Jeffery. The power of block-encoded matrix powers: Improved regression techniques via faster hamiltonian simulation. *46th International Colloquium on Automata, Languages, and Programming (ICALP 2019)*, 2019.
- [56] Jianer Chen, Xiuzhen Huang, Iyad A Kanj, and Ge Xia. Strong computational lower bounds via parameterized complexity. *Journal of Computer and System Sciences*, 72:1346–1367, 2006.
- [57] Lijie Chen and Roei Tell. Guest column: New ways of studying the BPP=P conjecture. *ACM SIGACT News*, 54:44–69, 2023.
- [58] Samuel Yen-Chi Chen, Chao-Han Huck Yang, Jun Qi, Pin-Yu Chen, Xiaoli Ma, and Hsi-Sheng Goan. Variational quantum circuits for deep reinforcement learning. *IEEE Access*, 8:141007–141024, 2020.
- [59] Ho Yee Cheung, Tsz Chiu Kwok, and Lap Chi Lau. Fast matrix rank algorithms and applications. *Journal of the ACM (JACM)*, 60:1–25, 2013.
- [60] Nai-Hui Chia, András Gilyén, Tongyang Li, Han-Hsuan Lin, Ewin Tang, and Chunhao Wang. Sampling-based sublinear low-rank matrix arithmetic framework for dequantizing quantum machine learning. *Proceedings of the 52nd Annual ACM SIGACT symposium on theory of computing*, 2020.
- [61] Andrew M Childs, David Gosset, and Zak Webb. The Bose-Hubbard model is QMA-complete. *International Colloquium on Automata, Languages, and Programming*, 2014.
- [62] Andrew M Childs, David Gosset, and Zak Webb. The bose-hubbard model is QMA-complete. In *International Colloquium on Automata, Languages, and Programming*. Springer, 2014.
- [63] Andrew Macgregor Childs. *Quantum information processing in continuous time*. PhD thesis, Massachusetts Institute of Technology, 2004.
- [64] Richard Cleve, Artur Ekert, Chiara Macchiavello, and Michele Mosca. Quantum algorithms revisited. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 454(1969):339–354, 1998.

- [65] David Cohen-Steiner, Weihao Kong, Christian Sohler, and Gregory Valiant. Approximating the spectrum of a graph. *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, 2018.
- [66] Corinna Cortes and Vladimir Vapnik. Support-vector networks. *Machine learning*, 20:273–297, 1995.
- [67] Marcos Crichigno and Tamara Kohler. Clique homology is qma1-hard. *arXiv preprint arXiv:2209.11793*, 2022.
- [68] Manlio De Domenico and Jacob Biamonte. Spectral entropies as information-theoretic tools for complex network comparison. *Physical Review X*, 6, 2016.
- [69] Ronald de Wolf. Quantum computing: Lecture notes. *arXiv:1907.09415*, 2019.
- [70] Edoardo Di Napoli, Eric Polizzi, and Yousef Saad. Efficient estimation of eigenvalue counts in an interval. *Numerical Linear Algebra with Applications*, 23:674–692, 2016.
- [71] Yan Duan, Xi Chen, Rein Houthooft, John Schulman, and Pieter Abbeel. Benchmarking deep reinforcement learning for continuous control. In *International conference on machine learning*, pages 1329–1338. PMLR, 2016.
- [72] Vedran Dunjko, Yi-Kai Liu, Xingyao Wu, and Jacob M Taylor. Exponential improvements for quantum-accessible reinforcement learning. *arXiv preprint arXiv:1710.11160*, 2017.
- [73] Vedran Dunjko and Peter Wittek. A non-review of quantum machine learning: trends and explorations. *Quantum*, 4:32, 2020.
- [74] Alicja Dutkiewicz, Barbara M Terhal, and Thomas E O’Brien. Heisenberg-limited quantum phase estimation of multiple eigenvalues with a single control qubit. *arXiv preprint*, 2021.
- [75] Art Duval, Caroline Klivans, and Jeremy Martin. Simplicial matrix-tree theorems. *Transactions of the American Mathematical Society*, 361:6073–6114, 2009.
- [76] Beno Eckmann. Harmonische Funktionen und Randwertaufgaben in einem Komplex. *Commentarii Mathematici Helvetici*, 17:240–255, 1944.
- [77] Talya Eden, Dana Ron, and Will Rosenbaum. Almost Optimal Bounds for Sublinear-Time Sampling of k-Cliques in Bounded Arboricity Graphs. *49th International Colloquium on Automata, Languages, and Programming – ICALP*, 2022.
