Editorial introduction: special issue on Gaussian queues

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EDITORIAL

Editorial introduction: special issue on Gaussian queues

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Gaussian queues form an important subdomain within queueing theory. They can be interpreted as Gaussian processes reflected at 0, or more precisely, Gaussian processes upon which the *Skorokhod reflection map* is imposed. This special issue contains a set of novel contributions in this field.

A natural motivation and formal justification for considering queues with Gaussian input stems from *central limit theorem*-type arguments: When aggregating a large number of input streams, their aggregate converges, after centering and scaling, to a Gaussian process. A notable early result of this nature can be found in the seminal paper by Iglehart [1]. There it was shown that an appropriately scaled compound Poisson process (corresponding to independent and identically distributed jobs that arrive according to a Poisson process), converges to Brownian motion (BM), under the condition that the variance of the job size distribution is finite. This convergence motivated the analysis of queueing systems that are fed by BM, in the literature referred to as *reflected Brownian motion* (RBM). Importantly, this type of *diffusion approximation* generalizes in various dimensions. Indeed, a large class of queueing processes is well approximated by RBM, and in addition more general functionals of a BM have been considered so as to analyze a broad range of queueing-related quantities. The interpretation of one-dimensional RBM as a limiting queueing process motivated the investigation of more sophisticated queueing models in the diffusion regime, such as tandem systems or generalized Jackson networks [2–4]. It in particular led to the

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elegant theory of RBM on a wedge, pioneered by Harrison [3] and further extended in various directions; see, e.g., [5] and references therein.

In the 1990s, with high-speed communication networks being introduced on a large scale, detailed traffic analyses were performed. Important new phenomena were discovered; most notably it was found that network traffic exhibits *long-range-dependence* and *self-similarity*. This led Norros [6] to propose to model traffic aggregates by a specific Gaussian process: fractional Brownian motions (FBM). The FBM process, often denoted by $B_H(t),\ t \geq 0$, for $H \in (0, 1)$, is a centered Gaussian processes with stationary increments, characterized through the variance function $\mathrm{Var}\, B_H(t) = t^{2H}, t \geq 0$. It is self-similar, and if the *Hurst parameter* $H > 1/2$, it is long-range dependent. Then, in Taqqu et al. [7], it was formally justified that the superposition of many On/Off alternating renewal processes with heavy-tailed On/Off periods (a natural model for traffic in communication networks) converges, after an appropriate time renormalization, to FBM. This key result was extended in many directions, including models where the number of On/Off processes scales simultaneously with time [8], parametrization sets that lead in the limit to more general classes of Gaussian processes [9], and a formal justification that the above stochastic process-limits carry over to the induced stationary queueing process [10]. The above findings initiated massive research on properties of various performance measures of Gaussian queueing systems. We do not provide an exhaustive overview, but refer to the monographs by Whitt [11] and Mandjes [12].

Due to the close connection between queueing processes, processes reflected at 0, and suprema of stochastic processes, the research on Gaussian queues stimulated the development of various techniques in extreme value theory for Gaussian processes, and vice versa. Also fueled by new results on Gaussian queues, significant progress was made in the analysis of the tail asymptotics of the supremum of non-centered Gaussian processes over an infinite time horizon. It is important to remark that virtually all available results in the literature are of an asymptotic nature, with a focus both on logarithmic and exact tail asymptotics [12–17]; see also the monographs on extremes of Gaussian processes [18–21]. Notably, when considering queueing systems that are just slightly harder than the standard single-node Gaussian queue, virtually no results are available. For example, even for two-node FBM-driven tandem queue with $H \neq 1/2$, the strongest results for the tail distribution of the downstream queue provide just logarithmic asymptotics [22], the exact asymptotics being a long-lasting challenge [23].

As mentioned, hardly any non-asymptotic results are known. Essentially only in the case of RBM, a detailed characterization of the corresponding queue's stationary and transient behavior has been established. In the context of the FBM-fed queue, a notable recent contribution is [24], presenting upper and lower bounds on the mean (as functions of the Hurst parameter H). A somewhat unorthodox, pragmatic approach has been followed in [25], proposing an experimental, regression-based procedure to fit the stationary workload moments of an FBM-driven queue.

As a consequence of the lack of closed-form exact expressions (and efficient computational approaches) for the workload distribution, simulation-based estimation techniques have been developed [26–28]. When it comes to estimating small tail probabilities, direct Monte Carlo techniques usually render infeasible, motivating the use of, e.g., importance sampling-based approaches.

With single-queue tail asymptotics being fairly well-understood by now, the current research lines on Gaussian queues are largely directed to the analysis of such tail asymptotics for Gaussian networks and related multi-node systems. Due to their intrinsic multi-dimensional nature, these are frequently related to extremes of vector-valued Gaussian processes. As techniques that work for one-dimensional settings typically fail in a multi-dimensional context, there is a strong need for new tools that can be used in the analysis of supremum-type functionals for multi-dimensional Gaussian processes. Other challenging themes of research concern the tail analysis of various interesting functionals of Gaussian queueing processes over a given time interval, including suprema, infima, and sojourn times (i.e., time spent above a given threshold).

In this special issue of queueing systems, we present a collection of state-of-the-art contributions on the above topics. The issue starts with two papers in which functional central limit theorems are established. Araman and Glynn consider a model with scheduled arrivals. While their earlier work identified cases in which the limiting process was of FBM type with $H \in (0, 1/2)$ (i.e., having a negative correlation structure), now a super-heavy-tailed case is studied in which there is convergence to conventional Brownian motion, as well as a case in which there is convergence to a degenerate FBM with $H = 0$. Fendick and Whitt analyze an open network of queues with path-dependent net-input processes. The main result is a Gaussian limiting model arising in a specific heavy-traffic regime.

