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Lasota, J.-P.; King, A.R.

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Ultraluminous X-ray sources are beamed

Jean–Pierre Lasota ^{1,2★} and Andrew King ^{3,4,5}

¹*Institut d’Astrophysique de Paris, CNRS et Sorbonne Université, UMR 7095, 98bis Bd Arago, F-75014 Paris, France*

²*Nicolaus Copernicus Astronomical Center, Polish Academy of Sciences, ul. Bartycka 18, 00-716 Warsaw, Poland*

³*Astrophysics Group, School of Physics & Astronomy, University of Leicester, Leicester LE1 7RH, UK*

⁴*Astronomical Institute Anton Pannekoek, University of Amsterdam, Science Park 904, NL-1098 XH Amsterdam, Netherlands*

⁵*Leiden Observatory, Leiden University, Niels Bohrweg 2, NL-2333 CA Leiden, Netherlands*

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ABSTRACT

We show that magnetar models for ultraluminous X-ray sources (ULXs) have serious internal inconsistencies. The magnetic fields required to increase the limiting luminosity for radiation pressure above the observed (assumed isotropic) luminosities are completely incompatible with the spin-up rates observed for pulsing ULXs. We note that at least one normal Be-star + neutron star system, with a standard (non-magnetar) field, is observed to become a ULX during a large outburst and return to its previous Be-star binary state afterwards. We note further that recent polarimetric observations of the well-studied binary Cyg X-3 reveal that it produces strong emission directed away from the observer, in line with theoretical predictions of its total accretion luminosity from evolutionary arguments. We conclude that the most likely explanation for ULX behaviour involves radiation beaming by accretion disc winds. A large fraction of X-ray binaries must pass through a ULX state in the course of their evolution.

Key words: accretion, accretion discs – black hole physics – binaries: close – pulsars: general – X-rays: binaries.

1 INTRODUCTION

Ultraluminous X-ray sources (ULXs) are defined by the two conditions

- (i) apparent luminosities (assumed isotropic) $L_X > 10^{39}$ erg s^{−1}, and
- (ii) locations away from galaxy centres.

These restrictions select a group of objects not straightforwardly explained either as accreting stellar-mass binaries, or as more massive accretors. Condition (i) requires L_X to exceed the Eddington luminosity for a $10 M_\odot$ black hole, i.e.

$$L_{\text{Edd}} = 1.3 \times 10^{38} m \text{ erg s}^{-1}, \quad (1)$$

with $m = M/M_\odot = 10$, which implies a corresponding Eddington accretion rate

$$\dot{M}_{\text{Edd}} \equiv \frac{L_{\text{Edd}}}{\eta c^2} = 1.4 \times 10^{18} \eta_{0.1}^{-1} m \text{ g s}^{-1} \quad (2)$$

$$= 2.2 \times 10^{-8} \eta_{0.1}^{-1} m M_\odot \text{ yr}^{-1}, \quad (3)$$

where $\eta = 0.1 \eta_{0.1}$ is the radiative efficiency of accretion. Condition (ii) rules out the central massive black holes in galaxies.

ULXs were identified as a separate class of objects at the end of the previous millennium (Colbert & Mushotzky 1999). By now, only two models of ULX behaviour remain under serious consideration.

The older of these two current models for ULX behaviour is disc-wind beaming (King et al. 2001). This asserts that the assumption of isotropic emission made in computing L_X from observations is

not valid for binary systems transferring mass at rates $\dot{m} \dot{M}_{\text{Edd}}$, with $\dot{m} \gg 1$, because in this case radiation pressure expels most of the transferred mass in quasispherical winds that are opaque except along narrow channels along the accretion disc axis (Shakura & Sunyaev 1973). This means that most of the emitted accretion luminosity $\sim L_{\text{Edd}}$ is beamed along these channels. ULXs are sources where the observer lies in one of the beams: the effect is that the apparent (assumed isotropic) luminosity inferred is

$$L_{\text{sph}} \sim \frac{1}{b} L_{\text{Edd}} \gg L_{\text{Edd}}, \quad (4)$$

where the total solid angle of the two channels is $4\pi b$.

