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## Computational speedups and learning separations in quantum machine learning

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# Computational speedups and learning separations in quantum machine learning

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