Hydrodynamical simulations of misaligned accretion discs in binary systems: companions tear discs

Doğan, S.; Nixon, C.J.; King, A.R.; Pringle, J.E.; Price, D.; Biskalo, D.; ... ; Boily, C.

The Predictive Power of Computational Astrophysics as a Discovery Tool
Proceedings IAU Symposium No. 362, 2023
D. Bisikalo, D. Wiebe & C. Boily, eds.
doi:10.1017/S1743921322001387

Hydrodynamical Simulations of Misaligned Accretion Discs in Binary Systems: Companions tear discs

S. Doğan[1], C. J. Nixon[2], A. R. King[2,3,4], J. E. Pringle[2,5] and D. Price[6]

[1]Department of Astronomy & Space Sciences, University of Ege, Bornova, İzmir, Turkey

[2]Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH, UK

[3]Anton Pannekoek Institute, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, Netherlands

[4]Leiden Observatory, Leiden University, Niels Bohrweg 2, NL-2333 CA Leiden, Netherlands

[5]Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, UK

[6]Monash Centre for Astrophysics (MoCA), School of Mathematical Sciences, Monash University, Vic. 3800, Australia
email: suzan.dogan@ege.edu.tr

Abstract. Accretion discs appear in many astrophysical systems. In most cases, these discs are probably not completely axisymmetric. Discs in binary systems are often found to be misaligned with respect to the binary orbit. In this case, the gravitational torque from a companion induces nodal precession in misaligned rings of gas. We first calculate whether this precession is strong enough to overcome the internal disc torques communicating angular momentum. For typical parameters, precession torque wins. To check this result, we perform numerical simulations using the Smoothed Particle Hydrodynamics code, PHANTOM, and confirm that sufficiently thin and sufficiently inclined discs can break into distinct planes that precess effectively independently. Disc tearing is widespread and severely changes the disc structure. It enhances dissipation and promotes stronger accretion onto the central object. We also perform a stability analysis on isolated warped discs to understand the physics of disc breaking and tearing observed in numerical simulations. The instability appears in the form of viscous anti-diffusion of the warp amplitude and the surface density. The discovery of disc breaking and tearing has revealed new physical processes that dramatically change the evolution of accretion discs, with obvious implications for observed systems.

Keywords. accretion, accretion discs — hydrodynamics — instabilities — black hole physics

1. Introduction

Accretion discs (e.g. Pringle 1981; Frank, King, & Raine 2002) are the essential ingredient for a vast range of astrophysical phenomena, including star and planet formation, X-ray binaries and active galactic nuclei (AGN). In most cases, discs are unlikely to be axisymmetric. The lack of symmetry produces a torque on misaligned rings of gas which makes their orbits precess differentially. Given a sufficiently strong viscosity communicating the precession between the rings, the disc warps. If the viscosity is too weak, or the external torque on the disc is sufficiently strong, the disc may instead break into distinct planes with only tenuous gas flows between them (Nixon & King 2012). If in addition these planes are sufficiently inclined to the axis of precession, they can precess until they are partially counterrotating, promoting angular momentum cancellation and rapid infall – disc tearing (Nixon et al. 2012; Nixon, King, & Price 2013).

The aim of this investigation is to find out if tearing can happen in circumprimary discs where the disc around one component is disrupted by the perturbation from a companion. This would have significant implications for all binary systems: e.g. fuelling SMBH during the SMBH binary phase and accretion outbursts in X-ray binaries. We first compare the disc precession torque with the disc viscous torque to determine whether the disc should warp or break. We check our findings by comparing our analytical reasoning with hydrodynamical simulations. Finally, we present an instability analysis which we perform to understand the underlying physics of 'disc breaking' (the case with no external torque) and 'disc tearing' (the case when the disc is subject to an external torque) observed in numerical simulations.