The more recent model for ULX behaviour, which we shall refer to as the ‘magnetar model’, was inspired by the discovery by Bachetti et al. (2014) that the source ULX-2 in the galaxy M82 is pulsed. This implies that the accretor is a magnetized neutron star.¹ The magnetar model asserts that unusually strong surface fields of neutron-star accretors reduce the electron scattering opacity defining the Eddington luminosity, making L_{Edd} numerically larger; the high luminosities of ULXs are actually subcritical in this picture. For magnetar-strength fields ($\gtrsim 10^{14}$ G), the modified L_{Edd} exceeds the assumed-isotropic luminosity L_X .

We note that the two models imply fundamentally different significances for the ULX phenomenon. Disc-wind beaming asserts that the ULX state is one that a large fraction of otherwise standard X-ray binaries pass through during a particular phase of their

¹This discovery had the effect of making an early model for ULXs invoking accretion on to more massive (‘intermediate-mass’, or ‘IMBH’) black holes relatively unattractive.

* E-mail: lasota@iap.fr

evolution, whereas the magnetar hypothesis reduces the ULX class to a relatively small subset of these systems defined by very strong magnetic fields.

The aim of this paper is to evaluate recent evidence allowing a clear decision between beaming and strong magnetic fields as the basic cause of ULX behaviour.

2 BEAMING

The suggestion by King et al. (2001) of beaming as the explanation for the high apparent luminosities of ULXs was motivated by the study of the X-ray binary Cyg X-2 (King & Ritter 1999). The neutron star in this system has evidently survived the companion star attempting to transfer $\sim 3 M_{\odot}$ to it at a highly super-Eddington rate, without retaining more than a small fraction of it.

This corresponds closely to the picture of how a disc deals with a super-Eddington mass rate suggested by Shakura & Sunyaev (1973). A radiation-pressure powered wind from the disc surface keeps the disc accretion rate at the local Eddington limit corresponding to each disc radius. This raises the true total emitted accretion luminosity only by a logarithmic factor to

$$L_{\text{acc}} \simeq L_{\text{Edd}}[1 + \ln \dot{m}], \quad (5)$$

so that even a huge (by X-ray binary standards) accretion rate of $\sim 10^4 \dot{M}_{\text{Edd}}$ would give a total accretion luminosity of only $10 L_{\text{Edd}}$, i.e. $\sim 2 \times 10^{39} \text{ erg s}^{-1}$ for a $1.4 M_{\odot}$ neutron star.

But importantly, the emission is now highly anisotropic: the outflowing wind is densest near the radius at which the full Eddington luminosity is attained, and has a large optical depth both along the disc plane and in the vertical direction. Thus, most of the disc radiation emitted within the wind region diffuses by scattering until it is able to escape through the central open funnels parallel to the disc axis. Since the funnel is tall and thin and has scattering walls, the escaping radiation is beamed by a factor $b \ll 1$, so that the apparent luminosity deduced by an observer in the beam, who assumes the luminosity to be isotropic, is

$$L_{\text{app}} = \frac{1}{b} L_{\text{Edd}}[1 + \ln \dot{m}] \gg L_{\text{Edd}}. \quad (6)$$

King (2009) showed that for $\dot{m} \gg 1$, the observed correlation $L_{\text{bb}} \propto T_{\text{bb}}^{-4}$ between ULX soft X-ray blackbody luminosity and temperature implies that

$$b \simeq \frac{73}{\dot{m}^2}. \quad (7)$$

This agrees with deductions from simple accretion disc theory, as conditions far from the disc centre are set by the mass supply rate, while those near the disc centre all converge to what is set by a near-Eddington central accretion rate.