The other four papers deal with the analysis of Gaussian queueing processes and related topics. In the first place, Anugu and Pang develop the large deviations principle for a model with generalized FBM input, with a focus on logarithmic asymptotics. Then, Ji and Peng present the precise asymptotics for the maximum of (finitely many) suprema of dependent Gaussian processes, with the objective to model the maximum of the queue lengths in Gaussian-driven fork-join networks. Bisewski and Jasnovidov derive bounds for the constants that appear in formulas for the exact asymptotics of suprema of Gaussian processes, in particular also those that appear in the paper by Ji and Peng. Finally, Dębicki, Hashorva, and Liu analyze asymptotic properties of the sojourn time in the context of the stationary FBM-driven queueing process.

References

1. Iglehart, D.: Diffusion approximations in collective risk theory. J. Appl. Probab. **6**(2), 285–292 (1969)
2. Iglehart, D.L., Whitt, W.: Multiple channel queues in heavy traffic. I. Adv. Appl. Probab. **2**(1), 150–177 (1970)
3. Harrison, J.M.: The diffusion approximation for tandem queues in heavy traffic. Adv. Appl. Probab. **10**(4), 886–905 (1978)
4. Harrison, J.M., Reiman, M.I.: Reflected Brownian motion on an orthant. Ann. Probab. **9**(2), 302–308 (1981)
5. Dieker, A.: Reflected Brownian Motion. Wiley Encyclopedia of Operations Research and Management Science. Wiley, New York (2010)
6. Norros, I.: A storage model with self-similar input. Queueing Syst. **16**, 387–396 (1994)
7. Taqqu, M.S., Willinger, W., Sherman, R.: Proof of a fundamental result in self-similar traffic modeling. ACM SIGCOMM Comput. Commun. Rev. **27**(2), 5–23 (1997)

8. Mikosch, T., Resnick, S., Rootzén, H., Stegeman, A.: Is network traffic approximated by stable Lévy motion or fractional Brownian motion? Ann. Appl. Probab. **12**, 23–68 (2002)
9. Dębicki, K., Palmowski, Z.: On-off fluid models in heavy traffic environment. Queueing Syst. **33**, 327–338 (1999)
10. Dębicki, K., Mandjes, M.: Traffic with an fBm limit: convergence of the stationary workload process. Queueing Syst. **46**, 113–127 (2004)
11. Whitt, W.: Stochastic-Process Limits: An Introduction to Stochastic-Process Limits and Their Application to Queues. Springer, Berlin (2002)
12. Mandjes, M.: Large Deviations for Gaussian Queues: Modelling Communication Networks. Wiley, New York (2007)
13. Dębicki, K.: A note on LDP for supremum of Gaussian processes over infinite horizon. Stat. Probab. Lett. **44**(3), 211–219 (1999)
14. Hüsler, J., Piterbarg, V.: Extremes of a certain class of Gaussian processes. Stoch. Processes Appl. **83**(2), 257–271 (1999)
15. Piterbarg, V.I.: Large deviations of a storage process with fractional Brownian motion as input. Extremes **4**, 147–164 (2001)
16. Dębicki, K.: Ruin probability for Gaussian integrated processes. Stoch. Processes Appl. **98**(1), 151–174 (2002)
17. Hashorva, E., Ji, L., Piterbarg, V.I.: On the supremum of γ-reflected processes with fractional Brownian motion as input. Stoch. Processes Appl. **123**(11), 4111–4127 (2013)
18. Piterbarg, V.I.: Asymptotic Methods in the Theory of Gaussian Processes and Fields, vol. 148. American Mathematical Society, Providence (1996)
19. Adler, R.J.: An introduction to continuity, extrema, and related topics for general Gaussian processes (1990)
20. Lifshits, M., Lifshits, M.: Lectures on Gaussian Processes. Springer, Berlin (2012)
21. Azaïs, J.-M., Wschebor, M.: Level Sets and Extrema of Random Processes and Fields. Wiley, New York (2009)
22. Mandjes, M., Mannersalo, P., Norros, I.: Gaussian tandem queues with an application to dimensioning of switch fabric interfaces. Comput. Netw. **51**(3), 781–797 (2007)
23. Dębicki, K.: Exact asymptotics of Gaussian-driven tandem queues. Queueing Syst. **100**(3–4), 285–287 (2022)
24. Bisewski, K., Dębicki, K., Mandjes, M.: Bounds for expected supremum of fractional Brownian motion with drift. J. Appl. Probab. **58**(2), 411–427 (2021)
25. Malsagov, A., Mandjes, M.: Approximations for reflected fractional Brownian motion. Phys. Rev. E **100**, 032120 (2019)
26. Dieker, A.B., Mandjes, M.: Fast simulation of overflow probabilities in a queue with Gaussian input. ACM Trans. Model. Comput. Simul. **16**(2), 119–151 (2006)
27. Blanchet, J., Li, C.: Efficient simulation for the maximum of infinite horizon discrete-time Gaussian processes. J. Appl. Probab. **48**(2), 467–489 (2011)
28. Adler, R., Blanchet, J., Liu, J.: Efficient Monte Carlo for high excursions of Gaussian random fields. Ann. Appl. Probab. **22**(3), 1167–1214 (2012)