2. Disc Tearing

We consider binary systems with an initially planar disc around one component, misaligned with respect to the (circular) binary orbit. We expect the disc to break when the precession induced in the disc is stronger than any internal communication in the disc. The disc precession caused by the presence of a binary companion is retrograde, and has frequency (Bate et al. 2000)

$$\Omega_{\rm p} = \frac{3}{4}\frac{M_0}{M_1}\left(\frac{R}{a}\right)^3 \Omega\cos\theta. \tag{2.1}$$

Here θ is the inclination angle between the disc plane and the binary orbital plane, M_0 & M_1 are the masses of each component of the binary with the disc around M_1, a is the binary separation, R (assumed $\ll a$) is the disc radius, and $\Omega = (GM_1/R^3)^{1/2}$ is the disc orbital frequency. Thus, the magnitude of the precession torque per unit area is

$$|\boldsymbol{G}_{\rm p}| = |\Omega_{\rm p}\times\boldsymbol{L}| = \frac{3}{4}\frac{M_0}{M_1}\left(\frac{R}{a}\right)^3 \Sigma R^2\Omega^2\cos\theta\sin\theta. \tag{2.2}$$

On the other hand, by using the α-viscosity prescription by Shakura & Sunyaev (1973) $\nu_i = \alpha_i H^2\Omega$, the total magnitude of the azimuthal and vertical viscous torques per unit area in a warped disc can be written as (Papaloizou & Pringle 1983)

$$|G_{\rm total}| = |G_{\nu_1}| + |G_{\nu_2}| = \frac{\Sigma R^2\Omega^2}{2}\frac{H}{R}\left[3\alpha_1 + \alpha_2\,|\psi|\right] \tag{2.3}$$

where $|\psi|$ is the warp amplitude and defined as $|\psi| = R\,|\partial l/\partial R|$ (Ogilvie 1999). To break the disc the precession must be stronger than its viscous communication, i.e. $|G_{\rm p}| \gtrsim |G_{\rm total}|$. This comparison gives an idea of where in the disc we expect breaking to occur:

$$R_{\rm break} \gtrsim \left[\frac{4\left(\alpha_1 + \frac{\alpha_2}{3}\,|\psi|\right)}{\sin 2\theta}\frac{H}{R}\frac{M_1}{M_0}\right]^{1/3} a. \tag{2.4}$$

It is not straightforward to evaluate (2.4) as both α_1 and α_2 are strong functions of the warp amplitude $|\psi|$ (Ogilvie 1999, 2000) and the warp amplitude itself is unknown before performing a full 3D calculation of the disc evolution. For large $\alpha \gtrsim 0.1$ it is reasonable

https://doi.org/10.1017/S1743921322001387 Published online by Cambridge University Press

to exclude the α_2 term. Proceeding with the method of the earlier papers on disc tearing we get

$$R_{\rm break} \gtrsim \left(\frac{4\alpha}{\sin 2\theta}\frac{H}{R}\frac{M_1}{M_0}\right)^{1/3} a. \tag{2.5}$$

We note that this equation is not relevant for $\alpha \ll 0.1$ as the vertical viscosity becomes important and small inclination angles where the strong vertical viscosity can result in rapid disc alignment. This tearing criterion is equivalent to requiring a minimum inclination of the disc to the binary orbit, $\theta_{\rm min}$, defined by

$$\sin 2\theta_{\rm min} \gtrsim 4\alpha \frac{H}{R}\frac{M_1}{M_0}\left(\frac{a}{R_{\rm break}}\right)^3. \tag{2.6}$$

We can simplify this formula in two limits. If the disc is around the less massive component we have $M_1 < M_0$ and the tidal limit on the disc size requires

$$\frac{a}{R_{\rm break}} > 2.5\left(\frac{M}{M_1}\right)^{1/3}, \tag{2.7}$$

where $M = M_1 + M_0$ is the total binary mass, so (2.6) becomes

$$\sin 2\theta \gtrsim 0.06\left(\frac{\alpha}{0.1}\right)\left(\frac{H/R}{0.01}\right) \qquad (M_1 < M_0) \tag{2.8}$$

since $M \simeq M_0$ in this case. If instead the disc is around the more massive binary component we have $M_1 > M_0$ and the disc size is approximately $0.6a$ (Artymowicz & Lubow 1994). In this case, breaking occurs if

$$\sin 2\theta \gtrsim 0.18\left(\frac{\alpha}{0.1}\right)\left(\frac{H/R}{0.01}\right)\left(\frac{M_1/M_0}{10}\right) \qquad (M_1 > M_0) \tag{2.9}$$