King & Lasota (2020) noted that when the accretor is a magnetized neutron star, its magnetic axis is not necessarily aligned with the disc (i.e. funnel) axis, and it is very common for the neutron star spin to be misaligned from the binary orbit defining the accretion disc plane. When these three axes are not aligned, the system appears as a pulsing ULX, or PULX. For a neutron-star spin axis strongly misaligned from the central disc axis at the spherization radius, large polar caps produce the sinusoidal pulse light curves observed in PULXs since a significant part of the pulsed emission can escape without scattering, giving a large pulse fraction. Using this disc-wind-beaming model, King, Lasota & Kluźniak (2017); King & Lasota (2019, 2020) (see also King, Lasota & Middleton 2023) were able to obtain self-consistent sets of parameters for the 10 known

PULXs, finding magnetic fields in the range of $\sim 2 \times 10^{10}$ – 10^{13} G, mass-transfer rates \dot{m} between ~ 10 and ~ 100 , and beaming factors from ~ 0.01 to ~ 0.5 .

3 MAGNETAR MODELS

But as we noted above, soon after the discovery of the first PULX, a different explanation of the apparent super-Eddington luminosities observed in these X-ray sources became possible. Dall’Osso, Perna & Stella (2015) and Eksi et al. (2015) assumed that the PULX magnetic fields had magnetar ($\gtrsim 10^{14}$ G) field strengths. These substantially reduce the scattering cross-sections and so increase the critical luminosity at which the radiation pressure force equals the pull of gravity. In this scenario, PULX luminosities are above the usual Eddington luminosity but actually subcritical, so that accretion proceeds in the same way as in other X-ray pulsars.

Indeed, very strong magnetic fields lower the Thomson and Compton scattering opacity (Canuto, Lodenguai & Ruderman 1971; Herold 1979) for photons with energies E_{γ} lower than the cyclotron frequency E_{cyc} :

$$\frac{\sigma_{\text{B1}}}{\sigma_{\text{T}}} \approx \sin^2 \theta + \left(\frac{E_{\gamma}}{E_{\text{cyc}}} \right)^2 \cos^2 \theta \quad (8)$$

$$\frac{\sigma_{\text{B2}}}{\sigma_{\text{T}}} \approx \left(\frac{E_{\gamma}}{E_{\text{cyc}}} \right)^2, \text{ for } \frac{E_{\gamma}}{E_{\text{cyc}}} \ll 1, \quad (9)$$

where indices 1 and 2 correspond to the two linear photon polarizations, σ_{T} is the Thomson cross-section, and θ is the angle between the directions of the magnetic field and light propagation. The opacities depend on the photon polarization but as shown by Paczynski (1992) their Rosseland means differ at most by a factor 2, depending on the angle between the direction of the photon propagation and the field lines. Therefore, in the presence of a very strong magnetic field, the critical luminosity corresponding to the equality of the radiation pressure and gravitational forces can be written as

$$L_{\text{crit}} \approx 2 B_{12}^{4/3} \left(\frac{g}{2 \times 10^{14} \text{ cm s}^{-2}} \right)^{-1/3} L_{\text{Edd}}, \quad (10)$$

where $g = GM/R^2$ (Paczynski 1992). Thus, in this picture, the apparent (assumed isotropic) PULX luminosities $\gtrsim 10^{40} \text{ erg s}^{-1}$ must be emitted by a plasma permeated by magnetar-strength fields $> 10^{14}$ G.

Although at first sight attractive, the idea of magnetars in PULXs faces the difficulty that these very strongly magnetized neutron stars have never been observed in binary systems (see King & Lasota 2019; King et al. 2023 and references therein). Accepting it requires belief in a cosmic conspiracy, making them detectable in binaries only when these have high mass transfer rates. We shall see in the next section that this idea disagrees with observations in any case.