- [78] Suguru Endo, Simon C Benjamin, and Ying Li. Practical quantum error mitigation for near-future applications. *Physical Review X*, 2018.
- [79] Joan Feigenbaum and Lance Fortnow. Random-self-reducibility of complete sets. *SIAM Journal on Computing*, 22:994–1005, 1993.

- [80] Xinlong Feng and Zhinan Zhang. The rank of a random matrix. *Applied mathematics and computation*, 185:689–694, 2007.
- [81] Joel Friedman. Computing Betti numbers via combinatorial Laplacians. *Algorithmica*, 21:331–346, 1998.
- [82] Sevag Gharibian, Ryu Hayakawa, François Le Gall, and Tomoyuki Morimae. Improved hardness results for the guided local hamiltonian problem. *arXiv 2207.10250*, 2022.
- [83] Robert Ghrist. Barcodes: the persistent topology of data. *Bulletin of the American Mathematical Society*, 45:61–75, 2008.
- [84] András Gilyén and Tongyang Li. Distributional property testing in a quantum world. *11th Innovations in Theoretical Computer Science Conference (ITCS 2020)*, 2020.
- [85] András Gilyén, Yuan Su, Guang Hao Low, and Nathan Wiebe. Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics. *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing*, 2019.
- [86] Timothy E Goldberg. Combinatorial laplacians of simplicial complexes. Master’s thesis, Bard College, 2002.
- [87] Shafi Goldwasser, Silvio Micali, and Po Tong. Why and how to establish a private code on a public network. In *23rd Annual Symposium on Foundations of Computer Science (SFCS 1982)*, pages 134–144. IEEE, 1982.
- [88] Google. Cirq: A python framework for creating, editing, and invoking noisy intermediate scale quantum circuits. URL: [github.com/quantumlib/Cirq](https://github.com/google/cirq), 2018.
- [89] Takahiro Goto, Quoc Hoan Tran, and Kohei Nakajima. Universal approximation property of quantum machine learning models in quantum-enhanced feature spaces. *Physical Review Letters*, 127(9):090506, 2021.
- [90] Kiya W Govek, Venkata S Yamajala, and Pablo G Camara. Clustering-independent analysis of genomic data using spectral simplicial theory. *PLoS computational biology*, 2019.
- [91] Evan Greensmith, Peter L Bartlett, and Jonathan Baxter. Variance reduction techniques for gradient estimates in reinforcement learning. *Journal of Machine Learning Research*, 5(Nov):1471–1530, 2004.
- [92] Anna Gundert and May Szegláky. Higher dimensional discrete Cheeger inequalities. *Proceedings of the 13th annual symposium on Computational Geometry*, 2014.
- [93] Sam Gunn and Niels Kornerup. Review of a quantum algorithm for Betti numbers. *arXiv preprint*, 2019.

- [94] Casper Gyurik and Vedran Dunjko. On establishing learning separations between classical and quantum machine learning with classical data. *arXiv 2208.06339*, 2022.
- [95] Casper Gyurik, Vedran Dunjko, et al. Structural risk minimization for quantum linear classifiers. *Quantum*, 7:893, 2023.
- [96] Jeongwan Haah, Matthew B Hastings, Robin Kothari, and Guang Hao Low. Quantum algorithm for simulating real time evolution of lattice hamiltonians. *SIAM Journal on Computing*, 2021.
- [97] Jeongwan Haah, Robin Kothari, and Ewin Tang. Optimal learning of quantum hamiltonians from high-temperature gibbs states. *arXiv 2108.04842*, 2021.
- [98] Jeongwan Haah, Robin Kothari, and Ewin Tang. Optimal learning of quantum hamiltonians from high-temperature gibbs states. In *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 135–146. IEEE, 2022.
- [99] Nathan Halko, Per-Gunnar Martinsson, and Joel A Tropp. Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions. *SIAM review*, 53:217–288, 2011.
- [100] Aram W Harrow, Avinatan Hassidim, and Seth Lloyd. Quantum algorithm for linear systems of equations. *Physical review letters*, 103:150502, 2009.
- [101] Aram W Harrow, Avinatan Hassidim, and Seth Lloyd. Quantum algorithm for linear systems of equations. *Physical review letters*, 103, 2009.
- [102] Vojtěch Havlíček, Antonio D Córcoles, Kristan Temme, Aram W Harrow, Abhinav Kandala, Jerry M Chow, and Jay M Gambetta. Supervised learning with quantum-enhanced feature spaces. *Nature*, 567:209–212, 2019.