For typical black hole disc parameters $\alpha = 0.1$, $H/R \lesssim 10^{-2}$ almost all discs should break unless they are aligned to the binary plane within a few degrees. However, a very large mass ratio $M_1/M_0 \gg 1$ makes the perturbation by the smaller companion so weak that breaking would occur only after a very long interval.

To check this analytical reasoning, we perform 3D hydrodynamical numerical simulations using the PHANTOM smoothed particle hydrodynamics code (Price et al. 2018). The disc is initially planar and extends from $R_{\rm in} = 0.1a$ to $R_{\rm out} = 0.35a$ with a surface density profile $\Sigma = \Sigma_0(R/R_{\rm in})^{-p}$ and locally isothermal sound speed profile $c_{\rm s} = c_{\rm s,0}(R/R_{\rm in})^{-q}$, where we have chosen $p = 3/2$ and $q = 3/4$. This achieves a uniformly resolved disc with the shell-averaged smoothing length per disc scale-height $\langle h \rangle / H \approx$ constant. Σ_0 and $c_{\rm s,0}$ are set by the disc mass, $M_{\rm d} = 10^{-3}M$ and the disc angular semi-thickness, $H/R = 0.01$ (at $R = R_{\rm in}$) respectively. Initially the disc is composed of 1 million particles, which for this setup gives $\langle h \rangle / H \approx 0.8$. The simulations use a disc viscosity with Shakura & Sunyaev $\alpha \simeq 0.1$ (which requires artificial viscosity $\alpha_{\rm AV} = 1.2$; cf. Lodato & Price 2010) and $\beta_{\rm AV} = 2$. We assume that the binary components, represented by two Newtonian point masses with $M_1 = M_2 = 0.5M$, accrete any gas coming within a distance $0.05a$ of them, and so remove this gas from the computation.

We perform our simulations for $\theta = 10°, 30°, 45°$ and $60°$. The complete results of our simulations are given in Doğan et al. (2015). When the initial inclination is small, the precession torque caused by the companion is weak. In this case, the disc evolves with a mild warp. The strength of the precession torque is higher when $\theta = 30°$, and it has its maximum value when $\theta = 45°$. As expected, we confirm disc breaking in our simulations

https://doi.org/10.1017/S1743921322001387 Published online by Cambridge University Press

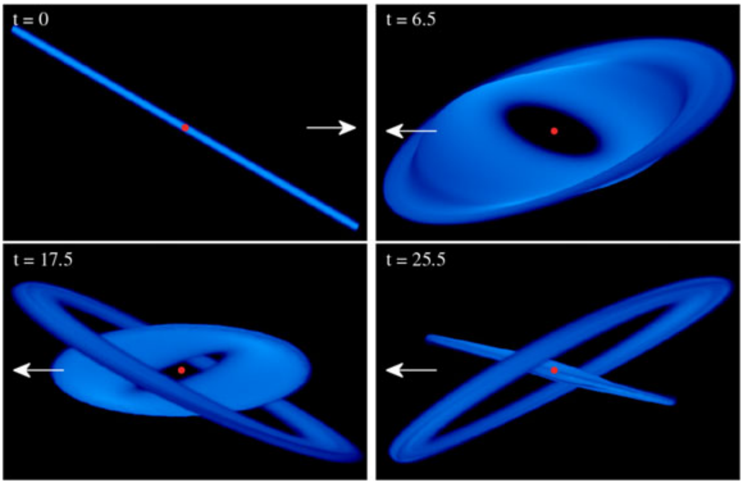

Figure 1. 3D surface rendering of the disc which was initially inclined at 30° to the binary plane with no warp. These snapshots are taken after 0, 6, 17.5 and 25.5 binary orbits. The disc is viewed along the binary orbital plane and the arrow points at the direction of the companion (see (Doğan et al. 2015) for details).

