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The power of one qubit in quantum simulation algorithms

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The power of one qubit in quantum simulation algorithms

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On the cover: *an artistic representation of quantum computing emerging as a new technology, inspired by the rising cities painted by the futurist movement.* — by Jeanne M. Viet [Miss J Art]

To those who brightened my hardest days

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