- [103] Vojtěch Havlíček, Antonio D Córcoles, Kristan Temme, Aram W Harrow, Abhinav Kandala, Jerry M Chow, and Jay M Gambetta. Supervised learning with quantum-enhanced feature spaces. *Nature*, 567, 2019.
- [104] Vojtěch Havlíček, Antonio D Córcoles, Kristan Temme, Aram W Harrow, Abhinav Kandala, Jerry M Chow, and Jay M Gambetta. Supervised learning with quantum-enhanced feature spaces. *Nature*, 567(7747):209–212, 2019.
- [105] Danijela Horak and Jürgen Jost. Spectra of combinatorial Laplace operators on simplicial complexes. *Advances in Mathematics*, 244:303–336, 2013.
- [106] He-Liang Huang, Xi-Lin Wang, Peter P Rohde, Yi-Han Luo, You-Wei Zhao, Chang Liu, Li Li, Nai-Le Liu, Chao-Yang Lu, and Jian-Wei Pan. Demonstration of topological data analysis on a quantum processor. *Optica*, 5:193–198, 2018.
- [107] Hsin-Yuan Huang, Richard Kueng, Giacomo Torlai, Victor V Albert, and John Preskill. Provably efficient machine learning for quantum many-body problems. *Science*, 377, 2022.

- [108] Hsin-Yuan Huang, Yu Tong, Di Fang, and Yuan Su. Learning many-body hamiltonians with heisenberg-limited scaling. *arXiv* **2210.03030**, 2022.
- [109] HY Huang, M Broughton, M Mohseni, R Babbush, S Boixo, H Neven, and JR McClean. Power of data in quantum machine learning (2020). *Nature Communications*, 2021.
- [110] Sofiene Jerbi, Lea M. Trenkwalder, Hendrik Poulsen Nautrup, Hans J. Briegel, and Vedran Dunjko. Quantum enhancements for deep reinforcement learning in large spaces. *PRX Quantum*, 2:010328, Feb 2021.
- [111] Ian T Jolliffe. *Principal components in regression analysis*, pages 129–155. Springer, 1986.
- [112] Stephen P Jordan, Hari Krovi, Keith SM Lee, and John Preskill. Bqp-completeness of scattering in scalar quantum field theory. *Quantum*, 2:44, 2018.
- [113] Abhinav Kandala, Antonio Mezzacapo, Kristan Temme, Maika Takita, Markus Brink, Jerry M Chow, and Jay M Gambetta. Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets. *Nature*, 549(7671):242–246, 2017.
- [114] Tosio Kato. *Perturbation theory for linear operators*, volume 132. Springer Science & Business Media, 2013.
- [115] Michael Kearns and Leslie Valiant. Cryptographic limitations on learning boolean formulae and finite automata. *Journal of the ACM (JACM)*, 1994.
- [116] Michael Kearns and Umesh Vazirani. *An introduction to computational learning theory*. MIT press, 1994.
- [117] Michael J Kearns and Robert E Schapire. Efficient distribution-free learning of probabilistic concepts. *Journal of Computer and System Sciences*, 48, 1994.
- [118] Iordanis Kerenidis and Anupam Prakash. Quantum recommendation systems. *Proceedings of the 8th Innovations in Theoretical Computer Science Conference*, 2017.
- [119] Iordanis Kerenidis and Anupam Prakash. Quantum gradient descent for linear systems and least squares. *Physical Review A*, 101:022316, 2020.
- [120] A Yu Kitaev. Quantum measurements and the abelian stabilizer problem. *quant-ph/9511026*, 1995.
- [121] Alexei Yu Kitaev, Alexander Shen, Mikhail N Vyalyi, and Mikhail N Vyalyi. *Classical and quantum computation*. American Mathematical Society, 2002.
- [122] Emanuel Knill and Raymond Laflamme. Power of one bit of quantum information. *Physical Review Letters*, 1998.

- [123] Timothy P Lillicrap, Jonathan J Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. *arXiv preprint arXiv:1509.02971*, 2015.
- [124] Lin Lin, Yousef Saad, and Chao Yang. Approximating spectral densities of large matrices. *SIAM review*, 58:34–65, 2016.
- [125] Yi-Kai Liu, Matthias Christandl, and Frank Verstraete. Quantum computational complexity of the N-representability problem: Qma complete. *Physical review letters*, 98:110503, 2007.
