# Non-Abelian metamaterials: emergent computing and memory

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# Summary

The research in this dissertation concerns the non-linear, sequence dependent response of mechanical metamaterials. Metamaterials are forms of designer matter whose properties (optical, acoustic, mechanical, topological, etc.) exceed typical natural materials [86, 87]. In particular, mechanical metamaterials feature advanced properties that are mechanical in nature, such as stiffness, elasticity, compressibility, and shape morphing [22]. We tap into the advanced properties of these metamaterials by mechanically actuating them, i.e., pushing, pulling, twisting, bending, or shearing such materials. These actions are collectively termed actuations.

Most early examples of such metamaterials involved using only a single actuation to obtain a single advanced functionality [7, 8, 33]. Later, metamaterials with multiple actuation sites were developed that could display multiple functionalities by actuating different combinations of these sites [40,71,88]. However, these newer metamaterials were not designed to be sensitive to the sequence in which you actuate these sites. For instance, imagine a metamaterial with two sites, A and B. Now, you actuate it in the sequence A first, B second. Then, you actuate it in the sequence B first, A second. The output of a traditional metamaterial would be the same in both cases. It only matters which sites you compress, not the sequence in which you do so. This is known as Abelian response [89].

But why do we care about sequence at all? Does it matter in real life? Yes, it does! Let's imagine your smartphone's password is 1234 and you wish to unlock it to text your friends about this cool dissertation you just read. You need to input the numbers in a specific sequence: 1 first, 2 second, 3 third, and 4 fourth. If you instead input the same numbers but in a slightly different sequence, say 1243, the smartphone won't open. In fact, sequence is a foundational concept in biology (e.g., protein folding), computers (e.g., information processing), robotics (e.g., locomotion), and many other areas essential to human progress.

In this thesis, we develop and study a metamaterial that is sequence sensitive. We call it a non-Abelian mechanical metamaterial. Concretely, it takes mechanical inputs and produces an output which is sensitive to the sequence of those inputs.

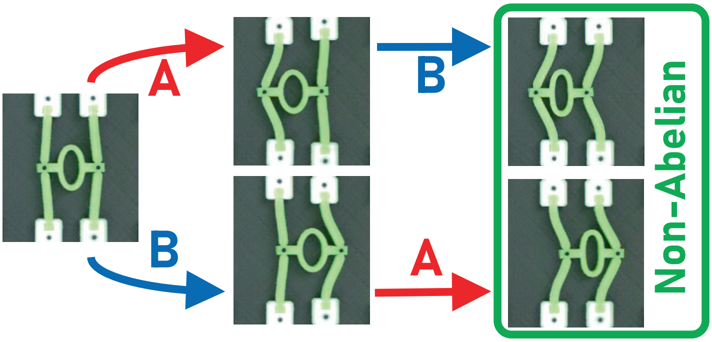


Figure 1: **Path dependence and sequence sensitive response in a non-Abelian unit cell.** This unit cell has two input channels, beam A on the left and beam B on the right. Unit cell response is distinct for the actuation sequence AB (compress A first, B second) and the actuation sequence BA (compress B first, A second).

This metamaterial is composed of building blocks which we call non-Abelian unit cells (Figure 1). Chapter 1 introduces the idea of sequence sensitive behavior and non-Abelian response in the context of mechanical metamaterials. While Chapters 2-5 focus exclusively on the non-Abelian unit cell, Chapters 6-8 focus on non-Abelian metamaterials composed of multiple unit cells.

In Chapter 2, we introduce a non-Abelian unit cell whose deformation output is sensitive to the sequence of actuation inputs. Modeling the unit cell as a network of linear springs, we develop a standardized actuation protocol which helps us explicitly characterize its non-Abelian behavior. We show that a single unit cell exhibits three different behaviors using the amount of actuation as a tuning parameter.

In Chapter 3, we experimentally realize the non-Abelian unit cell through Computer Aided Design, 3D printing, and molding, and develop an automated device to conduct actuation experiments. Through carefully designed experiments, we characterize the non-Abelian behavior of the unit cell sample and observe the three behaviors, or regimes, as predicted in Chapter 2. Our experimental findings are in excellent agreement with the numerical spring network model and demonstrate the feasibility of non-Abelian metamaterials.

In Chapter 4, we demonstrate the information processing capabilities of the non-Abelian unit cell by introducing two equivalent representations, the finite state machine and the digital electronic circuit. Specifically, we establish that the non-

Abelian unit cell is a simple mechanical computer which can process information using Boolean logic. We also establish that the unit cell can store one bit of information by showing its equivalence to a set-reset (SR) latch, the fundamental building block of digital memory.

In Chapters 2-4, we limited ourselves to homogeneous actuations of the unit cell: $\delta_A = \delta_B$, where $\delta_A$ and $\delta_B$ denote the amount of actuation of beams A and B respectively. In Chapter 5, we break this actuation symmetry by allowing distinct $\delta_A$ and $\delta_B$ values when both beams are actuated. This allows us to observe four additional "inhomogeneous" regimes, two Abelian and two non-Abelian, in addition to the three previously observed "homogeneous" regimes. This observation validates symmetry breaking as a relatively simple and viable strategy to extend the programmability of the non-Abelian unit cell.

In Chapters 2-5, we focused on isolated non-Abelian unit cells. In Chapter 6, we embed such unit cells in a long non-Abelian chain. We explore the chain's response to actuation along two adjacent input channels and explore its behavior for a range of $\delta$ values. We also define the distance of non-commutation which is a characteristic length scale that governs such non-Abelian metamaterials.

In Chapter 7, we numerically study a non-Abelian metamaterial composed of four vertical beams and explore its response to actuation sequences that involve all four input channels. Using a recursive algorithm and a directed graph representation, we show that the metamaterial can be programmed to exhibit a wide range of behaviors by tuning the amount of actuation $\delta$. To understand its complex response, we introduce the concept of two-loops: sub-graphs that show the response of the metamaterial for two active input channels and two passive tuning channels. We then numerically observe eight topologically distinct classes of two-loops. Finally, we demonstrate that the metamaterial's memory can be amplified by tuning the coupling between input channels.

In Chapter 8, we design and conduct experiments on the four-beam metamaterial guided by numerical observations from Chapter 7 and observe a wide variety of two-loop behaviors. Strikingly, we observe new classes of two-loops that go beyond those seen in our numerical observations, in particular featuring three stable states for a single actuation input. This observation suggests that it is possible to encode multiple bits of information in metamaterials using only two active input channels.

Such metamaterials can be leveraged for a range of applications including, but not limited to, mechanical computing and mechanical cryptography. While we have shown examples of computing in this thesis, we haves also filed a European patent for a mechanical cryptography device based on the non-Abelian metamaterial. Cryptography requires a lock-and-key mechanism for encryption and decryption. There already exist in the literature mechanical lock-and-key mechanisms based on metamaterials [71,88]. However, please note that most of them are not sequence sensitive. They unlock when provided with a fixed combination of mechanical inputs. In contrast, our unit cell can serve as a basis for a lock-and-key mechanism that unlocks only when a fixed combination of mechanical inputs is provided in a *specific order*. This novel sequence-sensitive lock-and-key mechanism serves as an ideal basis to implement a mechanical cryptography device, the details of which can be found in our patent (pending).