We now know that out of the ~ 1800 observed ULXs (see King et al. 2023 and references therein) at least 10 contain magnetized neutron stars, detected through their periodic pulses (PULXs). Four of them are transient: they are members of Be-X binary systems, which become X-ray sources when the eccentric orbit of the compact companion (in most if not all cases a neutron star) of the massive Be star crosses its circumstellar disc. In most cases, this disc-crossing produces sub-Eddington-luminosity outbursts (called ‘Type I’), but from time to time, most probably because of von Zeipel–Kozai–Lidov oscillations of the circumstellar disc (Martin et al. 2014), it results in a giant (super-Eddington; ‘Type II’) outburst.

Swift/*XRT* observations of galaxies NGC 4945, NGC 7793, and M81 suggest that although persistent ULXs dominate the high end of

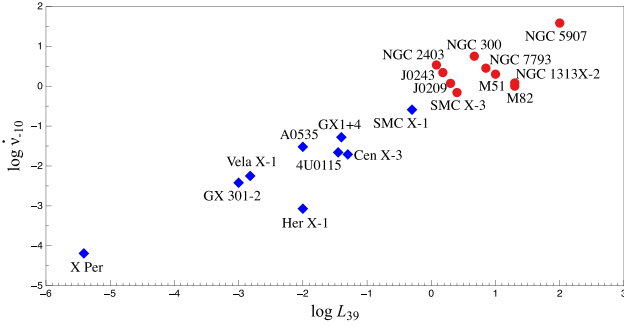


Figure 1. The $L_{39}-\dot{\nu}_{10}$ diagram for XRPs and PULXs. Red dots: the ten PULXs with known spin-up rates. Blue diamonds: selected (for comparison) sub-Eddington-luminosity X-ray pulsars (For details see King et al. 2023).

galaxy luminosity functions, the number of systems emitting ULX luminosities are probably dominated by transient sources. These transients are most probably not Be-X systems (Brightman et al. 2023).

4 MAGNETIC FIELDS IN ULXS CANNOT HAVE MAGNETAR STRENGTHS

There is a simple physical argument that rules out the presence of magnetars in observed PULXs. The argument is based on the value of their spin-up rate $\dot{\nu}$ (ν is the pulsar’s spin frequency).

After the discovery of the first PULX M82 ULX-2, Kluźniak & Lasota (2015) pointed out that it differs from other X-ray pulsars (XRP) not only through its higher luminosity but also in its extremely high spin-up rate.

It is immediately obvious that both ‘normal’ X-ray systems and PULXs lie on exactly the same strong correlation between spin-up rate $\dot{\nu}$ and X-ray luminosity L_X – see Fig. 1. This correlation extends more than seven orders of magnitude in luminosity, and arises because the spin-up results from the accretion torque on the neutron star:

$$\dot{\nu} = \frac{\dot{J}(R_M)}{2\pi I} = \frac{\dot{M}(GM R_M)^{1/2}}{2\pi I} \propto \dot{M}^{6/7} \mu^{2/7}, \quad (11)$$

where $R_M \propto \dot{M}^{-2/7} \mu^{4/7}$ (from equation (12)) is the magnetospheric radius, $\mu = BR^3$ the neutron star’s magnetic moment (with B the field and R the neutron-star radius), and I is the neutron star’s moment of inertia.

The magnetospheric radius is defined by the equation (Frank, King & Raine 2002)

$$R_M = 2.6 \times 10^8 q \left(\frac{\dot{M}}{10^{17} \text{ g s}^{-1}} \right)^{-2/7} \left(\frac{M}{M_\odot} \right)^{-3/7} \mu_{30}^{4/7} \text{ cm}, \quad (12)$$

where the factor $q \sim 1$ takes into account the geometry of the accretion flow at the magnetosphere and $\mu = 10^{30} \mu_{30} \text{ G cm}^3$.

Assuming $M \approx 1M_\odot$, $q \approx 1$, $I = 10^{45} \text{ g cm}^2$, and using equation (12), equation (11) gives

$$\dot{M} \approx 5.7 \times 10^{18} \dot{\nu}_{10}^{7/6} \mu_{30}^{-1/3} \text{ g s}^{-1} \quad (13)$$

as the accretion rate required to spin up a magnetized neutron star at the rate $\dot{\nu}$.

