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Paul Natorp's reformulation of the Kantian distinction between intuition and concept

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Chapter 4. Overcoming the Heterogeneity between Intuition and Concepts. The Deduction of Categories.

In the deduction of categories³⁵⁸, we find the clue to understanding the way in which Natorp conceives the relationship between intuitive and conceptual representations. The main task of his project is to explain how the totality of experience can be constructed by thinking. As we explained in the introduction, Natorp considers that it is necessary to reformulate the Kantian distinction between intuition and concepts. The crucial moment in order to overcome the heterogeneity between intuitions and concepts takes place in this deduction. We will show what we anticipated in the previous section (III.1). Thinking can build objectivity in all its determinations by its own means. As we studied, philosophy must show the way in which the object is constructed *in* and *by* thinking. This was introduced as a necessary task. Now, the specific development of this process will be exhibited. Natorp must show how concepts, as ways of organizing the multiplicity, can constitute the object completely determined. He must explain how thinking can concomitantly generate both unity and multiplicity. Natorp must clarify the way in which the processes of thinking can generate the object in all its determination. In the deduction of categories, it must be shown that thinking can constitute the object as it is presented in the experience without any reference to intuition. Natorp will show that the construction of objectivity is developed on four levels: quantity, quality, relation, and modality.

As we anticipated, in the deduction of categories, we find the core of the rupture with the Kantian proposal. Kant considers that the concepts have their origin in the understanding. Intuitions have their origin in sensibility. Concepts and intuitions are types of representation that arise from heterogeneous sources. As we explained in the first chapter, for Kant, the process of knowledge requires the contribution of concepts and intuitions. The reference of concepts to objects can only take place through intuition. The concepts provide the unity, while the intuition provides the multiplicity that must be gathered by the concept. Intuition cannot be the source of unity just as the concept cannot provide the multiplicity. As the object of experience is composed of the unity and

³⁵⁸ More precisely, Natorp develops a metaphysical deduction of categories. The question is to identify the categories. As there is no heterogeneity between sensibility and understanding, a transcendental deduction is not necessary at all. Hernán Pringe explains: “Without the distinction between a passive capacity (sensibility) and an active faculty (understanding), there is no *quid juris* question and therefore no necessity for a transcendental deduction”. Pringe, H., 2011, p. 210.

multiplicity, its conformation requires both: concepts and intuitions. Furthermore, as we studied in Chapter 1, pure multiplicity can be provided by pure intuition. However, the matter for the construction of experience must always be given *a posteriori*. The multiplicity that must be gathered is always provided by intuition, which in the case of man has the forms of space and time. The multiplicity can never be provided by the understanding. Therefore, in the Kantian proposal, the understanding has a relationship with objects only on the basis of this reference to intuition. As we explained in the first chapter, completely determined objects can only be given to intuition. Concepts cannot construct the object of knowledge. Knowledge always depends on a factor external to the understanding, i.e., intuition. For Natorp the understanding is the source of the totality of the determinations of the object. The understanding can provide both: the multiplicity and the unit. For this reason, Natorp's deduction of categories will be one of the fundamental points of disagreement with the Kantian system. However, as we studied in Chapter 3, Natorp considers that by showing how the understanding is the source and architect of nature, he is understanding Kant better than Kant understood himself.

The goal of this section is to carry out an analysis of Natorp's deduction of categories. We must show how thinking is constitutive of objectivity without any reference to intuition. The question that Natorp must answer is how thinking can constitute the object of experience. It will be exhibited how thinking can construct both the unity and the multiple that this unity contains, which means that thinking produces both the conceptual and intuitive aspects of the object. Natorp will show that thinking is synthetic, unifies a multiplicity, even if it has no reference to intuition. This is the only way in which philosophy can guarantee the overcoming of the dualism between intuitive and conceptual representations. In the first section of this chapter, we explained that objectivity must be constructed in and by thinking. Now, it will be exhibited how this process takes place. We will divide this section into two parts. First, we will analyze the categories of quantity and quality. Second, we will study the levels of relation and modality. In chapter five, we will draw the consequences that can be extracted from this deduction. We will study how the deduction of categories leads to the reformulation of the definition of intuitive and conceptual representations.

4.1. Main Features of the Deduction

The deduction of the categories will show that intuitions and concepts must be considered as moments of the process of thinking. As we explained in chapter 3, this guarantees the systematicity required by the very concept of knowledge, and the possibility of overcoming the Kantian dualism between intuitions and concepts. Natorp will show that the complete determination of the object of knowledge does not require a factor external to thinking. The deduction will show that all the determinations of the object are posited by thinking itself, proving that thinking can have reference to objects without any referee to intuition whatsoever, as the object is completely based on purely conceptual determinations. The question of how thinking can have access to the object in its singularity is now resolved in a brand-new way: concepts do not need a reference to intuition to have a reference to objects. The first step of this deduction is to exhibit the mathematical determinations of the object, the quantitative and qualitative determinations of objectivity. It will be shown that, on the one hand, the logical foundation of mathematics is grounded on thinking and, on the other, that logic as the science of thinking is expressed primarily in the legality of mathematics³⁵⁹. The deduction of the categories of quantity and quality will show that the most general determinations of objectivity, the mathematical determinations, are grounded on thinking. The laws of mathematic are deduced from the laws of thinking.

According to Natorp, the first germ of the idea of a systematic deduction of the categories can be found in Descartes. The author of the *Meditations* was the first to notice that a system of the fundamental functions of thinking should be developed. The unity of thinking is manifested in a coherent system, and this course is possible on the grounds of certain fundamental principles. These principles will be the expression of the synthetic unity of thinking, of its deductive chain. Under this systematic unity of thinking, science is one and indivisible³⁶⁰.

Descartes and Leibniz followed this conception initiated by Plato, and whose greatest exponent is Kant. The Kantian system of categories is the culminating point of

³⁵⁹ „Es schwebt also unmittelbar eine Mathematik der Qualitäten vor, wie sie Leibniz gefordert hat, und wie die jüngste Entwicklung der Mathematik sie der Verwirklichung näher zu führen scheint, wenn sie, allerdings nicht eine Arithmetik, aber wohl eine Algebra ohne Quantitätsbegriffe zu entwickeln wagt, ausdrücklich in dem Sinne, daß Mathematik ist nicht notwendig mit Quantität zu tun habe, sondern sich (wie einer der entschlossensten Vorkämpfer dieser Richtung, AN Whitehead, *Universal Algebra*, I, Cambridge, 1898, sagt) auf Alles erstreckt worin, „die Folge der Gedanken oder der Ereignisse in bestimmter Weise ausgemacht und präzise festgesetzt werden kann (Preface, pg. VIII). “According to Natorp, in this way, the ideal of Leibnizian philosophy would be concreted. Cf. Natorp, P, PILb, p. 439.

³⁶⁰ For Dufour, this is one of the ruptures of Natorp with Cohen, For Cohen, the science division is a *factum a posteriori* grounded. Cohen considers the division of sciences as a given fact. Natorp believes that this division is exhibited *a priori* in the foundation of science in the logical law. Dufour, É., 2003, p.104.

this task initiated by Platonic philosophy³⁶¹. The proposal of Cartesian idealism has pointed in the direction of overcoming the dualism between intuition and concepts. Kant has followed this tendency. However, neither Descartes nor Kant were deep enough in their approaches. According to Natorp, Descartes begun the path of overcoming the heterogeneity between intuition and concepts, “but after all Descartes fell into a gross dualism”³⁶². Even in the Cartesian idealist philosophy, there is always an element in the experience that thinking cannot provide from its own source. The object is an external element for knowledge. Cartesian idealism recovers the task begun by Plato. However, in the system of Descartes “the naive belief in the existence of the object, given in itself before all knowledge, and to be grasped by knowledge remains unchanged”³⁶³. The overcoming of dualism between the given and what is thought is not carried out either by Cartesian or by Kant’s proposal. Kantian idealism failed to show the way in which thinking is the producer of objectivity. The task of constructing a coherent idealism remains. This is the task that must be accomplished by the deduction of categories, where it is shown how thinking can truly be the source of objectivity.

³⁶¹ „Der wesentliche, rein objektive Sinn der Einheit des Intellects ist die deductive Verkettung der gesamten menschlichen Erkenntnis, kraft deren sie von den ersten, einfachsten Elementen an in, kontinuierlicher, nirgends unterbrochener Gedankenbewegung“ (Reg. III) gewonnen werden kann. Die Forschung nach jenen, in Grundbegriffen und Grundsätzen zu definierenden Elementen des deductiven Zusammenhanges der einen unteilbaren Wissenschaft hat eigentlich Descartes zu Ehren gebracht. Leibniz ist darin sein Nachfolger; die Kategorien und Grundsätze Kants sind die spät gereifte Frucht dieser langen Vorarbeit.“ Natorp, P., DED, p. 16.

³⁶² „Aber schließlich fällt Descartes in den groben Dualismus zurück.“ Natorp, P., DED, p. 18.

³⁶³ „Vielleicht wendet man ein, dass selbst bei Kant dieser Dualismus keineswegs ganz überwunden sei, dass neben dem in der Erkenntnis rein erzeugten doch noch etwas wie ein „gegebener“ Gegenstand übrig bleibe. Allein das ist bei ihm bloß ein rudimentärer Rest einer in der Hauptsache verlassenen Stufe des Philosophirens; ein Rest, der auf der Höhe des Systems verschwindet. Bei Descartes im Gegenteil ist der erste Ansatz rein und folgerecht, aber daneben wuchert das naive Vorurteil des an sich vor aller Erkenntnis vorhandenen und nun zu erfassenden Gegenstandes ungestört weiter, er endlich auch jenen richtigeren Ansatz zu überwuchern und sich auf der Höhe der Entwicklung des Philosophen, in seiner Metaphysik, zum System zu verhärten.“ Natorp, P., DED, p.19.

Natorp introduces the deduction of the categories³⁶⁴ for the first time in *Quantität und Qualität*³⁶⁵. The core of the deduction was focused on the categories of quantity and quality. The deduction reappeared in 1900 in *Nombre, temps, espace, dans leurs rapports avec les fonctions primitives de la pensée*. The point of departure was the definition of thinking as synthetic unity. After briefly considering the problems of quantity and quality, Natorp focused on the core of his presentation: the problems of arithmetic and geometry. The explanation of the categories of relation and modality is only briefly sketched. The reference to the quantitative and qualitative procedure of thinking is required to clarify the three main issues of his presentations: number, space, and time. According to Natorp, the explanation of the procedure of quantification and qualification sheds light on the operation of thinking that constitutes numbers and the fundamental relations among them. The process of quantity and quality grounds the mathematical determinations of objectivity. From the process of quantity and quality, Natorp draws the properties of numbers and, from the properties of numbers he obtains the determinations of space and time. His article of 1901, *Zu den logischen Grundlagen der neueren Mathematik* follows the same line. Thus, the first deduction that Natorp proposed is focused on the categories of quantity and quality. In his *Philosophical Propaedeutics* (1903), Natorp incorporated a more detailed account of the categories of relation and modality. From 1904 on, the deduction already had the full development that it will exhibit in LGEW, his most systematic work. For his reason, we will take this text as a point of reference.

