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Computational speedups and learning separations in quantum machine learning

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Computational speedups and learning separations in quantum machine learning

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Contents

Acknowledgements	vii
1 Introduction	1
1.1 Problem statement and research questions	1
1.2 List of contributions	3
2 Background and definitions	6
2.1 Quantum computing	6
2.1.1 Basics of quantum algorithms	10
2.1.2 Quantum complexity theory	13
2.1.3 Quantum linear classifiers	15
2.2 Topological data analysis	18
2.2.1 Betti numbers as features	18
2.2.2 Quantum algorithm for Betti number estimation	21
2.2.3 Classical algorithms for Betti number estimation	26
2.3 Structural risk minimization	27
2.4 Reinforcement learning	31
2.5 Computational learning theory	33
2.5.1 Learning separations in the PAC learning framework	33
2.5.2 Complexity theory	39
3 Towards quantum advantage via topological data analysis	44
3.1 Problem definitions	44
3.1.1 Relationships between the problems	48
3.2 Classical intractability of LLSD	49
3.2.1 The one clean qubit model of computation	51
3.2.2 Hardness of LLSD for the one clean qubit model	51
3.2.3 Closing the gap for classical intractability of ABNE	53
3.2.4 Graphs with quantum speedup	55
3.3 Quantum speedups beyond Betti numbers	59
3.3.1 Numerical rank estimation	59
3.3.2 Combinatorial Laplacians beyond Betti numbers	62
3.4 Possibilities and challenges for implementations	66

4	Structural risk minimization for quantum linear classifiers	70
4.1	Complexity of quantum linear classifiers	70
4.2	Expressivity of quantum linear classifiers	74
4.3	Structural risk minimization in practice	76
5	Parametrized quantum policies for reinforcement learning	81
5.1	Parametrized quantum policies	81
5.1.1	The RAW-PQC and SOFTMAX-PQC policies	81
5.1.2	Learning algorithm	83
5.1.3	Efficient policy sampling and policy-gradient evaluation	84
5.2	Performance comparison in benchmarking environments	85
5.2.1	RAW-PQC v.s. SOFTMAX-PQC	85
5.2.2	Influence of architectural choices	87
5.3	Quantum advantage of PQC agents in RL environments	87
5.3.1	Quantum advantage of PQC agents over classical agents	88
5.3.2	Quantum advantage of PQC agents over DNN agents	89
6	Exponential separations between classical and quantum learners	92
6.1	Learning separations with efficient data generation	93
6.1.1	A learning separation based on a worst-case to average-case reduction	94
6.1.2	A learning separation based on obfuscation	95
6.1.3	A learning separation with efficiently evaluable concepts	97
6.1.4	A learning separation with a fixed hypothesis class	99
6.2	Learning separations without efficient data generation	100
6.2.1	Learning separations from physical systems	103
6.3	Connections to other works on (quantum) learning tasks	104
6.3.1	Provably efficient machine learning with classical shadows	105
6.3.2	Power of data	106
6.3.3	Physically-motivated PAC learning settings with fixed hypothesis classes	107
7	Conclusion	110
7.1	Research overview	110
7.2	Limitations	112
7.3	Future work	112
	Bibliography	114
	Summary	128
	Samenvatting	130
	Curriculum Vitae	133

A	Towards quantum advantage via topological data analysis	134
A.1	LLSD is DQC1-hard	134
A.2	Quantum algorithms for SUES and LLSD	137
A.2.1	Quantum algorithm for SUES	137
A.2.2	Quantum algorithm for LLSD	139
A.3	Betti number and spectral gap calculations	143
A.4	SWES is DQC1-hard	144
B	Structural risk minimization for quantum linear classifiers	146
B.1	Proofs of Section 4.1	146
B.1.1	Proofs of Proposition 12 and Lemma 13	146
B.1.2	Relationship Proposition 12 and ranks of observables	147
B.1.3	Proof of Proposition 14	149
B.2	Proofs of propositions Section 4.2	151
B.2.1	Proof of Proposition 15	151
B.2.2	Proof of Proposition 16	157
B.2.3	Proof of Proposition 17	158
C	Parametrized quantum policies for reinforcement learning	160
C.1	Derivation of the log-policy gradient	160
C.2	Efficient implementation of SOFTMAX-PQC policies	160
C.2.1	Efficient approximate policy sampling	160
C.2.2	Efficient estimation of the log-policy gradient	161
C.3	Role of trainable observables in SOFTMAX-PQC	163
C.3.1	Training the eigenbasis and the eigenvalues	163
C.3.2	The power of universal observables	163
C.4	Environments specifications and hyperparameters	164
C.5	Deferred plots and shape of policies PQCs vs. DNNs	165
C.5.1	Influence of architectural choices on RAW-PQC	165
C.5.2	Shape of the policies learned by PQCs v.s. DNNs	168
C.5.3	Additional simulations on CognitiveRadio	168
C.6	Supervised learning task of Liu <i>et al.</i>	172
C.7	Proof of Theorem 20	172
C.8	Proof of Lemma 46	175
C.8.1	Upper bound on the value function	176
C.8.2	Lower bound on the value function	177
C.8.3	Bounds on classical- vs. and quantum- learnability	177
C.9	Proof of Lemma 47	179
C.9.1	Proof of classical hardness	179
C.9.2	Proof of quantum learnability	181
C.10	Construction of PQC agent for the DLP environments	181
C.10.1	Implicit vs. explicit quantum SVMs	182
C.10.2	Description of the PQC classifier	182
C.10.3	Noisy classifier	183

C.11 Proof of trainability of PQC agent in the SL-DLP	184
D Exponential separations between classical and quantum learners	189
D.1 Details regarding definitions	189
D.1.1 Constraining hypothesis classes to those that are efficiently eval- uatable	189
D.1.2 Proof of Lemma 3	189
D.1.3 Proof of Lemma 4	190
D.2 Proof of Theorem 24	191
D.2.1 Discrete cube root assumption for moduli of Definition 19 . . .	193
D.3 Proof of Theorem 25	194
D.4 Proof of Theorem 26	195
D.4.1 Proof of Lemma 27	196
D.4.2 Proof of Lemma 28	197
D.5 Proof of Theorem 30	197

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