- [126] Yunchao Liu, Srinivasan Arunachalam, and Kristan Temme. A rigorous and robust quantum speed-up in supervised machine learning. *Nature Physics*, 2021.
- [127] Yunchao Liu, Srinivasan Arunachalam, and Kristan Temme. A rigorous and robust quantum speed-up in supervised machine learning. *Nature Physics*, 17(9):1013–1017, 2021.
- [128] Yunchao Liu, Srinivasan Arunachalam, and Kristan Temme. A rigorous and robust quantum speed-up in supervised machine learning. *Nature Physics*, 17(9):1013–1017, 2021.
- [129] Seth Lloyd. Universal quantum simulators. *Science*, pages 1073–1078, 1996.
- [130] Seth Lloyd, Silvano Garnerone, and Paolo Zanardi. Quantum algorithms for topological and geometric analysis of data. *Nature communications*, 7:1–7, 2016.
- [131] Owen Lockwood and Mei Si. Reinforcement learning with quantum variational circuit. In *Proceedings of the AAAI Conference on Artificial Intelligence and Interactive Digital Entertainment*, volume 16, pages 245–251, 2020.
- [132] László Lovász et al. Very large graphs. *Current Developments in Mathematics*, 2008:67–128, 2009.
- [133] Guang Hao Low and Isaac L Chuang. Optimal hamiltonian simulation by quantum signal processing. *Physical review letters*, 118:010501, 2017.
- [134] Slobodan Maletić and Milan Rajković. Combinatorial Laplacian and entropy of simplicial complexes associated with complex networks. *The European Physical Journal Special Topics*, 212:77–97, 2012.
- [135] Sam McArdle, András Gilyén, and Mario Berta. A streamlined quantum algorithm for topological data analysis with exponentially fewer qubits. *arXiv:2209.12887*, 2022.
- [136] Sam McArdle, Xiao Yuan, and Simon Benjamin. Error-mitigated digital quantum simulation. *Physical review letters*, 2019.
- [137] Kosuke Mitarai, Makoto Negoro, Masahiro Kitagawa, and Keisuke Fujii. Quantum circuit learning. *Physical Review A*, 98(3):032309, 2018.

- [138] Volodymyr Mnih, Adria Puigdomenech Badia, Mehdi Mirza, Alex Graves, Timothy Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous methods for deep reinforcement learning. In *International conference on machine learning*, pages 1928–1937. PMLR, 2016.
- [139] Mehryar Mohri, Afshin Rostamizadeh, and Ameet Talwalkar. *Foundations of machine learning*. MIT press, 2018.
- [140] J.W. Moon and Moser L. On a problem of turan. *Publ. Math. Inst. Hung. Acad. Sci.*, 1962.
- [141] Tomoyuki Morimae. Hardness of classically sampling the one-clean-qubit model with constant total variation distance error. *Physical Review A*, 2017.
- [142] Tomoyuki Morimae, Keisuke Fujii, and Joseph F Fitzsimons. Hardness of classically simulating the one-clean-qubit model. *Physical review letters*, 2014.
- [143] Sayan Mukherjee and John Steenbergen. Random walks on simplicial complexes and harmonics. *Random structures & algorithms*, 49:379–405, 2016.
- [144] Ken Nakanishi, Keisuke Fujii, and Synge Todo. Sequential minimal optimization for quantum-classical hybrid algorithms. *Physical Review Research*, 2, 2020.
- [145] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, 2011.
- [146] Thomas O’Brien, LevC Ioffe, Yuan Su, David Fushman, Hartmut Neven, Ryan Babbush, and Vadim Smelyanskiy. Quantum computation of molecular structure using data from challenging-to-classically-simulate nuclear magnetic resonance experiments. *arXiv 2109.02163*, 2021.
- [147] Thomas E O’Brien, Stefano Polla, Nicholas C Rubin, William J Huggins, Sam McArdle, Sergio Boixo, Jarrod R McClean, and Ryan Babbush. Error mitigation via verified phase estimation. *Physical review X Quantum*, 2021.
- [148] Thomas E. O’Brien, Brian Tarasinski, and Barbara Terhal. Quantum phase estimation of multiple eigenvalues for small-scale (noisy) experiments. *New Journal of Physics*, 2019.
- [149] Bryan O’Gorman, Sandy Irani, James Whitfield, and Bill Fefferman. Electronic structure in a fixed basis is qma-complete. *Physical review X Quantum*, 2021.
- [150] Bryan O’Gorman, Sandy Irani, James Whitfield, and Bill Fefferman. Electronic structure in a fixed basis is qma-complete. *arXiv 2103.08215*, 2021.