with initial inclinations of 30° and 45°. Fig. 1 shows a simulation with an initial inclination of 30°. Here the disc becomes significantly warped after a few orbits. Then the outer disc breaks off to form a distinct outer ring. The accretion rate through tearing discs is generally significantly enhanced. In the $\theta = 60°$ simulation, the disc quickly becomes a narrow and quite eccentric ring (through a strong interaction induced by tearing). The remaining eccentric disc then goes through strong Kozai-Lidov cycles producing a highly variable accretion (Doğan et al. 2015; Martin et al. 2014).

3. Instability Analysis of Warped Discs

The dynamical behavior of disk tearing has been explored by a series of numerical investigations in different contexts. Disc tearing has been shown to occur in (*i*) discs inclined to the spin of a central black hole (Nixon et al. 2012), (*ii*) circumbinary discs around misaligned central binary systems (Nixon, King, & Price 2013; Facchini, Lodato, & Price 2013; Aly et al. 2015), (*iii*) circumprimary discs misaligned with respect to the binary orbital plane (Doğan et al. 2015). In these studies, the criterion for disc tearing has been derived simplistically by comparing the viscous torque with the precession torque induced in the disc. Initially, the precession torque was compared to the torque arising from azimuthal shear (Nixon et al. 2012). This criterion is clearly insufficient as it does not account for the (more important) torque attempting to smooth the disc warp which arises from vertical shear. For small α and small disc inclination angles, the simulations of Doğan et al. 2015 showed that the inclusion of the vertical viscous torque, which depends on the disc structure through the warp amplitude, was required.

Further to these simulations of disc tearing, it has also been shown that warped discs may break without any external forcing (Lodato & Price 2010). When there is no external forcing, the disc evolution must be driven by the dependence of the effective viscosities on the disc structure. This instability occurs in a simpler environment than disc tearing, and likely underpins that process. Motivated by the breaking and tearing behaviour observed in numerical simulations of warped discs, we perform a stability analysis of the warped disc equations to connect these behaviours with disc instabilities and determine a general criterion for discs to break.

Here we provide a brief description of the relevant equations and refer the reader to Ogilvie (2000) and Doğan et al. (2018) for more details and Doğan & Nixon (2020) for the non-Keplerian case. The governing evolutionary equations are the conservation of mass equation,

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\bar{v}_r\Sigma) = 0\,, \tag{3.1}$$

and the conservation of angular momentum equation,

$$\frac{\partial}{\partial t}\left(\Sigma r^2 \Omega l\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[Q_1 \Sigma c_{\rm s}^2 r^2 l + Q_2 \Sigma c_{\rm s}^2 r^3 \frac{\partial l}{\partial r} + Q_3 \Sigma c_{\rm s}^2 r^3 l \times \frac{\partial l}{\partial r} - \left(\frac{\partial}{\partial r}\left[Q_1 \Sigma c_{\rm s}^2 r^2\right] - Q_2 \Sigma c_{\rm s}^2 r \left|\psi\right|^2\right)\frac{h}{h'} l\right]. \tag{3.2}$$

Here $\Sigma\,(R,t)$ is the disc surface density, $\bar{v}_r$ is the mean radial velocity, $\Omega\,(R)$ is the orbital angular velocity of each annulus of the disc, $l\,(R,t)$ is the unit angular momentum vector pointing perpendicular to the local orbital plane, $c_{\rm s}$ is the sound speed, $h = r^2\Omega$ is the specific angular momentum, $h' = {\rm d}h/{\rm d}r$ and Q_i are the dimensionless torque coefficients.

