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The power of one qubit in quantum simulation algorithms

Polla, S.

Citation

Polla, S. (2024, February 22). *The power of one qubit in quantum simulation algorithms*. *Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/3719849>

Version: Publisher's Version

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Note: To cite this publication please use the final published version (if applicable).

The power of one qubit in quantum simulation algorithms

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof. dr. ir. H. Bijl
volgens besluit van het college voor promoties
te verdedigen op donderdag 22 februari 2024
klokke 15:00 uur

door

Stefano Polla

geboren te Bergamo, Italië
in 1994

Promotor: Prof. dr. C. W. J. Beenakker
Co-promotor: Dr. T. E. O'Brien

Promotiecommissie: Prof. Dr. J. Aarts
Dr. A. Roggero (Università di Trento)
Prof. Dr. K. E. Schalm
Prof. Dr. B. M. Terhal (Technische Universiteit Delft)
Prof. Dr. L. Visscher (Vrije Universiteit Amsterdam)

Casimir PhD series, Delft-Leiden 2023-42

ISBN 978-90-8593-587-2

An electronic version of this thesis can be found
at <https://openaccess.leidenuniv.nl>

On the cover: *an artistic representation of quantum computing emerging as a new technology, inspired by the rising cities painted by the futurist movement.* — by Jeanne M. Viet [Miss J Art]

To those who brightened my hardest days

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