Before studying the process of the deduction of categories, we can already highlight some interesting aspects in relation to how Natorp introduced the deduction of categories in his philosophical development. Regarding the modifications of his

³⁶⁴ In ZLGM, Natorp defines categories as the fundamental procedures of thinking. The categories are the way in which the legality of objectivity can be conceived. Natorp defines the concept of categories as follows: „Der Mathematiker, auch der logisch interessierte Mathematiker mag sich dabei beruhigen, solche letzten Prämissen zu „postulieren“; die Logik fordert für sie, als synthetische Sätze, wie Kant sagt, „wo nicht einen Beweis“ (der hier in der That ausgeschlossen ist) „doch wenigstens eine Deduktion der Rechtmässigkeit ihrer Behauptung“, sie fordert, nach Plato, den Rückgang auf „voraussetzungslose“, d. h. auf solche letzte Voraussetzungen, von denen es möglich ist, sich zu überzeugen, dass sie nicht wiederum andere, fundamentalere voraussetzen, nämlich auf die schlechthin fundamentalen Verfahrensweisen des „Denkens“, d. i. gesetzmässigen Vorstellens der Gegenstände überhaupt, die sie in einer begrenzten Zahl reiner Grundfunktionen des Denkens (Kategorieen) festzulegen sucht.“ Natorp, P. ZLGM, p. 383. Natorp uses both, the concept of levels (*Stufen*) and of categories (*Kategorieen*). For example: In NTE, L, LGEW, we find the concept of *Stufen*, But in ZLGM, EGM, Natorp talks about categories. As Holzhey, explains Natorp prefers to talk about logical functions rather than of categories. We will analyze the problem of the definition of categories in chapter 4.

³⁶⁵ *Quantität und Qualität in Begriff, Urtheil und gegenständlicher Erkenntnis. Ein Kapitel der transcendentalen Logik*. Helmut Holzhey explains the development of Natorp's position. He considers that the main differences are to be found between 1903 and the *Logik* of 1904. Cf. Holzhey, H., 1986 p. 107 ss.

presentation, we consider that two significant issues must be stressed. In the first place, when Natorp introduced the problem for the first time in the article of the *Philosophische Monatshefte*, he focused on the deduction of quantity and quality. The full development of the system was presented by 1900. This highlights the crucial importance that deduction of categories has in relation to the problem of the mathematical determinations of the object. As we saw, when Natorp first introduced the problem, he was focused on the mathematical determinations of objectivity. Second, Natorp differentiates two possible ways of how the deduction could take place. We could carry out the deduction in two ways. Both would lead to the same point. In the *Erkenntnistheoretische Grundlagen der Mathematik*, Natorp maintains:

Es handelt sich um die letzten gemeinsamen Grundlagen der Arithmetik und Geometrie, deren Blosslegung nichts geringeres bedeuten würde, als eine rein logische Deduktion des Raumes wie auch der Zeit. Die bezüglichen Untersuchungen sind niedergelegt in zwei Abhandlungen, die eine aus Anlass des internationalen philosophischen Kongresses bei der Pariser Weltausstellung, daher in französischer Sprache veröffentlicht: *Nombre, temps et espace*; die andere „Zu den logischen Grundlagen der neueren Mathematik“, im „Archiv für systematische Philosophie“. Ich werde aber hier einen etwas anderen Weg einschlagen, da ich glaube, dass auf diesem neuen Wege der Beweisgang logisch strenger wird, obgleich er zu keinem anderen Ergebnis führt.“ [...] „Ich ging dort so zu Werke, dass ich zu nächst die Gesetze der Zahl herleitete aus den Grundgesetzen der „quantitativ – qualitativen Synthesis“, d. h. aus den beiden, überhaupt fundamentalsten, von einander untrennbaren Denkverfahren, durch die wir, einerseits ein Mannigfaltiges als solches, andererseits jene Einheit eines Mannigfaltigen, die einen Denkinhalt konstituiert, gedanklich erzeugen.“³⁶⁶

In this remark, we appreciate these two interconnected aspects. In the first place, the problem that Natorp had in mind when he introduced the problem of the categories of quantity and quality. The core of the problem here is the mathematical determinations of objectivity. He wants to show that both the properties of numbers and of space and time,

³⁶⁶ Natorp, P., EGM, p.2.

can be drawn from the fundamental forms of thinking. That is to say: arithmetical and geometrical determinations of objectivity have the same root, and the nature of the properties of space and time can be derived from the nature of numbers³⁶⁷. It would prove that mathematics is purely grounded on thinking. Second, Natorp claims that the determinations of number and time can be obtained in two ways. On the one hand, the most general laws for the conformation of objectivity, the categories of quantity and quality, can be taken as a starting point. However, he affirms that another possibility consists in starting directly from the very concept of synthetic thinking. This is the path that he will take, says Natorp, in EGM. In contrast, both in ZGNM and in NTE, the properties of numbers and arithmetic relationships were derived from the categories of quantity and quality. However, this deduction of the properties of numbers could have been carried out directly, starting from the very concept of thinking. Kant calls this way of proceeding synthetic method. This is the method of the *Critique of pure reason*³⁶⁸. Thus, on the one hand, it can be clearly seen the crucial importance of deduction of categories for the development of the concept of number. On the other hand, it is evident how the problem of the deduction grows until it occupies the heart of Natorp's proposal.

The deduction of categories shows the stages in the constitution of objectivity. However, for some commentators, the deduction of the categories of quantity and quality does not represent a relevant element within the Natorp system. Morris Cohen argues that this moment is only part of a modern category deduction that does not affect the core of

³⁶⁷ This question is briefly and clearly exposed particularly in NTE and EGM. The shortness of the exposition makes it much easier to see the relation between the deduction of categories and the problem of the mathematical determinations of objectivity.

³⁶⁸ The method is synthetic or progressive. The synthetic method is the method that Kant follows in *Critique of Pure Reason (Proleg., AA 4: 274 ss.)*. The synthetic method is progressive. It starts from a first representation that is conceived as confuse and unclear and seeks to gain clarity and distinction. In this process, the elucidation of the elements that constitute each part of the representation leads to an elucidation of the other parts. Knowledge is organic. The way in which each part of knowledge operates determines the operation of the remaining areas. This allows the application of the progressive method. This organic conception of knowledge demands a synthetic method of exposition. The synthetic method allows exhibiting this organic structure of knowledge (*Proleg., AA 4: 263 ss.*). This procedure is that it has no empirical assumptions (*Proleg., AA 4: 275.*). The only assumption is the possibility of the very concept of thinking. We only depart from confused representation of what thought is. For this reason, the synthetic method is progressive, it advances by gaining determinations as they are required by the investigation, i.e., by the elucidation of the representation that is being analyzed. Kant explains: „In der *Kritik der reinen Vernunft* bin ich in Absicht auf diese Frage synthetisch zu Werke gegangen, nämlich so, dass ich in der reinen Vernunft selbst forschte und in dieser Quelle selbst die Elemente sowohl, als auch die Gesetze ihres reinen Gebrauchs nach Principien zu bestimmen suchte. Diese Arbeit ist schwer und erfordert einen entschlossenen Leser, sich nach und nach in ein System hinein zu denken, was noch nichts als gegeben zum Grande legt ausser die Vernunft selbst und also, ohne sich auf irgend ein Factum zu stützen, die Erkenntnis aus ihren ursprünglichen Keimen zu entwickeln sucht.“ Kant, I., *Proleg.*, AA, pp. 275ss.

the system³⁶⁹. Helmut Holzhey, in this same direction, affirms that the very concept of “category” has a merely historiographical function to refer to the Kantian system³⁷⁰. However, for others, the quantitative-qualitative synthesis is the most important step in Natorp’s philosophical system. For some commentators, such as André Laks and Éric Dufour, the deduction of the categories of quantity and quality represents the philosopher’s rupture with the Cohenian proposal. Dufour focuses his analysis on the double front of the debate: against Kantianism and Cohenianism.³⁷¹ In fact, Natorp would find himself, at this point, distant from Cohen and close to Cassirer.³⁷²

For Natorp, within the primary categories of thought, quantity and quality have traditionally been accepted as the most essential. This is due to the fact that the conceptualizing functions of quantity and quality “... represent the original process of the synthetic unity of a multiplicity in general ...”³⁷³. Natorp will show that these functions allow the logical progression. The relationship between these two functions (quantitative and qualitative) is so close that the separation is only an abstraction of thinking that allows

³⁶⁹ In his review to the LGEW Morris Cohen holds: “In the second chapter, we have a modernized deduction of the categories. The dry bones of the Kantian framework receive a great deal of flesh and blood. In the end, however, they turn out to be our old friends the Twelve, marching in four groups of three each. If it were not for the fact that students at our colleges do not read German, this chapter could profitably be recommended to those who are reading Kant for the first time and who generally cannot grasp what these categories are about.” Cohen, M., 1911, p. 694.

³⁷⁰ “In his book *Die logischen Grundlagen der exakten Wissenschaften* of 1910, Paul Natorp employed the concept of 'category' only in a historical sense when referring to Kant.” Holzhey, H., 2005, p. 70.

³⁷¹ “Cependant, en critiquant la thèse de Cohen et en plaidant pour un retour à Kant, donc en affirmant que toute synthèse est synthèse de la diversité, Natorp n’est-il pas obligé d’admettre ce à quoi Cohen voulait précisément échapper, à savoir la présupposition d’un divers qui relève d’autre chose que de la pensée? Ce n’est pourtant pas le cas (...) Il ne s’agit pas pour Natorp de réhabiliter, contre Cohen, une passivité primordiale qui équivaldrait à la donation d’un divers que l’activité de la pensée aurait ensuite à penser. Car c’est la pensée elle-même qui pose, dans sa propre activité, un divers qu’elle a pour tâche d’unifier. Dès lors, contre Kant, le divers relève bien de la pensée et non de la sensibilité, mais, contre Cohen, la synthèse est bien synthèse du divers et non de l’unité.” Dufour, É., 2002, p. 337.

³⁷² “Il faut remarquer combien Natorp est proche de Cassirer et combien tous deux s’éloignent de Cohen”. Dufour, É., 2002, p. 338. André Laks focuses on the dispute with the Cohenian system. He highlights: “Cohen claims to follow the Kantian principle of the division of the forms of judgment; yet this is not all the case. The Kantian table of judgments places at its head the judgments of quantity and quality ... For Cohen, on the contrary, the table of judgments is entirely related to the analytic – i.e. purely logical- use of judgment, with no reference to content at all. (...) But Natorp while accepting this programme (which is the programme of neo-Kantianism) nonetheless rejects the way in which Cohen in fact puts it into action. Instead of subsuming the (analytic) principles of traditional logic under the (synthetic) principles of objective cognition, one must recognize that the two series of principles correspond to each other (...) for this reason it is necessary to start, like Kant, from the judgments of quantity and quality, i.e. from mathematical judgments, and not form a purely formal principle like the principle of contradiction. For the commentator, this divergence represents the break within Marburg neo-Kantianism. Laks, A., 2004, p. 481, 482. Also, for Dufour, this is Natorp’s most important criticism of Cohen’s system. Dufour, É., 2002, p. 338 n. 62.

³⁷³ Natorp, P., LGEW, p. 52.

us to delineate each of the moments that are part of the whole. Although they are characterized separately, they are part of a unique process.