The stability is considered with respect to linear perturbations in $\delta\Sigma$ and δl. This yields a third-order dispersion relation:

$$\begin{aligned} s^3 - s^2 \left[aQ_1 - 2Q_2 + |\psi|\left(aQ_1' - Q_2'\right)\right] \\ -s\left[2aQ_1Q_2 - Q_2^2 - Q_3^2 + |\psi|\left(aQ_1Q_2' - Q_2Q_2' - Q_3Q_3'\right)\right] \\ -a\left[Q_1(Q_2^2 + Q_3^2) + |\psi|\left(Q_1Q_2Q_2' - Q_1'Q_2^2 + Q_1Q_3Q_3' - Q_1'Q_3^2\right)\right] = 0. \end{aligned} \tag{3.3}$$

Here, the prime on Q_i represents differentiation with respect to $|\psi|$, $a = h/rh' = {\rm d}\ln r/{\rm d}\ln h = 1/(2-q)$. We note that $a = 2$ for a Keplerian disc with $q = 3/2$. The dimensionless growth rate, s, is defined by

$$s = -\frac{{\rm i}\omega}{\Omega}\left(\frac{\Omega}{c_s k}\right)^2. \tag{3.4}$$

We note that validity of the equations requires $k \lesssim 1/H$. Full solutions of (3.3) are provided in Doğan et al. (2018) with a more detailed investigation. The disc becomes unstable if any of the roots of (3.3) has a positive real part, i.e. $\Re(s) > 0$, as the perturbations then grow exponentially with time. The simplified criteria for instability can be expressed as follows: If

$$\left[a\frac{\partial}{\partial \psi}\left(Q_1\left|\psi\right|\right) - \frac{\partial}{\partial \psi}\left(Q_2\left|\psi\right|\right)\right] > 0, \tag{3.5}$$

the disc is unstable, or if

$$\left[a\frac{\partial}{\partial \psi}\left(Q_1\left|\psi\right|\right) - \frac{\partial}{\partial \psi}\left(Q_2\left|\psi\right|\right)\right] < 0, \quad \text{and} \quad 4{\rm a}\left[({\rm Q}_1{\rm Q}_2 + ({\rm Q}_1{\rm Q}_2' - {\rm Q}_1'{\rm Q}_2)|\psi|)\right] > 0 \tag{3.6}$$

the disc is also unstable. The instability criterion simply implies that if the maximum diffusion rate is not located at maxima in warp amplitude, then local maxima in warp amplitude will grow, and the disc will break. As this will result in rapid transfer of mass out of this region due to the large warp amplitude implying large torques, this will also be realized by a significant drop in local surface density. This resembles the Lightman-Eardley viscous instability but for a warped disc with the warp amplitude playing the

https://doi.org/10.1017/S1743921322001387 Published online by Cambridge University Press