Now we can calculate the luminosity produced by this accretion rate. Supercritical luminosities are not proportional to the accretion rate (see equation (5)). But very strong magnetic fields make the critical luminosity much larger than the Eddington value, i.e. $L_{\text{crit}} \gg L_{\text{Edd}}$ (see equation (10)). So for $L_{\text{crit}} > L_X \gtrsim L_{\text{Edd}}$, the standard

formula $L_X = 0.1 \dot{M} c^2$ applies, even though L_X exceeds the usual Eddington value.

Then for magnetar PULXs ($\mu \gtrsim 10^{31} \text{ G cm}^3$), we get from equation (13) the luminosity

$$L_X \approx 2 \times 10^{38} \dot{\nu}_{10}^{7/6} \mu_{31}^{-1/3} \text{ erg s}^{-1} \approx L_{\text{Edd}}. \quad (14)$$

But in deriving this equation, we assumed $L \gtrsim L_{\text{crit}} \gg L_{\text{Edd}}$, which would require a much smaller field (i.e. $\mu_{31} \ll 1$).² This contradiction shows that magnetars cannot be present in systems with both $L_X > 10^{39} \text{ erg s}^{-1}$ and $\dot{\nu} \gtrsim 10^{-10} \text{ s}^{-2}$.

In other words, PULXs cannot contain magnetars.

This in turn means that the super-Eddington luminosity observed in PULXs is not intrinsic and must presumably be anisotropic, i.e. beamed.

Importantly, since $L_{\text{crit}} \sim B^{4/3}$ in a dipole field, the critical luminosity decreases radially outwards as R^{-4} . So at radius ~ 100 stellar radii, all of the cross-section suppression is lost, well inside the magnetosphere. Then a hyper-Eddington luminosity emitted near the neutron-star surface would blow away all of the gas in the upper part of the accretion column, thus cutting off the mass supply and supposedly producing the posited hyper-Eddington emission. This rules out the interpretation of the CRSF observed in the magnetized, non-pulsing ULX-8 in M51 as an effect of protons orbiting a $9 \times 10^{14} \text{ G}$ magnetic field, as envisaged by Brightman et al. (2018), and provides another strong argument against the presence of magnetars in ULXs.³

5 MAGNETIC FIELDS IN PULXs

We conclude from the last section that neutron stars in PULXs have magnetic fields spanning the same range as the usual XRP – from 10^8 G to several 10^{13} G (Revnivtsev & Mereghetti 2018). They are evidently normal XRP observed in a special phase of the evolution of their parent binary systems, as is implicit in the original suggestion by King et al. (2001). We can see examples of this in real time in observations of Be-star PULXs. These are normal XRP for most of their lifetimes and become PULXs only during their occasional giant outbursts. This allows one to follow the transformation of an XRB into a PULX and its return to ‘normal’ again.

The best studied case is that of the binary SMC X-3. It shows that as the system enters the ULX phase, the neutron-star spin evolution becomes dominated by the accretion torque, as assumed in equation (11). Between giant outbursts, this source is an XRP that spins down. In Fig. 2 (Townsend et al. 2017), this corresponds to the time right up to the beginning of a giant outburst on MJD 57599; then a significant spin-up is observed. From SMC X-3’s long-term spin history, Townsend et al. (2017) deduce that the angular momentum transferred by accretion during the 5-month giant outburst was larger than the total angular momentum lost by magnetic braking over the previous 18 years of the spin-down phase. The long-term spin-down rate of SMC X-3 is about 500 times lower than the rate of spin-up observed during the giant outburst, showing that the torques acting during this outburst are far larger than during the out-of-outburst phases. During weaker (Type I) outbursts, the spin period continues to increase, but during the giant outburst, the spin-up rate is tightly correlated with the X-ray luminosity through the super-Eddington

²Equation (14) explains why sub-Eddington-luminosity XRP ($\mu_{31} \sim 0.001-1$) have $\dot{\nu} < 10^{-10} \text{ s}^{-2}$.