- [151] OpenAI. Leaderboard of openai gym environments. URL: github.com/openai/gym/wiki, 2020.
- [152] Braxton Osting, Sourabh Palande, and Bei Wang. Spectral sparsification of simplicial complexes for clustering and label propagation. *Journal of Computational Geometry*, 2017.

- [153] Jae-Eun Park, Brian Quanz, Steve Wood, Heather Higgins, and Ray Harishankar. Practical application improvement to quantum svm: theory to practice. *arXiv preprint*, 2020.
- [154] Ori Parzanchevski and Ron Rosenthal. Simplicial complexes: spectrum, homology and random walks. *Random Structures & Algorithms*, 50:225–261, 2017.
- [155] Filippo Passerini and Simone Severini. Quantifying complexity in networks: the von Neumann entropy. *International Journal of Agent Technologies and Systems (IJATS)*, 1:58–67, 2009.
- [156] Jordi Pérez-Guijarro, Alba Pagès-Zamora, and Javier R Fonollosa. Relation between quantum advantage in supervised learning and quantum computational advantage. *arXiv 2304.06687*, 2023.
- [157] Adrián Pérez-Salinas, Alba Cervera-Lierta, Elies Gil-Fuster, and José I Latorre. Data re-uploading for a universal quantum classifier. *Quantum*, 4:226, 2020.
- [158] Adrián Pérez-Salinas, David López-Núñez, Artur García-Sáez, Pol Forn-Díaz, and José I Latorre. One qubit as a universal approximant. *Physical Review A*, 104(1):012405, 2021.
- [159] Stephen Piddock and Ashley Montanaro. The complexity of antiferromagnetic interactions and 2d lattices. *Quantum Information & Computation*, 17:636–672, 2017.
- [160] John Preskill. Quantum computing in the NISQ era and beyond. *Quantum*, 2, 2018.
- [161] TensorFlow Quantum. Parametrized quantum circuits for reinforcement learning. <https://tinyurl.com/ycy58267>, 2021.
- [162] Christian Reiher. The clique density theorem. *Annals of Mathematics*, 2016.
- [163] Mathys Rennela, Sebastiaan Brand, Alfons Laarman, and Vedran Dunjko. Hybrid divide-and-conquer approach for tree search algorithms. *Quantum*, 7:959, 2023.
- [164] Bernhard Schölkopf, Alexander J Smola, Francis Bach, et al. *Learning with kernels: support vector machines, regularization, optimization, and beyond*. MIT press, 2002.
- [165] Maria Schuld. Supervised quantum machine learning models are kernel methods. *arXiv preprint*, 2021.
- [166] Maria Schuld, Ville Bergholm, Christian Gogolin, Josh Izaac, and Nathan Kilorian. Evaluating analytic gradients on quantum hardware. *Physical Review A*, 99(3):032331, 2019.
- [167] Maria Schuld, Alex Bocharov, Krysta M Svore, and Nathan Wiebe. Circuit-centric quantum classifiers. *Physical Review A*, 101:032308, 2020.

- [168] Maria Schuld and Nathan Killoran. Quantum machine learning in feature Hilbert spaces. *Physical review letters*, 122, 2019.
- [169] Maria Schuld and Nathan Killoran. Quantum machine learning in feature hilbert spaces. *Physical review letters*, 122(4):040504, 2019.
- [170] Maria Schuld, Ryan Sweke, and Johannes Jakob Meyer. Effect of data encoding on the expressive power of variational quantum-machine-learning models. *Physical Review A*, 103(3):032430, 2021.
- [171] Maria Schuld, Ryan Sweke, and Johannes Jakob Meyer. Effect of data encoding on the expressive power of variational quantum-machine-learning models. *Physical Review A*, 103, 2021.
- [172] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*, 2017.
- [173] Rocco Servedio and Steven J Gortler. Equivalences and separations between quantum and classical learnability. *SIAM Journal on Computing*, 2004.
- [174] Shai Shalev-Shwartz and Shai Ben-David. *Understanding machine learning: From theory to algorithms*. Cambridge university press, 2014.
- [175] John Shawe-Taylor, Peter L. Bartlett, Robert C. Williamson, and Martin Anthony. Structural risk minimization over data-dependent hierarchies. *IEEE Transactions on Information Theory*, 1998.
- [176] Peter Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM review*, 41, 1999.
- [177] Peter W Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM review*, 41(2):303–332, 1999.