Throughout his works, Natorp identifies synthesis as the most essential operation of thinking. The first element of the logical is defined by the synthetic unity. Thinking is synthesis. The task of thinking consists in generating relationships. The parts of the relationship are the terms of the relation. To overcome the separation between intuition and thinking, it must be exhibited how thinking by its own means generates multiplicity and the unity that unites it³⁷⁴. The deduction of the levels of quantity and quality will arise by means of a synthetic procedure³⁷⁵. The analysis of one of the parts will lead to revealing another necessary element required by the concept. The primitive notion is the very concept of thinking. Thinking must be defined as synthesis, as an activity whose main task is generating relationships. To think is to establish relationships between the whole and the parts, between multiplicity and unity. The first step is to define what a multiplicity consists of. The concept of multiplicity necessarily implies a plurality of differentiable moments. The quantity and quality levels will emerge as the necessary moments implied by the definition of thought. Thinking consists of producing relationships. The possibility of the position of relations implies the union of multiplicity in a unity. Multiplicity is a plurality of differentiable moments. Then, the conformation of the plurality requires the position of units. Thus, the function of quantitative synthesis depends on the correlation of two fundamental logical moments: unity and multiplicity. The multiplicity leads to the concept of unity because it needs a unity to conform a multiplicity. Without the unity, it would be a mere rhapsody of elements. The unity requires the multiplicity to have a content. The deduction of categories does not need more than this because, as we pointed out, the development of the levels of thinking follows the synthetic method. It is this definition of thought that ‘pulls’ the deduction of the categories of quantity and quality. Thinking consists of the link between unity and multiplicity. Multiplicity, by representing a plurality of differentiable moments requires first a quantitative unity.³⁷⁶

4.2 The Level of Quantity.

³⁷⁴ Natorp considers that all the relations of thinking can be grounded on this fundamental relation of the unity and the multiplicity. Natorp, P., L, §9.

³⁷⁵ Cf. Kant, I., *Proleg*, AA, pp. 275ss.

³⁷⁶ Natorp, P., LGEW, p. 53.

The fundamental act of thinking consists in correlating. The action of the synthetic unity requires three fundamental moments: unity, plurality and totality. When we claim that the concept A contains the elements x_1, x_2, x_3 , etc., we assume the three moments: the units, the plurality and the unity in a totality. The concept A is the unity that brings together an undetermined plurality. In the expression $A = (x_1, x_2, x_3, \dots)$, it is included the unity (A), the plurality of elements that this unity contains (x_1, x_2, x_3, \dots) and the elements that compose the plurality: $(x_1), (x_2), (x_3), \dots$. The relation of these moments requires the position of the plurality and an articulating unity. The moment of the unity (A) is empty *per se*. Being empty means that it is meaningless as such without the multiplicity it contains. It makes no sense to state A is a unity of a plurality of differentiated moments if one does not think in concomitance with it the terms that this unity contains (x_1, x_2, x_3, \dots). Conversely, the elements gathered can only be posited as such in the relationship that constitutes them. The constitution of the plurality of elements of A can only take place as these units are differentiated in relation to each other. The analysis of the concept of thinking as a correlation leads to the discovery of three fundamental actions of thinking in the position of quantity: 1) position of the one, 2) Repetition of the position, 3) Totality. The function of quantitative synthesis depends on the correlation of three fundamental logical moments: units (*Einheiten*), the plurality (*Mehrheit*), and the quantitative totality (*Ganze*). In this case, the multiplicity represents a plurality of differentiable moments. Natorp explains:

Eine Mehrheit ist als solche notwendig Mehrheit aus Einheiten. Die Einheit im Sinne des numerisch Einen, des Einzelnen der Zahl nach, ist also der unvermeidliche Ausgang, das unerläßliche Fundament jeder quantitativen Setzung. Es bedeutet den Einsatz des quantitativen Verfahrens selbst, als des Verfahrens der Diskretion. Was in jedem Falle als Eines gelte, ist hierfür gleichgültig. Eine (der Zahl nach) ist die Welt, eins das Atom, oder was sonst man als Letztes (der Teilung) oder Erstes (der Zusammensetzung — auch das gilt hier gleichviel), als letzte Eins gleichsam, mit der die Natur zähle, ansetzen mag. Stellt man ein solches auf, so ist solche Hypothese selbst diktiert durch das Gesetz jenes Denkverfahrens, welches vorschreibt, von irgendetwas als Erstem zu beginnen, einen Anfang überhaupt zu setzen, d. h. aber in

quantitativer Hinsicht: ein letztes Eines, etwas, dem unser Gedanke diesen Charakter der Einsheit erteilt³⁷⁷.

The concept of unity is the indispensable logical requirement for the characterization of plurality. This is the unavoidable origin of the quantitative process: the establishment of the unitary. The establishment of the numerically one is the beginning of this function, and what is considered as one is completely indifferent. 'One' is both the atom and the triangle and, for quantitative judgment, the determined content of what is established as unitary is indistinct. This establishes the discretion as the first moment of the quantitative synthesis. At this level, the unit is the point of departure. The possibility of a plurality (*Mehrheit*) requires the position of the units (*Einheiten*). It is posited an indeterminate "x" that must be conceptualized under a general concept, for example: A. The first judgment we obtain is: "This particular x is A". A second moment is required necessarily. Each element differentiated as a unit is only relative to something else. The distinction of an x_1 requires an x_2 to constitute a distinct unity. However, this x_2 is nothing considered independently. The x_2 is always in relation to an x_1 . The concept of plurality starts from the unity and generates a plurality as a repeated one-to-one position. The position of x_2 can only be repeated (a second position) if x_1 is retained as already posited. This is the way in which an indeterminate plurality is conceived as a multiplicity. In this way, the open series expressed in the judgment is obtained: "These (individuals) $x_1, x_2, x_3 \dots$ are A". The plurality is the mediating element between individuality and totality as it represents the possibility of repetition one by one infinitely. This second stage consists in the repetition of the units. Thus, we obtained pluralities and units as correlated moments, the units are units of a plurality, and the plurality is a plurality of differentiated units. However, in this second moment the series is still undetermined, a third articulating form that constitutes the unity of the series is required. This is given by the third moment: the conformation of a totality, unity as the unity of many. This third moment is expressed in the judgment: "Every x is A". This judgment contains the previous two moments as its condition. In the third moment, we obtain the totality of the units³⁷⁸. The beginning of the position is always a relative beginning. The element that is posited as the initial moment

³⁷⁷ Natorp, P., LGEW, p. 54.

³⁷⁸ „auch der letzte notwendige Schritt des quantitativen Verfahrens: die Zusammennehmung allemal einer bestimmten, durch diesen neuen Akt eben sich bestimmenden Folge von Einzelsetzungen zu einem Ganzen, d. h. wiederum einer Einheit, aber im neuen Sinn der Einheit aus den Mehreren, ihrer Vereinigung in einem Totale.“ Natorp. P., LGEW, p.55.

may contain within it a multiplicity. Likewise, the whole can be placed as a unit in relation to a superior synthesis of thinking. This process of thought enables the development of the progression. It is possible to conform more comprehensive units every time. This possibility of thinking to determine more and more its object to reach higher units allows progression. The symbolic representation of the quantity levels would be³⁷⁹:

I
II
III ...
(I) (II) (III) ...

The number is the scientific expression of this natural operation of thinking that includes these three moments: the setting of the numerical one; the establishment of the unlimited plurality, and the generation of the determined plurality of the totality. From the point of view of the concepts, the category of quantity results in these three stages:

one (this one) a,
several (these many) a,
all (these all) a,

And it is introduced the possibility of these three types of judgments:

this (one) x is a,
these (several) x1, x2, ... are a,
all of these ... are a

4.3. The Level of Quality.

The categories of quantity are insufficient in themselves to guarantee a differentiated objectification. The functions of the quality categories that objectify the sensation are required to conform the object. This function is intended to distinguish one thing from another in order to understand it from a higher point of view (from a comprehensive

³⁷⁹ Natorp. P, NTE, pp. 345ss.

unity). Quality is the synthetic function of unity that provides a central understanding, an original unity. This synthetic function has, as in quantitative synthesis, three differentiable moments. First, a plurality of differentiation must be put on the basis of a qualitative identity. In the same way as with the numerical unity, in this case, the identity is the first basis, regardless of what is considered as the identically one. However, an allusion to an alterity is inevitably found in every identity judgment. The “this” something can only be defined in relation to an “other” something, and the “other” something can only be defined in relation to a “this”. Both terms are required by the comparison itself. In this qualitative relationship, the one is set as qualitative opposite of the other. There must be at least one differentiating characteristic that establishes the one with respect to the other. This is the basis of the identity position series. However, there must be a third moment where what was separated is reunified from a point of view, under a higher unity of understanding. This point of view is required by thought as that from which it is compared. In this way, the qualitative function represents the synthetic unity of diversity on which a genus is grounded. Genus (*Genos*) is the logical name for this new qualitative unity of uniformity of diversity (*Einerleiheit des Mehrererlei*). The quality, as a production of the diverse from the unity, sets the condition for the exercise of the quantitative function; this is: homogeneity. The establishment of something liable to numbering occurs thanks to the quality function that gives something differentiable that can be measured by number. Only the procedure of enumeration of elements allows to define ‘the what’, while allowing not only a mere description of its attributes but the differentiation of one entity from others. In this way, qualitative synthesis constitutes a unity of understanding that differs from mere composition, allowing the identity to be constituted in diversity. This comprehensive totality based on qualitative synthesis should not be confused with quantitative totality, which is a composition. The qualitative unity is the unity of understanding, an original unity. The synthetic-qualitative function constitutes unity as identity. Natorp concludes:

Denn das entscheidende Moment im Begriff der Gattung ist nicht die äusere Umfassung (Die Einheit des Begriffsumfangs), sondern die innere, zentrale Vereinigung unter einem gemeinsamen Gesichtspunkt des Denkens (dem „Gattungsmerkmal“, als der Einheit des

Begriffsinhalts), das heißt in einer neuen, man pflegt zu sagen, höheren Identität³⁸⁰.

Unlike the quantitative unity that establishes a purely compositional totality (*Allheit*); qualitative synthesis constitutes a comprehensive (*komprehensive*) whole (*Ganzheit*). However, if the number was the scientific expression for quantity, the quality does not have a mathematical expression that reflects its function. Only the expression of the number can serve as a basis for expressing the quality. Therefore, quality is measured by quantity. The degree is defined as the number applied to the quality that can indicate the intensity of something qualitatively characterized. Thus, it is *a priori* established that all content of thinking will have a degree expressed in a *quantum*.

The constitution of the object in general is made in the correlation between qualitative and quantitative synthesis. The synthesis of quantity and quality together represent the two fundamental forms of logical development of thought. The function of synthesis is characterized by the type of judgment A is B, unlike the mere tautology A is A. The judgment A is B, expresses the progression of thinking. For this type of judgment, it is necessary, firstly, the introduction of an A, identical to itself (A = A) and, also, a B identical to itself (B = B); finally, a general point of view from which to postulate a superior identity³⁸¹.