³We are grateful to the anonymous referee of the present paper for suggesting this line of argument.

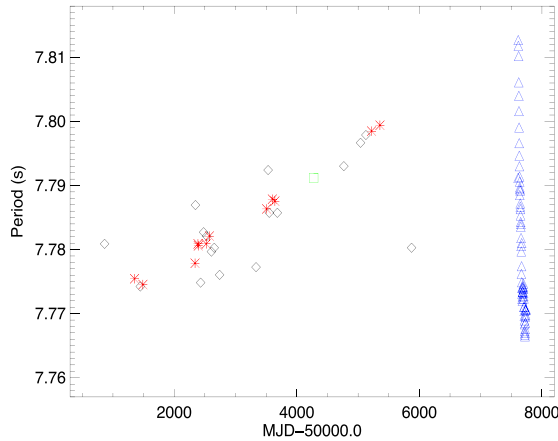


Figure 2. X-ray derived pulsed period history of SMC X-3. Black diamonds and red stars denote RXTE period detections above the 99 and 99.99 per cent confidence levels respectively. Blue triangles denote Swift detections of the pulse period during the current outburst. A single *XMM-Newton* detection at MJD 54 274 was found in the literature and is denoted by a green square. (From Townsend et al. 2017).

phase (Weng et al. 2017), in agreement with equation (11). This means that in PULXs, the spin-up rate is strongly correlated with the X-ray luminosity, both in time and over the population. There are no magnetars in PULXs.

6 CYG X-3: THE SECOND HIDDEN ULX IN THE GALAXY

Quite recently, Veledina et al. (2023) performed X-ray polarimetry, indicating ‘unambiguously’ that the Wolf–Rayet X-ray binary Cyg X-3, consisting of a helium star transferring mass to a black hole on its thermal time-scale is a ULX with a beaming factor⁴ $b \approx 1/65$, but seen from the side. This system is assumed to contain a black hole. Earlier inferred examples of ‘sideways’ ULXs notably include the extreme source SS433 (cf Begelman, King & Pringle 2006; King & Muldrew 2016).

From equation (7), we find that this requires an Eddington factor $\dot{m} \simeq 69$. This is consistent with the estimates of the mass transfer rate found by Lommen et al. (2005) on evolutionary grounds. Together with the similar estimates for SS433 (Begelman et al. 2006; King & Muldrew 2016), this appears to be explicit confirmation that compact binaries with mass transfer rates exceeding the Eddington rate produce beamed emission, as first suggested by King et al. (2001). Moreover, the estimate (7) appears to be in reasonable agreement with observation.

The good match between the luminosities of some ULX nebulae and the luminosity of their ULX irradiators has been used as an argument against strong geometrical beaming in these ultraluminous sources since, in these cases, the nebula would see an isotropic emission. However, the sources in question are spectrally soft, and it is not surprising that the irradiating luminosity inferred from photoionization modelling is consistent with the observed luminosity (see King et al. 2023 for a detailed discussion).

⁴We use here the symbol b as defined in King (2009); by contrast Veledina et al. 2023 use b to denote his $1/b$.

7 CONCLUSION

We have shown that magnetar models for ULX behaviour have serious internal inconsistencies. In particular, the field strengths required to increase the radiation pressure luminosity limit above the observed (assumed isotropic) luminosities are completely incompatible with the spin-up rates observed for PULXs. In addition, we note that at least one normal Be-star system, with a standard (non-magnetar) field, is observed to become a ULX during a large outburst.

In contrast, recent polarimetric observations of the well-studied binary Cyg X-3 reveal that it produces strong emission beams away from the observer.

We conclude that ULXs are beamed.

DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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