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On quantum transport in flat-band materials

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On quantum transport in flat-band materials

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The cover shows an artistic representation of some results presented in this thesis: the dynamical gap generation in a flat-band material on the front cover, the spectral functions of a bilayer dice lattice on the back cover; the background shows the dice lattice itself, as a platform that hosts these physical effects. The cover is a digital painting made by Oleg Mykhailov, cousin of the author of the thesis.

To my family.

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