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## Non-Abelian metamaterials: emergent computing and memory

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### Citation

Singh, A. (2023, September 28). *Non-Abelian metamaterials: emergent computing and memory*. Casimir PhD Series. Retrieved from <https://hdl.handle.net/1887/3642534>

Version: Publisher's Version

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**Note:** To cite this publication please use the final published version (if applicable).

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*Emergent computing in a non-Abelian metamaterial*,  
In preparation.
- [2] Amitesh Singh, Matthieu Labousse and Martin van Hecke,  
*Device implementing a configurable finite state machine*,  
European patent (pending).