However, the understanding does not stop at the position of universality. The universality cannot take place without the category of infinity that includes the three stages of quantitative synthesis. This infinity should not be understood as a mere absence of an end, that is, in a purely negative sense. If so, it could be conceived as the mere denial of the end, through the mere non-thinking (*Nichtdenken*) of the end. On the contrary, it is the concept of finitude that expresses something merely negative as long as “something has an end means that somewhere the being-nothing (*Nichtsein*) takes place”³⁸². In contrast, it is the absence of the end that states that something is continually. This sense of relative negativity, expressed through quantitative infinity, is the origin of the thinking

³⁸⁰ Natorp, P., LGEW, p. 62

³⁸¹ „Bevor ich sagen kann: A ist (identisch mit) B, muss ich ein Identisches = A und ein Identisches = B haben. Schon der Gebrauch die Begriffszeichen ist ja bedingt durch eine im Gedanke gesetzte Identität, welche durch die des Symbols vertreten wird.“ Natorp, P., QQ, p. 9. This is: Identity is established through synthesis and in no way given. Natorp, P., QQ, p. 8

³⁸² Natorp, P., QQ, p. 19.

process as it represents the possibility of its unlimited progression³⁸³. Infinity is constituted in a purely positive concept by representing the continuing possibility of a quantitative position. Natorp concludes:

Unendlichkeit bedeutet nicht blosse Unbestimmtheit des Endes; sie ist nichts rein Negatives, in dem Sinne, dass sie schon gedacht wäre durch das blosse Nichtdenken des Endes. Es ist tausendmal gesagt und noch immer wahr: nicht der Begriff des Unendlichen ist negativ, sondern der des Endes. Etwas hat ein Ende, heisst: es macht irgendwo dem Nichtsein Platz; es hat kein Ende, heisst: es ist immerfort. Also wird die Unendlichkeit der Quantität die immer fortbestehende Möglichkeit quantitativer Setzung bedeuten müssen³⁸⁴.

Then, the category of infinity is an indispensable condition to ensure the continuation of the task of thinking. The category of infinity expresses the totality of the three categories of quantity. Qualitative infinity, as an inclusive condition of the three moments of quantitative synthesis, is the condition of existence of thought in general and, through its scientific expression, that reflects its most proper legality. The category of infinity allows the continuous limitation of the unlimited that is the proper task of thinking. It also guarantees the very existence of thinking given that “the limiting process itself must be applicable without limitation”³⁸⁵. Quantitative synthesis, in the pursuit of unity and progression of thought, are the true origin of it. The infinity category, which allows the perennial search for unity and progression, is the ultimate legal origin of thought in general as the origin (*Ursprung*) is the ultimate unity pursued³⁸⁶.

In this way, quantitative-qualitative synthesis establishes a transcendental logical concept of infinite progression, which moves away from both a generative-psychological and logical-formalistic conception. The infinite progression in thinking does not consist

³⁸³ For André Laks, the concept of “nothing” is another point of rupture with Cohen. The commenter notes: “Nothingness (the *Nichts*) does not have the absolute sense implied by the original Cohenian concept, but must be interpreted, within Natorp’s conceptual Framework of synthetic unity, as alterity.” Laks, André; (2004), p.483. In this same direction, Éric Dufour observes that Natorp’s central criticism of Cohen at this point is that denial establishes an alterity as a relative negativity and not as an absolute negativity. Cf. Éric, Dufour, 2009, p.41. Coinciding on this point with Laks and Dufour, we believe it is necessary to emphasize that the notion of relative negativity as an infinite possibility of progression in the determination of thought is, strictly speaking, attributed by Natorp to Cohen. Cf. Natorp, P., LGEW, p. 25.

³⁸⁴ Natorp, P., QQ, p. 19.

³⁸⁵ Natorp, P., QQ, p. 20.

³⁸⁶ Natorp, P., LGEW, p. 27.

of empirical progression nor does it establish a logical series concept merely applicable to a special region of objects, as mathematical objects. On the contrary, by means of qualitative and quantitative synthesis, a logical-transcendental notion of progression in knowledge is established on which the empirical genesis and the formation of the series of numbers depend³⁸⁷. The logical-transcendental synthesis is a condition of legitimacy and regulations regarding the empirical experience. Thus, the first laws of thinking, which determine the essentially relational character of it, are progressively grounded through the deduction of the categories of quantity and quality.³⁸⁸ The laws of number are derived from the logical process of quantity and quality³⁸⁹. These logical functions build the numerical series. Therefore, the numerical series holds the properties of the logical position. The fundamental operations of numbers are also obtained by virtue of this mode of thinking. Regarding the fundamental series of numbers, Natorp points out:

Comme pure expression du procédé pris en lui-même et généralement, elle est — nécessaire et universelle — unique et la même dans toutes ses applications—indéfiniment prolongeant, car le procédé qui la fonde a des ressources illimitées (6) — homogène, car ses termes d'après leur détermination même ne sont que des numérateurs et tous produits également par juxtaposition, ils ne se distinguent donc pour la pensée par aucun autre signe que leur rang dans la série ; de plus chaque terme de la série enveloppe cette série tout entière, puisque le procédé de la numération est déterminé dès le commencement et conséquemment à partir de n'importe quel terme donné par une régression vers le commencement ou une progression à l'infini.³⁹⁰

³⁸⁷ This is also stated by Young: “The author regards as a primitive faculty of the mind the power of conceiving any mental act to be repeated indefinitely. He thus obtains essentially what mathematicians would call the abstract form of an unlimited sequence”. Young, JW, 1913, p. 370.

³⁸⁸ Faced with the correlativity of qualitative and quantitative moments, in some instances, Natorp seems to grant a preeminence to the level of quality. Natorp states in 1900: “Thus, the two fundamental processes of quantity and quality correspond in all their evolution and that they raise in a narrow interdependence the fundamental signs of the progress of knowledge, its peripheral extension and its central deepening; that is, its unification. But what is first in itself is the primitive unity of the third level of quality. For in thought, the content determines the continent, the height of the point of view establishes the extent of the horizon.” Natorp, P., NTE, p.347.

³⁸⁹ Cf. Natorp, P., EGM, p.2; NTE, p.355; LGEW, p.98ss.

³⁹⁰ Natorp, P., NTE, p. 355.

Number is the purest expression of thinking³⁹¹. The series of numbers is built thanks to the processes of quantity and quality. The relationship of the series with its members is determined based on these fundamental logical processes. What distinguishes the members of the series is their place. The distinctive feature of each term is defined by the place they occupy in the series. To be a particular number is to occupy a certain place in the series. The number is the term of the relationship. The number cannot be defined separately from the relationship it establishes with the other members of the series. The determined number is the term in the set of relationships.

The series is generated in the iteration of the quantitative and qualitative process in which each term placed is considered as a counter-term in relation to a previous position. First of all, there is the position of the one, the position of an individual as the first element to form multiplicity. Second, a repetition of this initial position is necessary. This repetition must ensure that the previous moments are preserved, a repetition of the initial position is generated while retaining the previous positions. The second term is presented as a counter-term with respect to the previous one. The third moment generates the possibility of taking the terms as new initial moments. That which was put as 'the other' with respect to an initial position, can be considered as a new position in relation to another position. The unity of one and the other can be considered as a totality with respect to a later moment. The unity of the one and the other can also be a unity. Each of the terms can be either first term or a counter-term. This process is infinitely iterable. That which was a whole with respect to its parts can also be considered as a unit for the conformation of higher order totalities. There is no absolute beginning of the position of one, but there is an iterable structure where nothing is itself a unit or a totality in an absolute sense. This possibility of positing the terms in different relationships can lead to the mistaken conviction that the terms can subsist regardless of the relationship itself. This possibility of putting the terms of the relationship in one or another relationship gives the appearance of independence. Each of the terms has the appearance of independence by the possibility of being put into different relationships. The possibility of positing each of the terms in different relationships can generate the appearance that these terms can exist in itself and that only in one instance further are placed on relationships. On the contrary, the terms can be posited in different relationships because their determination only consists in being a term

³⁹¹ Natorp, P., LGEW, p. 98.

of a relationship. The terms do not have any other determination beyond these positions in which they are placed. An element can be term or counter-term depending on the relationship considered. This possibility gives the appearance of independence of the terms. The possibility of entering into multiple relationships generates the appearance of the independence of the terms, from the possibility that they have of entering into a relationship in general. However, this apparent independence is nothing more than the possibility that the terms have to establish different relationships. Each member of the series is defined by virtue of the position it occupies. The iterated position from term to term generates the series along with the possibility of directing the plus and minus as oppositional relations. The quantity and quality allow a positional relationship according to the before and after³⁹².

The properties of the numerical series are derived from the operation of these fundamental logical processes. The properties of the series are concrete expressions of the general operation of thinking. As an expression of pure thought, this series is: necessary and universally valid, unique, infinite, homogeneous and continuous. It is universally valid because it is grounded on the pure process of thinking. It is unique because the permutation of values only alters the position in relation to the same set of relationships. The exchange of values does not generate a new series as long as the determination of the value is only based on its position in the series. The function of each value is always interchangeable³⁹³. As the process is always iterable, the series is infinite and open. This iteration results in the open infinite series on both sides, from the plus side and the minus side. In the series, each fundamental member of a first relationship can turn into a counter-member, and each counter – member can become a fundamental member in relation to another counter - member in a new relationship³⁹⁴. This iteration allows a term to always be considered as counter-term and vice versa. No term can constitute an absolute beginning, but every term can adopt the function of beginning. The series is homogeneous because its values are equivalent. The direction of the plus and minus can be reproduced anywhere in the series by taking a moment as 0 and expressing in relation to it, again, a term and counter-term relation. The difference of

³⁹² „Die Beziehung der Position oder der Ordnung des Vor und Nach erwies sich als das letzte Gattungsmerkmal der Zahl, welches aller Maßbedeutung derselben logisch vorhergeht Sein mathematischer Ausdruck ist das Plus und Minus, welches eine immer gleiche Art der Relation von Glied zu Glied unserer Urreihe“ Natorp, P., LGEW, p. 225.

³⁹³ Natorp, P., LGEW, p. 113.

³⁹⁴ Natorp, P., L, p. 31.

the members of the series is only in relation to the place they occupy, so the series is homogeneous. Each determination of a value is relative to the function it occupies.

The quantity allowed indefinite positions and guaranteed the possibility of considering the plurality of differentiated positions in a total quantitative unity. Meanwhile, the quality allows the continuous transit from one magnitude to another. The possibility of this continuity is not given but is based on the fundamental operation of thought³⁹⁵. Each unity will be a moment of provisional rest. The relationship of a term with another will be the place where a term is positioned in relation to a position selected as instance '0'. The 1 is the zero point of the numbering. The position of before and after is, in relation to this initial position, always relative. The definition of the numbers in the series is done by virtue of this term-to-term relationship in the fundamental series with respect to the function assigned as '0'. In this way, the 'absolute' need for an initial position and the relative character of what is set as the beginning is recognized. The 0 can be defined as "a reference point or comparison point in view of the position of a given value"³⁹⁶.

Concomitantly, the fundamental operations of numbers can be defined by virtue of the position of quantity and quality. Just like the moment 0, the position of 1 is a necessary moment for the beginning of numbering. The 1 will be defined as the addition of a unit to what has been considered as moment 0. Thus, operation $1 + 1$ is logically equivalent to operation $0 + 2$ ³⁹⁷. In this way, the fundamental series finds its foundation in the operative of thinking. The process of quantity and quality are expressed in the numerical relationship.