- [178] Peter W Shor and Stephen P Jordan. Estimating Jones polynomials is a complete problem for one clean qubit. *Quantum Information & Computation*, 8:681–714, 2008.
- [179] David Simmons, Justin Coon, and Animesh Datta. The quantum Theil index: characterizing graph centralization using von Neumann entropy. *Journal of Complex Networks*, 6:859–876, 2018.
- [180] Rolando D Somma. Quantum eigenvalue estimation via time series analysis. *New Journal of Physics*, 2019.
- [181] Sathyawageeswar Subramanian and Min-Hsiu Hsieh. Quantum algorithm for estimating renyi entropies of quantum states. *Physical review A*, 2021.
- [182] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.

- [183] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.
- [184] Yasunari Suzuki, Yoshiaki Kawase, Yuya Masumura, Yuria Hiraga, Masahiro Nakadai, Jiabao Chen, Ken M Nakanishi, Kosuke Mitarai, Ryosuke Imai, Shiro Tamiya, et al. Qulacs: a fast and versatile quantum circuit simulator for research purpose. *arXiv preprint arXiv:2011.13524*, 2020.
- [185] Ryan Sweke, Jean-Pierre Seifert, Dominik Hangleiter, and Jens Eisert. On the quantum versus classical learnability of discrete distributions. *Quantum*, 5, 2021.
- [186] Ewin Tang. A quantum-inspired classical algorithm for recommendation systems. *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing*, 2019.
- [187] Kristan Temme, Sergey Bravyi, and Jay M Gambetta. Error mitigation for short-depth quantum circuits. *Physical review letters*, 2017.
- [188] Barbara M Terhal and David P DiVincenzo. Adaptive quantum computation, constant depth quantum circuits and arthur-merlin games. *Quantum Information & Computation*, 4, 2004.
- [189] Shashanka Ubaru, Ismail Yunus Akhalwaya, Mark S Squillante, Kenneth L Clarkson, and Lior Horesh. Quantum topological data analysis with linear depth and exponential speedup. *arXiv preprint*, 2021.
- [190] Shashanka Ubaru, Yousef Saad, and Abd-Krim Seghouane. Fast estimation of approximate matrix ranks using spectral densities. *Neural computation*, 29:1317–1351, 2017.
- [191] Johan Ugander, Lars Backstrom, and Jon Kleinberg. Subgraph frequencies: Mapping the empirical and extremal geography of large graph collections. *Proceedings of the 22nd international conference on World Wide Web*, 2013.
- [192] Gregory Valiant and Paul Valiant. Estimating the unseen: an $n/\log(n)$ -sample estimator for entropy and support size, shown optimal via new clts. *Proceedings of the forty-third annual ACM symposium on Theory of computing*, 2011.
- [193] Dyon van Vreumingen. Quantum feature space learning: characterisation and possible advantages. Master’s thesis, Leiden University, 8 2020. <https://studenttheses.universiteitleiden.nl/handle/1887/2734545>.
- [194] Vladimir N Vapnik and A Ya Chervonenkis. On the uniform convergence of relative frequencies of events to their probabilities. In *Measures of complexity*. Springer, 2015.
- [195] Rui Wang, Duc Duy Nguyen, and Guo-Wei Wei. Persistent spectral graph. *International journal for numerical methods in biomedical engineering*, 36:e3376, 2020.

- [196] Jordi Weggemans, Marten Folkertsma, and Chris Cade. Guidable local hamiltonian problems with implications to heuristic ansatz state preparation and the quantum pcg conjecture. *arXiv* **2302.11578**, 2023.
- [197] Tzu-Chieh Wei, Michele Mosca, and Ashwin Nayak. Interacting boson problems can be qma hard. *Physical review letters*, 104, 2010.
- [198] Lilian Weng. Policy gradient algorithms. URL: lilianweng.github.io/lil-log, 2018.
- [199] Ronald J Williams. Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine learning*, 8(3-4):229–256, 1992.
- [200] Pawel Wocjan and Shengyu Zhang. Several natural BQP-complete problems. *arXiv preprint*, 2006.
- [201] Michael M Wolf. Mathematical foundations of supervised learning. https://www-m5.ma.tum.de/foswiki/pub/M5/Allgemeines/MA4801_2020S/ML_notes_main.pdf, 2020.
- [202] Shaojun Wu, Shan Jin, Dingding Wen, and Xiaoting Wang. Quantum reinforcement learning in continuous action space. *arXiv preprint arXiv:2012.10711*, 2020.