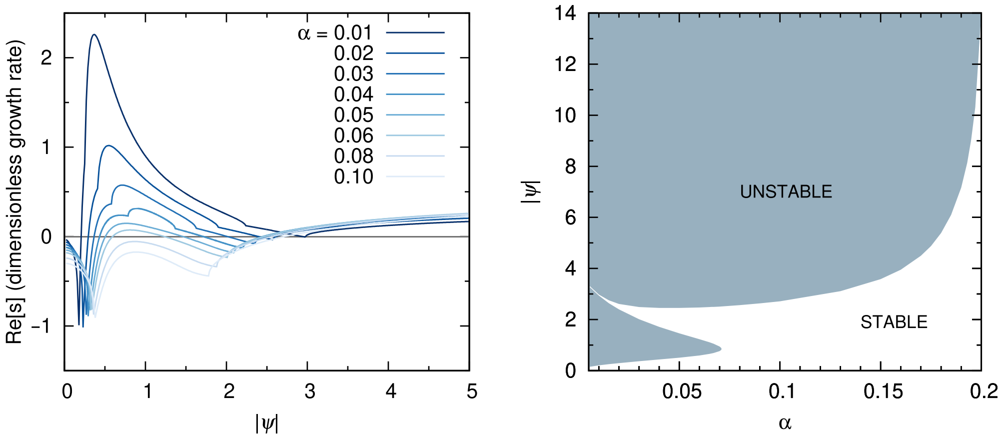

Figure 2. Left-hand panel shows the dimensionless growth rates $\Re(s)$ as functions of $|\psi|$ for different values of α. The grey line represents zero growth rate. The disc becomes unstable for sufficiently large warp amplitudes. The critical warp amplitudes where the disc becomes unstable are smaller and the growth rates of the instability are higher for low values of α. Right-hand panel shows the stable (white) and unstable (blue) regions in the (α, $|\psi|$) parameter space. This plot represents the critical warp amplitudes for instability to occur in discs with various α values. For a given value of α, there is always a minimum value of the warp amplitude, which gives rise to instability (see Doğan et al. 2018 for details).

role of the surface density. In Doğan et al. (2018), we showed that there is always a critical warp amplitude, $|\psi|_c$, which gives rise to instability for the parameters we have explored, with the exception of nearly flat discs with $|\psi| \lesssim 0.1$. The dimensionless growth rates and the critical warp amplitudes for the instability are shown in Fig. 2. The growth rates of the instability can be comparable with the dynamical rate ($\Re\,[s] \sim 1$).

4. Conclusion

We have shown that tilted discs inside a binary are susceptible to tearing from the outside in, because of the gravitational torque from the companion star. We have also connected the disk breaking and tearing behavior observed in numerical simulations with the instability of warped disks that was derived by Ogilvie (2000) through a local stability analysis of the warped disk equations. The necessary conditions for disk "breaking" have been derived. The instability occurs physically due to viscous anti-diffusion of the warp amplitude. This underlies the process of disc tearing which has the capacity to dramatically alter the instantaneous accretion rate and the observable properties of the disc on short time-scales.

Acknowledgements

SD is supported by the Turkish Scientific and Technical Research Council (TÜBİTAK – 117F280).

References

Aly H., Dehnen W., Nixon C., King A., 2015, MNRAS, 449, 65. doi:10.1093/mnras/stv128
Artymowicz P., Lubow S. H., 1994, ApJ, 421, 651.
Bate M. R., Bonnell I. A., Clarke C. J., Lubow S. H., Ogilvie G. I., Pringle J. E., Tout C. A., 2000, MNRAS, 317, 773.
Doğan S., Nixon C., King A., Price D. J., 2015, MNRAS, 449, 1251.
Doğan S., Nixon C. J., King A. R., Pringle J. E., 2018, MNRAS, 476, 1519.
Doğan S., Nixon C. J., 2020, MNRAS, 495, 1148.

Facchini S., Lodato G., Price D. J., 2013, MNRAS, 433, 2142.

Frank J., King A., Raine D. J., 2002, Accretion Power in Astrophysics: 3rd Edn. Cambridge Univ. Press, Cambridge

Lodato G., Price D. J., 2010, MNRAS, 405, 1212.

Martin R. G., Nixon C., Lubow S. H., Armitage P. J., Price D. J., Doğan S., King A., 2014, ApJL, 792, L33.

Nixon C. J., King A. R., 2012, MNRAS, 421, 1201.

Nixon C., King A., Price D., Frank J., 2012, ApJL, 757, L24.

Nixon C., King A., Price D., 2013, MNRAS, 434, 1946.

Ogilvie G. I., 1999, MNRAS, 304, 557.

Ogilvie G. I., 2000, MNRAS, 317, 607.

Papaloizou J. C. B., Pringle J. E., 1983, MNRAS, 202, 1181.

Price D. J., Wurster J., Tricco T. S., Nixon C., Toupin S., Pettitt A., Chan C., et al., 2018, PASA, 35, e031.

Pringle J. E., 1981, ARA&A, 19, 137.

Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337.