4. 4. The Level of Relation.

The categories of quantity and quality determined the object as a magnitude³⁹⁸. Natorp exhibited that the object, as an object of thinking, is already completely created by

³⁹⁵ Natorp, P., NTE, p. 365. LGEW, p. 180. At the foundation of continuity, Natorp seems to give a preeminence to the category of quality over that of quantity. He states: „Kontinuität ist ein so ursprüngliches, unverbrüchliches Gesetz des Denkens, dass überhaupt irgendwelche Diskretion sich nur als Diskretion eines Kontinuums will denken lassen. Also gibt es für das reine Denken das Kontinuum der Beziehungssine oder Richtungen ebenso wie das Kontinuum der Werte.“ Natorp, P., LGEW, p.237.

³⁹⁶ Natorp, P., LGEW, p.181.

³⁹⁷ Natorp, P., LGEW, p.135.

³⁹⁸ ³⁹⁸ „Durch die beiden eng Verbunden Verfahren der Quantität und Qualität ist für die mögliche Bestimmung eine Gegenstand eine erste gesetzmäßige Grundlage gegeben. Sie enthalten das zureichende

thinking itself by the categories of quantity and quality. The quantitative and qualitative determinations were capable of constructing objectivity in its most general determinations. Natorp argues that one could be easily led to conclude that a further moment is not necessary, as it was already exhibited how the object of thinking can be created by thinking alone. However, the task is the constitution of experience, and experience is more than a single object. For this reason, the constitution of the isolated object is not enough for our purposes, i.e., to explain how objectivity can be constituted purely conceptually. The determination of the object in general is insufficient. Natorp explains:

Wir haben vielleicht den Gegenstand, aber noch nicht die Gegenstände, nämlich nach ihren gegenseitigen Verhältnissen der Abhängigkeit, dass heißt nach der Art, wie sie nicht bloß jeder für sich als bestimmt überhaupt gedacht, sondern als in einer Erfahrung sich untereinander bestimmend erkannt werden. Erfahrung mag noch so sehr auf dem Denken beruhen, sie ist doch etwas mehr als nur überhaupt Denken; jedenfalls mehr als das Denken, welches nur Denken der Quantität und Qualität wäre³⁹⁹

Experience exceeds the determinations of quantity and quality. To construct the experience, it is necessary not only to determine the object but also to connect the objects. Concomitantly, to know is, precisely, to put objects in relation. To know is not to determine an object but to establish the relations among objects. For this reason, it is necessary to have another level of determination. The object created by thinking must be related to other objects. It must be shown how the object can be put into relationships with other objects. Now, it is necessary to establish a system of objects. It must be guaranteed the possibility of an interconnection of objects. For this reason, it is necessary to have another level, the level of relation. In the level of relation, it must be established the conditions of possibility of the interconnection among objects. At this point, the constitution of the plurality of objects and the relations among them is required. Natorp

logische Fundament für denn Gegenstand als Größe, und dies Fundament reicht aus für grenzenlos verschiedene Setzungen von Größen, unter denen auch wieder Beziehungen denkbar und, sofern sie in einer Erkenntnis zusammen bestehen sollen, zu denken notwendig sind.“ Natorp, P., L , p. 24.

³⁹⁹ Natorp, P., LGEW, p. 65.

must show how thinking can determine not only objects but the system of objects. It must guarantee the condition of the possibility of the relation of objects among themselves. Thus, thinking seeks every time higher levels of relations to accomplish what was posited as a task: the constitution of experience. The method demanded that the determinations cannot be considered as if they were given. On the contrary, it must be shown how they are produced⁴⁰⁰. On these grounds, a further level is required. Thinking must generate the system that enables the interconnection of these systems, generated by quantity and quality, among themselves. The synthesis of quantity and quality gave us a multiplicity of series. The simple synthesis generated by the first levels is insufficient. This second-order of relation will not be just a synthesis but a ‘synthesis of synthesis’⁴⁰¹. Indeed, every act of thinking can be conceived as a way of relating. To think is to relate. To relate is the fundamental action of thinking. However, the general mode of relating, present in every act of thinking, must be distinguished from the determined action of relating that enables the interconnection of the series of the magnitudes generated by the process of quantity and quality. This is a specific mode of relation that generates higher levels of determination. The mathematical determination of the object must be completed by the dynamical determination.

Thinking takes as its starting point the moments that it generated. It does not depart from any given factum to conform its objects. Rather, it departs from its own creations. This determination can only come from thinking. The relation present in the interconnection of the objects is the interconnection according to law, the determined connection. The act of determination according to the relation is expressed in the law. The law is the expression of the determination of the connection⁴⁰². Natorp remarks:

Dieses Verfahren wird beruhen müssen nicht auf einer einfachen
Synthesis eines Mannigfaltigen (diese ist in dem Doppelverfahren

⁴⁰⁰ „Nicht als hätte die Logik hier nun doch zum „gegebenen“ Wirklichen zu fluchten; das hieße den Weg des reinen Denkens schlechthin verlassen. Sondern was Erfahrung mehr enthält gegenüber dem Denk[^]i, wie wir es bis dahin kennen lernten, dem Denken der Quantität und Qualität, muß in seiner Möglichkeit doch wiederum als Denken, als eine höhere Stufe, gleichsam eine höhere Potenz des Denkens sich verstehen lassen. Wirklichkeit selbst, Gegebenheit ist Denkbestimmung, und zuletzt Leistung reinen Denkens. Aber auch noch nicht diese (die erst die Modalität zu vertreten hat) steht hier in Frage, wohl aber die bisher noch nicht erbrachten methodischen Vorbedingungen dazu.“ Natorp, P., LGEW, p. 66.

⁴⁰¹ „Diese können nur gesucht werden, nicht in der einfachen, sondern in der gegenseitigen Bestimmung; in Wechselseitigen Abhängigkeitsbeziehungen also, gemäß der ersten Stufe welchen Gegenstände (d.h. quantitativ-qualitativ bestimmte) sich gegenseitig-bestimmen. Also nicht mehr in einer einfachen Synthese, sondern einer neuen in Synthesen von Synthesis, synthetischen oder synthetischer Einheit Einheiten.“ Natorp, P., LGEW, p. 66.

⁴⁰² Natorp, P. LGEW, p. 66.

der Quantität und Qualität erschöpft), sondern auf einer Synthesis von Synthesen. Dies ist es, was Kant als das Verfahren der Relation bezeichnet.⁴⁰³

Quantitative-qualitative synthesis left a system of order. The order of the determined series is now required to generate a total order. The result of this total order will be nature. Nature is this system of interconnected laws. This interconnection can be called function (mathematically) or law (in connection with nature). The connection according to a law will allow articulating each separate series in a system of series. It is a demand of thinking that each generated series is not left indeterminate. It must be connected in a higher order. Each quantitative-qualitative series must be incorporated into the total order, under its subsumption to the law expressed in the function. The process of thinking itself demands that nothing remains undetermined and then, that every series can be connected with every other. The law expresses the necessary connection of any term with any other by conditionally indicating that if one condition is met, another necessarily follows (“if A, then B”). Through this connection, the function generates a new superior concatenation. The requirement is a total ordering, a requirement that remains as a regulatory idea for thinking in each of the progressions of its actions. As Natorp already showed, it belongs to the essence of thinking that this process can never be exhausted⁴⁰⁴. For a limited cognition, the accomplishment of the complete determination of experience is a demand that can never be fully accomplished.

The final objective of this act of relating is a totality in which each of the particular series can relate to another in the conformation of a system. The law is the expression of these connections. What is sought is the total connection of each of the particular laws in a total system. This is the guiding idea of thinking, articulating a coherent order

⁴⁰³ Natorp, P., L, p. 25.

⁴⁰⁴ „Aufgabe ist: Ordnung des Einen nach (d.h. gemäß) dem Anderen, wodurch ein System von Ordnungen, das heißt eine Gesamtordnung entstehe. Eine solche ist, in der Sprache der Mathematik: die Funktion, in der Sprache der Naturwissenschaft: das Gesetz. Die Glieder, unter denen solche Ordnung herzustellen, sind, wie gesagt, Ergebnisse einfacher, quantitativ-qualitativer Synthesen, also Größenreihen, je far sich aufgebaut nach den Gesetzen der quantitativ-qualitativen Synthesis. Die Ordnung dieser Reihen aber, gemäß welcher sie sich untereinander bestimmen, wird dann bestehen müssen in solchen Beziehungen unter ihnen« welche eine gesetzmäßige Verknüpfung von Glied zu Glied der verglichenen Reihen herstellen. Man kann es foglich bezeichnen als Ordnungs Synthese, wobei zu denken ist nicht bloß an eine irgendwie geordnete Fortschreitung von Glied zu Glied in jeder Einzelreihe; dazu würde die quantitativ-qualitative Synthesis für sich ausreichen; sondern vielmehr daran, daß die Art der Ordnung, die an sich auf vielfache Weise möglich ist, für jede Einzelreihe sich bestimme durch eine gesetzmäßige Beziehung zu irgendwelchen, schließlich allen parallelen Reihen; das heißt, es wird die Ordnung in jeder Einzelreihe determinierbar, indem sie an die Bedingung einer bestimmten gesetzmäßigen Beziehung zu den Parallelreihen gebunden wird.“ Natorp, P., LGEW, p. 69.

system. This is the requirement of thinking in its demand for unity. In this progression, thinking aspires to generate higher-order units. This demand for unity is the guiding idea of the progression of thinking, i.e., searching for every time higher stages of unity. This context of the total order will be the way in which the coherent total order of the various series systems is established. The requirement consists in the prosecution of an order of the multiplicity of laws in an articulated system. This total articulation will be the reality, the goal that thinking proposes as a task. Thinking seeks a unique order that can contain all the laws in an articulated system. This search is the eternal task of thinking, which can only be approached in its progression. However, this idea operates as a guiding thread. Thinking operates with this requirement as a goal. The consummation of this requirement, as we will see in detail in the next section, can never be accomplished⁴⁰⁵.

The starting point of the moments of the relation is those that thinking created itself. The terms of the relation are generated by thinking. In this case, the starting point is the object generated by the process of quantity and quality. The relationship will also have three moments: an initial position, a continuation of the position, and a closure. As in the previous levels, there will be an initial unit, a multiplicity, and a whole as a temporary total closure. Natorp claims that he will arrive at the same result that Kant did. There will be three levels: a) substantiality, b), causality c) community.

The initial act consists in the position of a *Grundreihe* as the first term established by thinking⁴⁰⁶. This is the equivalent to the Kantian category of substance. As the unit was the first moment of quantity and the identity of the quality, the *Grundreihe* is the first moment, as a 'unit' to be connected. It is a ground for every possible relation. Natorp calls it a *fundamentum relationis*. This first moment consists of the establishment of a reference system that allows the series to be articulated with each other. This will be the fundamental series.

The various relationships series will be possible under this common reference to this basic series. The first requirement is the establishment of a system of positions, of a scale, in which the course of interconnected series of variables can be articulated. Natorp recognizes a coincidence with the Aristotelian position. Indeed, as Aristotle noted, it is necessary to establish a subject that is invariant in relation to its determinations. A subject must be established with respect to which these determinations are taken as accidents. The

⁴⁰⁵ Natorp, P. LGEW, p.69.

⁴⁰⁶ „Die Möglichkeit einer Reihenordnung der verlangten Art erfordert als Erstes eine feste Grundreihe, als Fundament der ganzen Reihenordnung.“ Natorp, P. LGEW, p.70.

possibility of a dynamic system of connections demands the establishment of something constant. Any possibility of determining the change would be lost if nothing could be established as constant⁴⁰⁷. Thinking demands the invariance in respect of which every change can be determined. However, this is only a logical requirement, not something real permanent given to thinking. The fundamental series is not a thing but a process⁴⁰⁸. Aristotelian naive realism confuses a logical requirement with the postulation of an entity: the substance. This is the mistake of naive realism. Thinking requires taking something as invariant, but that invariant is only provisionally determined by virtue of this requirement. It is not something given to thinking but a logical demand. This demand can only be satisfied in provisional stays in which thinking takes a certain stage as invariant in relation to a series of changes. Naive realism confuses the requirement with its satisfaction and transforms this satisfaction into a metaphysical entity. This absolute instance of the fulfilment of the requirement of an invariant is hypostasized in a metaphysical entity: the substance. This is the mistake of the Aristotelian position. The substance is not an entity that remains invariant in contrast to changes. The substance consists, rather, in the procedure of taking as invariant a certain moment to think from there the series of changes. It is an action and not a thing. This distinction between the variant and the invariant is always relative, as long as something is taken to be an invariant only by virtue of articulating certain determinations at a certain stage. The substance is not a metaphysical entity but a way of establishing a relationship, a logical condition. This is the first logical condition to establish the whole system required by the very concept of nature. As we pointed out, the very establishment of nature demands to assume this stage of invariance as a logical requirement. But this demand must remain a requirement. It may vary what is considered invariable. Only the requirement remains. It must remain as a logical requirement of thinking. The substance will be this requirement of thinking of establishing an invariant to put in relation the quantitative-qualitative series.

As we exhibited in Chapter 3, the leading thread of the task is the definition of thinking as correlation. Relating is the most proper task of thinking⁴⁰⁹. To think is to relate. As we saw, the general function will always be to distinguish and relate what was distinguished. The second moment will consist in the generation of an act that allows each

⁴⁰⁷ Natorp, P. LGEW, p.71.

⁴⁰⁸ „Unsere Reihe bedeutet ja nicht ein Ding, sondern ein Verfahren“. Natorp, P., EGM, p.6.

⁴⁰⁹ „Aber der wahrhaft letzte Grundbegriff des mathematischen und alles strengen Denkens überhaupt ist viel mehr die Relation.“ Natorp, P., EGM, p.6.

term of the series to be ordered in the relation among themselves. The quantity and quality levels showed the series with ordered members: $(x_1 x_2 \dots)$ $(y_1 y_2 \dots)$. Now, the series $(x_1 x_2 \dots)$ must be able to be connected to the series $(y_1 y_2 \dots)$, so that a unitary system can be generated between the heterogeneous. The question that must be answered is how can the indexes $(1,2,3, \dots)$ be connected when they belong to different series (x,y,\dots) . Time will be the form that allows the series to order each other and thus the ordering of events. The series formed by the level of quantity and quality had an internal order. Now it is necessary to connect a series with another in search of the total order. The first step would require the possibility of establishing a link between each member of one of the series with the members of the other. The members of the series of the x must be able to be posited in relation with the members of the series of the y . An order must be arranged that articulates the x_1, x_2, x_3 , with $y_1 y_2 y_3$, etc., so that each first member of one of the series is connected to the first member of the other series. Time will be the universal scale that allows this arrangement. In every moment of time, everything that is the first member of each series will be contained. Thus, it will be possible to claim in time 1 we find x_1, y_1 , etc; in time 2, x_2, y_2 , in time 3, x_3, y_3 , etc. Time is the condition of the possibility of establishing this order. Thinking requires the articulation of the series, and time will be the condition that allows a first answer to this demand. Time is the most basic order of occurrences. It is the first requirement for the series to be articulated in a common higher-ordered system. This scale will allow the order of the simultaneous. However, it is also necessary that what is contained in each moment can be distinguished. An order is necessary in this 'being together' (*Miteinandersein*). One must establish the order of these elements of each of the series contained in each moment. This second condition will be the spatial order. The space will be the order of the successive because it will be what allows us to distinguish what it is at the same time. The expansion of the simultaneous generates the spatial places. The possibility of this establishment of places allows the 'expansion' of what is contained in the instant. If the function of thinking is to distinguish and gather the distinguished, the specific expression of this function is now the distribution in places of what is contained in each moment. The establishment of spatiality allows to conserve the multiplicity contained in each moment of time. Space and time together enable an order that posits the series in a common system. Space and time are specifications of the function of thinking as correlation. They are functions of thinking; more precisely, functions of the relation. The movement establishes the relation between the instants and the places of space. The movement will allow coordinating the space

points with the time points⁴¹⁰. In this way, it is established a system of variance and invariance. Time and space are connected in the concept of movement. For this reason, the movement expresses the basic form of change in nature. The variant will always be in relation to what is considered to be permanent. Conversely, everything permanent will always refer to something that varies. Each moment necessarily demands the other. Change can only be understood in relation to permanence and vice versa. That something changes implies that there is also something that remains the same in relation to what changes. This is the relation between the substance and its accidents. It is the product of this logical requirement of thinking, the relationship between the invariant and what varies. Time could allow an order between the series. However, “the state of something that changes at a given time can only be defined through its dynamic relations with the whole with which it is disposed according to a law”⁴¹¹. The task of the function of causality will be to gather the moments that were considered at first independently. In this level, the relationship of the change from moment 1 to moment 2 is indicated. The final goal is ordering in a total system⁴¹².

As long as a systematic interconnection is required as a whole, the principle of causality will be insufficient unless each series is connected with every other. This is only possible thanks to the determination of the reciprocal action. Natorp argues that the relation among series would be insufficiently grounded if it is not established a more fundamental law according to which the relations between series can be made. Thinking cannot create anything fortuitous but a lawful integrated system. Natorp holds:

In der Tat würde die gesetzliche Beziehung von Reihe zu Reihe so lange zufällig, d.h. unzureichend begründet sein, als nur auf eine beliebige, zufällige Mehrheit, nicht auf eine durch eine gemeinsame und zwar fundamentale Beziehung begründete Allheit paralleler Veränderungsreihen die Forderung der gesetzmäßigen Entsprechung von Glied zu Glied sich erstrecken würde.⁴¹³

⁴¹⁰ „Die bestimmte Zuordnung aber bestimmter Raumpunkte zu bestimmten Zeitpunkten im Verlauf einer Veränderung ist es zugleich, welche den Verlauf dieser Änderung selbst, fundamental also als Bewegung (räumliche Änderung in der Zeit).“ Natorp, P. LGEW, p.74.

⁴¹¹ „... der Zustand eines Veränderlichen zu gegebener Zeit läßt sich nur define durch seine dynamischen Beziehungen zum Ganzen, dem es sich gesetzmäßig einordne.“ Natorp, P. LGEW p.79

⁴¹² „Die Synthesis der Erscheinungen nach dem Verhältnis der Ursache und Wirkung beruht darauf, daß eine Mehrheit von Vorgängen zu einander im Denken in eine solche Beziehung gesetzt werden, daß allemal der Wechsel in einer Reihe des Geschehens nach dem Wechsel in einer andern Reihe von Moment zu Moment sich bestimmend gedacht wird. Nur dadurch können die verschiedenen parallelen Veränderungsreihen sich in die eine gemeinschaftliche Zeitfolge bestimmt ordnen und also in der einen Erfahrung zusammenstehen.“ Natorp, P., PP, p. 29.

⁴¹³ Natorp, P., L, p. 26.

The modes of relationship between the series are established in a necessary way thanks to the generation of a total lawful system that contains the particular interconnections. It will be a task for thinking to establish the whole of these possible relationships, the totality of relations of a series of changes with every other. This is the task of thinking. This third level enables us to comprehend all events in nature in a coherent integrity. The series is determinable only in relation to the total system. The system as the global coordination of series is the final condition of the determination of each series as such. Each series is thought of as jointly determining and determined by the others. For this reason, the complete system of the coordinated series must be presupposed. Natorp presents the example of how the sun heats a stone, the same example that Kant took in his *Prolegomena*⁴¹⁴. Suppose that the sun illuminates the stone and heats it. At time 1 the stone has a certain temperature and at time 2 the temperature increases. The subject remains identical and, however, its state varies. To substantiate how the sun heats the stone, the causal relationship establishes that given condition a, consequence b occurs. However, for the sun to heat the stone, a whole series of conditions not specified in the judgment must be met. The level of causality is insufficient to explain the circumstances that led to the heating of the stone. For this reason, the problem of the relationships to other parallel changes inevitably arises⁴¹⁵. In this third level, which unites the three moments of the relation, we reach the highest point of the synthetic unity⁴¹⁶. In this way, the determination of experience required three interconnected processes. First, the

⁴¹⁴ Kant, I., *Proleg.*, AA 4, p. 301.

⁴¹⁵ Natorp explains: „Der Stein wurde eine gewisse Zeit von der Sonne beschienen, davon wurde er warm; d. h.: im Momente 1 zeigte er einen bestimmten Wärmegrad, im Momente 2 einen anderen, höheren; woher kam diese Änderung des Zustandes in dem übrigens der Voraussetzung nach identisch bleibenden Subjekt; d. h. rein methodisch gesprochen: wonach ist diese Änderung auf gesetzmäßige Weise bestimmt? Das Gesetz der Kausalität antwortet hierauf nur, dass eine Ursache dafür sein mußte, d. h. etwas, irgendein Umstand oder eine Summe von Umständen (Bedingungen) im Zeitpunkt 1, welche diese Änderung bis zum Zeitpunkt zum Ergebnis haben mußten, d. h. aus welchen dieses Ergebnis für den Zeitpunkt 2 nach einem Gesetze bestimmt ist. Fragt es sich dann aber weiter, welcher Art solche bestimmenden Momente seien, so kommen, wie das Beispiel klar zeigt, unumgänglich die Beziehungen zu anderen parallelen Veränderungen in Frage. Die Sonne traf vorher den Stein nicht, sei es weil die Achsendrehung der Erde noch nicht die dazu erforderliche Lage der Sonne gegen den Stein herbeigeführt hatte, oder eine Wolke den Zutritt der Sonne zum Stein hinderte oder dergleichen. Kurz es mußte etwas, nicht im Stein für sich genommen, sondern in sonstigen, aber ihn irgendwie mitberührenden Umständen sich geändert haben. Das Gesetz der Kausalität sagt nur: Unter gleichen Bedingungen im Zeitpunkt 1 gleiches Ergebnis im Zeitpunkt 2; es sagt für sich nichts darüber, welche und welcher Art diese Bedingungen seien; es behauptet nur eine Gesetzmäßigkeit der Zuordnung überhaupt eines Consequens zu einem Antecedens, eine Gesetzmäßigkeit also, die als solche und unmittelbar nur die Ordnung der Sukzession betrifft.“ Natorp, P., LGEW, p. 80.

⁴¹⁶ „Damit ist die Einheit des Gesamtgegenstandes der Erfahrung, und also die Einheit der Erfahrung selbst oder der „Natur“ in abschließender Weist methodisch ermöglicht, und so der höchste Punkt erreicht, zu dem alle einzelnen Leistungen der synthetischen Funktion zusammenstreben.“ Natorp, P., PP, p. 30.

establishment of an invariance, the moment of substantiality as a logical requirement. Second, the causality, as a condition of the establishment of a relation among objects. Third, to search a total articulated system of relations. The complete determination is a task for thinking, something that must be achieved, not a datum⁴¹⁷.

4. 5. The Level of the Modality.

According to Natorp, as it was for Kant, the level of modality does not add a new determination to objectivity. There is no direct determination of the object but an indication of the ways in which the object can be related to thinking⁴¹⁸. The categories of quantity, quality, and relation determine objectivity. However, it is necessary to establish the link between the object and thought. It has not been determined whether the object is merely a possible object, an actual object, or a necessary object. It has to be decided whether this object is possible, real, or necessary. The fact that the object is possible, real, or necessary does not determine the object itself. The object does not change its determinations being possible, real, or necessary. Only the statutes in relation to knowledge changes but the determinations remain. At the level of the modality, there is added a consideration regarding the way in which the object is considered with respect to the faculty of knowledge. It is possible to claim that the object is possible, real, or necessary without adding any determination. This problem could not be raised at the levels of the determination of objectivity because it is not a question that refers only to the constructed object, but it is a problem that concerns the relation between the object and thinking. Therefore, following Kant, Natorp states that the levels of the modality do not add any determination to the concept of the object⁴¹⁹.

⁴¹⁷ „Der logische Grund dieser Supposition ist zuletzt kein anderer als die Notwendigkeit, das Wirkliche auf einzige Art bestimmt zu denken; also muß is jedenfalls bezogen werden auf eine in einziger Art bestimmte Ordnung der miteinander in einer Natur Zusammenstehenden Erscheinungsreihen. Daß eine solche empirisch gegeben weder ist noch je werden könnte, macht es nur um so fühlbarer, daß diese Ansetzung eine reine Denkleistung ist und kein Datum.“ Natorp, P., LGEW, p. 72.

⁴¹⁸ „Dies nun betont gerade Kant: die Modalitätsstufen betreffen direkt nicht den Gegenstand, wohl aber sein Verhältnis zur Erkenntnis, ihre Gegenständlichkeit Nachdem wir uns aber überzeugt haben, wie sehr der Gegenstand überhaupt nur in der Gesetzmäßigkeit des Denkens, des Erkennens wurzelt, hat es wohl Sinn, dies Verhältnis auch noch besonders ins Auge zu fassen.“ As Éric Dufour observes, in what has been considered as the second period of his production, Natorp profoundly modifies his conception regarding the status of the modality category. The modality has in the *Philosophische Systematic* constitutive role of objectivity. Cf., Dufour, É, 2010, p. 181.

⁴¹⁹ „Es sind also nicht neue Leistungen der synthetischen Einheit, die in den Modalitätsstufen formuliert werden, sondern es ist die Gesamtleistung des synthetischen Prozesses der Gegenstandserkenntnis, wie er in der Quantität, der Qualität und der Relation nach seinen Grundrichtungen sich auseinanderlegte.“ Natorp P., LGEW, p. 86.

For this reason, unlike the levels of quantity, quality, and relation, the level of modality cannot be derived purely and exclusively from the principle of synthetic unity. Like Kant, Natorp considers that the determinations of the object are exhausted with the categories of quantity, quality, and relation. The modality does not add anything to the construction of the object but defines how the objects are related to thinking. The level of the modality characterizes how thinking relates to the object constituted in the previous levels. The level of the modality receives this name because, precisely, it 'moulds' the experience. It establishes how thinking is related to the object. In a judgment of modality, what is shaped is not the object itself but the way in which thinking conceives the object. The modality is not a necessary feature for the construction of the objectivity itself⁴²⁰. Therefore, it cannot be derived directly from the concept of synthetic unity.

For this reason, the levels of the modality are the clearest expression of the ideality of the object. The levels of the modality show in a paradigmatic way the dependence of the object on thinking. The qualification of possible, real or necessary is completely meaningless without its relation to the process of thinking. The modality determinations only make sense when considering the relationship of the object with the act of knowledge. These determinations cannot be attributed to the object regardless of the consideration of the way in which the object is known. Certainly, the categories of quantity, quality, and relation are also determinations that arise from pure thinking. However, in the modality this aspect is seen more clearly. In the consideration of the modality of the object, the possibility of thinking the determinations of the object independently of the act of thinking is banned from the beginning. Therefore, Natorp argues that the ideality of the object is most clearly seen in the consideration of the modality. In the modality, thinking can establish those determinations of objectivity that cannot be defined independently of the thinking process. The determination of the objects as possible, real or necessary only arises as a problem with regard to the relation of the process of thinking to the objects.

A further peculiarity of the modality is that it describes the general structure of the three category levels, quantity, quality and relation. This structure that we were emphasizing as common for the three categories - starting from a first position, the generation of a multiplicity, and the unity of this multiplicity- is the basic structure that describes the modality. The level of quantity, quality, and relation will have the tripartite

⁴²⁰ „Gehören sie dem Gegenstande direkt vielleicht nicht an, dann um so mehr dem Denken, dem Erkennen - immerhin des Gegenstandes.“ Natorp. P., LGEW, p.82.

structure characterized by the levels of the modality: a first arbitrary position as an initial hypothesis, a second moment of repetition of the initial position preserving the previous moments, and a provisional closure in the form of a totality. Thus, for example, the first moment of the quantity is the establishment of a unit that will operate as a reference point in the measurement, a quantitative unit in this case. This arbitrarily established measurement pattern is the initial hypothesis indicated by the modality. The second moment is the repetition of that unit a certain number of times. This is the moment of multiplicity: the repetition of the measurement pattern. Thirdly, a provisional closure is made in which the entire measure is indicated. At this moment, that multiplicity is thought as a unity. This structure of the modality is also repeated for the levels of the relation and the quality. In the case of the relation, the first moment is the adoption of one of the series as a hypothetical unit. The association of one of the series with other series is the moment of multiplicity, the construction of a series system in the union of one series with another. Finally, the third moment consists of the union in a system with the final goal of connecting that set of series in one total unity. Thus, each of the moments of the modality corresponds to each of the moments of the relation, as well as with each level of the rest of the categories. In this case, it can be clearly appreciated that the modality does not propose new determinations of objectivity. It does not describe new specificities of the synthetic unity. The modality indicates the path of the synthetic unity in general. It shows how the procedure of the synthetic unity is expressed in each of the fundamental directions of thinking: quantity, quality, and relation. In the modality, the path of the synthesis is reflected. While describing the relation of thinking with the object, the modality establishes the ways in which thinking created objectivity. Therefore, it is also present in each construction of the experience because it describes the path that thinking follows in the construction of its objects. The modality describes the universal course of synthesis levels.

Natorp argues that the problem of modality is introduced with the concept of nature. The distinction between the possible, the real, and the necessary is not present in mathematics. In mathematics, the construction of the object itself affirms its existence and, with it, its possibility and its necessity. Every object of mathematics is real, possible, and necessary. The object that the mathematician names as existing does not distinguish a real entity from a possible or a necessary one. In mathematics, the demonstration that an object is possible guarantees its existence, and this existence is never merely contingent but always a necessary existence. With the assumption of the existence of

a mathematical object, it is affirmed that the object has been constructed mathematically, i.e., with the means of mathematics. Mathematically created objects all have a logical necessity. Therefore, the object thus grounded is immediately possible, real, and necessary. In mathematics, the thought of the object does not require the distinction between possibility, existence, and necessity. Natorp notes:

Es ist sehr bemerkenswert, daß es innerhalb der bloßen Mathematik diesen Unterschied der Modalität nicht gibt. Zwar reden die Mathematiker von einer Existenz ihrer Begriffe (etwa des Irrationalen, des Imaginären), aber diese Existenz unterscheidet sich in nichts von der Möglichkeit und der Notwendigkeit. Was als mathematischer Begriff möglich, ist damit für die Mathematik sofort auch existent und sofort auch notwendig. Ist der Begriff erwiesen als in den Methoden der Mathematik begründet, so ist er damit gesichert nicht als bloß möglich, sondern mit dieser Möglichkeit für die Mathematik auch existierend, und mit dieser Existenz für sie zugleich notwendig. Dagegen in der Naturwissenschaft ist es wahrlich ein Unterschied,...⁴²¹

Thus, for example, the existence of a number implies that this number is a possible mathematical object. The number, as an object derived from mathematical laws, is also a necessary object of thought. A triangle constructed from the laws of formation of geometric figures is a possible, real, and necessary object. In mathematics, no additional tests are required to show the necessity of objects constructed by thought. In the modality, the transition from pure mathematics to physical-mathematical science is carried out, since the distinction between possibility, reality and necessity occurs only in the science of nature. Therefore, by virtue of the levels of the modality, the object of mathematical thought can be distinguished from the object of nature. In the modality, the distinction between mathematics and physics can be made. This step could not take place in the previous levels as long as the quantity determinations, the quality determinations, and the relation determinations concern both the mathematical object and the object of nature. Only in the modality are the conditions of possibility defined to think of an object as an object of experience. In mathematics, the distinction between the possible, the real, and the necessary is not relevant at all. On the contrary, the knowledge of nature requires

⁴²¹ Natorp. P, LGEW, p. 84.

that it can be decided whether the object is a possible fact, a real or necessary one. The affirmation of a hypothesis requires an additional test to show that the object thus considered is also a fact of experience. The fact of experience is considered as necessary if it can be proved that it is the case of a law. In the experience, the possibility, the reality, and the necessity are well distinguished as different ways in which the object is related to thinking. The levels of quantity, relation, and modality are insufficient to show how thinking posits the object. The synthesis of the modality will be in charge of this task. The modality will indicate how the object is thought, and the way in which the object is thought shows how a mathematically constructed object can be thought as an object of experience. The modality thus contributes to the true transition from mathematics to physics.

The levels of the modality

The possibility expresses the first moment of the modality. This moment will open the way to the rest of the modal moments. The possibility is a first initial estimation (*Ansatz*). This starting point considers the object as a possible object. In the first instance, the object is constructed as a possible object for thinking. This starting point is provisional, since the course of the investigation may show that this object is only possible, a real one or a necessary one. This initial estimation is always provisional, and it is this provisional nature of the estimation that allows the rest of the moments of the modality. Natorp compares the stage of the possibility with the formulation of a question in which the formulation itself sets the beginning of the investigation. The question regarding what this something unknown is can only be posited by thinking itself. For something to be an object of thinking it must, first of all, be something for thinking. As Natorp explains:

Die Möglichkeit steht sehr nahe der Frage, aber sie geht über diese hinaus, indem sie den Prozeß zur Entscheidung der Frage wenigstens einleitet. Was als möglich angesehen wird, wird damit allerdings zur Frage gestellt, aber es wird zugleich schon der erste Schritt zur Beantwortung der Frage getan. Dieser besteht darin, daß man setzt, es

sei so; so muß dann dieser Ansatz in der Durchführung sich bewähren,
oder aber seine Undurchführbarkeit sich herausstellen⁴²²

Each determination that thinking establishes is the answer that the thinking itself gives to a question raised. The determination of that something can only be given by thinking itself. The reference to something that is not the object of thinking makes no sense. As it was already established, nothing is beyond thinking. Natorp argues that, in general, every question of knowledge is based on this type of relationship. The question about the object of knowledge is based on the fundamental structure of thinking, on the possibility of thinking of something. Each determination that the thought introduces is the answer that the thought itself gives to a question raised. The idea of knowledge as a task is reflected in the fundamental structure that has the same function of inquiring. One question has three dimensions. On the one hand, it has a prospective moment in a “not knowing”. What is unknown is what is investigated. The goal of the question is to provide content to what is defined, in the first instance, as the ‘no’ of knowledge. It is the moment of indeterminacy. The question sets the task. This is the condition that makes it possible to ask about something in general. What is asked about is the unknown, it is an x that must be determined. However, there must be elements that allow, at least, to understand what one is asking for. Without this first step, the necessary presuppositions for the question itself would be missed: the establishment of the task. Three moments can be distinguished. First, forwards, the presentation of what is undetermined, what is to be determined. Second, backward, are those conditions that allow a primary identification of the object that is investigated. The object of investigation cannot be a pure nothing. It must be something for thinking so as to be something to be determined. In the middle, there is the knowledge of not knowing, where the previous two moments are combined. This is the basic structure of thinking. The source of the process is this possibility of the understanding to generate new determinations in its object. Every new determination is a provisional answer. The purpose of the process of determination is to progressively determine the object of experience. This is the first step in the path of knowledge, the establishment of an estimation. This estimation expressed in a hypothesis can be corroborated in successive levels of thinking. The possibility represents a first initial hypothesis with which the investigation begins. As we exhibited, in the quantity

⁴²² Natorp, P., LGEW, p. 87.

category, this initial estimation corresponds to the unit. This unit is a provisional support point to begin the process. This unit is taken as an initial proposal to form the scale that will operate as a measure of multiplicity. It must be checked that the initial selection of the scale is suitable for the measurement of the object. The initial moment is this provisional generation of a tentative scale whose effectiveness must be verified. The same process also takes place in the quality and in the relation, where the initial estimation is a first question that opens a horizon of possible answers. The question itself represents the level of possibility. The moment to verify the answer to the initial question is the level of reality. The beginning at the level of possibility shows that the process is always ongoing and that there is no absolute beginning. The process is infinite because it does not start with a first data whose origin is unknown. There is no initial data for thinking, but the path of thinking always begins with a question that opens a horizon of possible answers. For Natorp, there is no beginning in a pre-logical data that operates as an absolute beginning in the construction of the experience. On the contrary, at the stage of possibility, it becomes clear that the beginning is always relative. Reality will be a continuation of the process initiated by the position of possibility that is always a position of thinking.

The second level consists of the accreditation of what was initially set as merely possible. It must be confirmed that the tentative answer to the question does indeed take place. This 'taking place' that is expressed in the accreditation of the initial estimation constitutes the fact. It represents the moment of reality. The reality requires 'making a decision'. It is decided that what was initially considered as only possible is real. The problematic judgment is replaced by the assertive judgment. It is argued that what in the first instance may or may not be the case actually takes place. That is why Natorp calls existence, the *complementum possibilitatis*. The existence allows determining what the possibility left undetermined. The moment of the reality consists in the accreditation of the determination. At this moment, it is shown that what was set as possible actually takes place and that it is, consequently, a fact. The proof of experience is the proof of the fact. Proving that something exists means that what was considered as possible is now part of the experience. Therefore, the hypothesis test is also the proof that something is the subject of experience; that is, it exists. It is shown that what was raised as merely possible is a fact. This process is the path of thinking. The complete determination of experience is an infinite process. Experience can be defined as this process that has no closure because the path of determination continues infinitely. As in the case of

possibility, reality as a moment of the modality is also exhibited in each of the other categories. In the case of the quantity, the arbitrarily unit must be suitable for counting. At the level of possibility, an arbitrarily selected unit was taken to measure a multiplicity. Now it must be proven that the unit chosen can effectively operate as a measurement standard. In the multiplication of the unit, it is evidenced whether this arbitrarily selected unit is suitable or not to operate as a scale of what is measured. In the second moment of the quantity, that of multiplicity, it is tested whether this estimation of the unit is satisfactory or not. The initial estimation process is successful when it is proven that the unit is suitable for measurement. The moment reality is tested by showing that the unit functions as a unit of measurement. In the quality, the second moment of the modality corresponds to the comparison. The objectivity considered can be compared with another from the finished point of view of the genus⁴²³. In the case of the relation, this is particularly evident. In the relation, a first order is taken as tentative. It must now be shown that this order can operate as a pattern for the rest of the systems. This second moment of reality consists in showing that the selected pattern is indeed suitable. Thus, the initial estimation for the relation is the proposal of a fundamental order, of an order that operates as the basis for the rest of the systems. In the search for a complete system of order, the possibility leaves the way open for the selection to be corrected or not, and the initial proposal becomes real if it is verified. What was established as merely possible was that which should be determined in some way. Reality is the continuation of the process that the possibility left open. It is proved that the objectivity already determined in a quantitative and qualitative process is real. The objectivity test is performed when it is shown that the series can be integrated under a common series. This evidence establishes a fact on a provisional basis since in the course of the investigation this fact can be posited again as a question. The initial estimation becomes a hypothesis that must be tested. This test is the proof of reality.

This course is exhibited in scientific research in which after the formation of a hypothesis, it is shown that the initial approach was correct through an experiment. The initial hypothesis is thus corroborated. The experiment is always oriented according to a specific question - which left the level of possibility. The experiment seeks to verify the hypothesis. The demonstration gives a solution whose necessity must be proved. The evidence constitutes only a provisional moment of detention. Therefore, the path of the

⁴²³ Cf. Natorp, P., L, p. 29.

experiment, the *fiat experimentum*, is the clearest testimony of the second level of the modality⁴²⁴. Even in the induction, the objective is the gain of a provisional conclusion that can operate as a major premise in a deduction. The initial test allows a provisional answer whose need must be accredited later. The deductive proof will allow passing from the real thing to the necessary when it is shown that the accredited is the case of a law.

The necessity represents a provisional closure of the process in the recognition that what was considered a fact is the case of a law. What was held to be contingently becomes necessary when it is considered the result of a deductive process. What is necessary is what is taken as a consequence of the process of deduction. To maintain that a fact is necessary means that this fact is considered as having been established by a law. Ultimately, this fact is incorporated in a system of laws through which it can be considered as the conclusion of a deduction⁴²⁵. The establishment of a fact implies the possibility of finding the law from which it can be considered an instance. It is concluded that a fact is necessary when it is presented as the case of a law. Indeed, the premises are held provisionally and may themselves be subject to revision. The need for the conclusion is always relative as the premises themselves can be tested. They are considered necessary if they are the result of another deductive process. It can be requested that the premises of the deduction be subjected to a new revision, and a new initial estimation is required to show the necessity for the premise that operated to ground what was held as necessary. Certain fact that is considered as a necessary fact may become hypothetical by questioning the premises on which it was sustained. Thus, knowledge affirmed as necessary is only provisionally. The point reached at this stage can also operate as a provisional starting point in the search for a further conclusion. This moment can also be the starting point for the beginning of a new three-level cycle. The point reached may turn into a new beginning. Natorp argues:

Die dritte Stufe eines jeden synthetischen Prozesses aber betraf allemal den *Abschluß* des durch die erste nur eingeleiteten, auf der zweiten Schritt um Schritt weiter verfolgten Verfahrens, sozusagen den Rechnungsabschluß, der aber nur sicheren Grundlagen dienen soll für neue Prozesse von gleichem allgemeinem Stufengang. Die zweite und

⁴²⁴ „Der Weg des Experiments, das *Fiat experimentum*, das ist daher das deutlichste Zeugnis des allgemeinen Sinns der zweiten Modalitätsstufe.“ Natorp. P., LGEW, p. 89 .

⁴²⁵ „Die Notwendigkeit der Tatsache bedeutet nichts anderes als ihre Feststellung im Gesetz.“ Natorp, P., LGEW, p. 91.

dritte Stufe unterscheiden sich also als der Weg, insofern man im Gange ist, ihn zu verfolgen, und der vorläufig erreichte Haltpunkt, auf dem man stillsteht, nicht um darauf stehen zu bleiben, sondern des Gewonnenen sich zu versichern und auf der soweit gesicherten Grundlage dann weiterzuschreiten.⁴²⁶

This provisional closure is also expressed in each of the categories. In quantity, the determined totality can always operate again as a unit for the formation of another plurality. The necessity corresponds to the consummation of the function in the process of counting, this is the totality determined as a provisional total closure. This particular unity, the whole, can be a relative unity for a new counting process. In the quality, this third level represents the possibility of establishing a subsequent genus of a higher order. The first genus becomes a species of this higher-order genus. The totality that represented the genus thus becomes a new unity of a multiplicity. The genus is now a qualitative unity in the multiplicity of species now subsumed in a superior genus. In the relation, it is sought the total concatenation in a series system.

The starting point is always provisional and conditional. The initial moment, which was considered as a hypothesis, is presented as a result of a demonstration. The path of thinking consists of the pursuit of this task, to prove that the original estimations are necessary facts. The function of a first data is to be a first step for new questions. This process is always relative. Indeed, the establishment of the fact as a case of a law proves this fact to be necessary. However, this need is always relative because new variables can always be introduced. The emergence of a new hypothesis introduces a new process. The whole path of knowledge expressed in science is oriented in this direction. The purpose of science is to seek universal laws that explain the particular facts based on them. The general goal of the science is the creation of laws through this process that involves induction and deduction. The purpose of induction is the possibility of a deduction. Induction seeks the establishment of a general law that can operate as a major premise. The induction seeks the general laws that allow to deduce the fact and show it as necessary. Therefore, induction and deduction are two poles of the same procedure. The knowledge process is based on this tripartite structure of the modality, which puts a hypothesis as possible, takes the fact as a tentative response, and, finally,

⁴²⁶ Natorp, P., LGEW, p. 90 .

shows its necessary character through the deductive process. This end of the investigation, establishing the fact as necessary, is always provisional as this need may be the starting point of a new start.

Conclusion

In this deduction of categories, Natorp shows how thinking can by its own means construct objectivity. The concepts do not need anything given. On the contrary, the deduction of the categories exhibited how thinking is capable of producing the object without reference to intuition. For Natorp, thinking is the source of the totality of the determinations of the object. It provides the multiplicity and the unity. In the deduction of quantity and quality, it was exhibited that the act of thinking can concomitantly generate both unity and multiplicity. In this way, it was proved that thinking can constitute the object without any reference to intuition. As we exhibited, this construction was developed in four levels: quantity, quality, relation, and modality. In this way, Natorp explains how thinking can constitute the object of experience. In this way, it can be overcome the dualism between intuitive and conceptual representations as it is exhibited that there is no gap between the rules of objectivity and the concreteness of the object.