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## **Paul Natorp's reformulation of the Kantian distinction between intuition and concept**

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## Chapter 2. Contemporary Approaches to the Problem of the Distinction between Intuition and Concepts

### Introduction

As we exhibited, the problem of the relationship between intuitions and concepts has a long tradition that results in the Kantian formulation of the problem. The question of the relation between intuitive and conceptual representations is the core of Kantian philosophy. In Chapter 1, we exhibited that one of the central problems of knowledge – on what grounds rests the relation of our representations with the objects - can be formulated in terms of the relation between intuition and concepts. We studied the problems involved in the distinction and how they were revisited by Kantian contemporaries. They considered that Kant could not give a satisfactory solution to the problem that he posed. As we saw in the previous section, the relation between intuition and concepts was one of the most discussed aspects of the Kantian proposal. In contemporary philosophy, the problem of the relationship between intuitions and concepts emerges as one of the central issues. Philosophers argue about what elements thinking introduces and which ones are given to it. The Kantian question remains unsolved. As we analyzed in the first chapter, the question of the relationship between intuition and concept was also presented by Kant in the following way: How can thought legitimately represent the object? How can the universality of the concept relate to the singularity of the object? Natorp's proposal is grounded on the Kantian paradigm. However, his position is presented in a context where different schools had already tried to give a solution to these problems. The aim of this chapter is to study how Natorp approaches the problem of the relationship between intuitions and concepts in dialogue with his own contemporaries. We will analyze how Natorp presents his proposal in dialogue with the philosophical tendencies of the time: psychologism and logicism. We will exhibit that both currents have a wrong conception of the relationship between intuitions and concepts based on methodological errors.

In the first place, we will study the proposal of psychologism and then that of logicism. Natorp will show that both positions are two types of dogmatic proposals, unsuccessful to explain the issues raised by Kant. In chapters 3 to 5, we will focus on Natorp's own position. However, it is necessary first to explain why the solutions of the time were infertile for him.

## 2. 1. Against Psychologism

### Introduction

This section aims to analyze Natorp's criticism of psychologism. Natorp will argue that the main confusion of psychologism is due to a methodological error. More precisely, Natorp considers that an accurate conception of the relation between intuitions and concepts demands abandoning the standpoint of subjectivity. The subjective point of view will lead to considering the object as a fact given to intuition. The object will be determined in advance. According to this point of view, the task of concept formation consists in a process of abstraction. The concepts are abstractions of the marks that belong to the object that is given to intuition. The position will receive the name of psychologism. As we shall see, psychologism will have a misconception of the relation between intuitions and concepts due to a methodological error. Natorp will exhibit that the problems of this perspective are grounded on the assumption of the subjectivity standpoint. Psychologism takes subjectivity as the starting point of the investigation and considers the object as what is opposed to it. Natorp will show that psychologism starts from an incorrect understanding of the philosophical method. Particularly, the mistake of psychologism consists in grounding logic on psychology. Psychologism confuses the study of the laws of knowledge with the study of the legality of psychical life. The problem of the genesis is confused with the problem of validity. From this methodological error, psychologism considers the data given to intuition as a first element in the formation of knowledge. Starting from the problem of the formation of knowledge, psychologism conceives the immediate data as what is truly real, as the first for the act of knowing and the foundation of all objectivity. For psychologism, what is given to intuition is the starting point for the production of concepts. According to this perspective, starting from a given initial data would guarantee the possibility of objective knowledge. However, Natorp will show that from this perspective, concepts are merely abstractive. From the finite human standpoint, the intuitive representation is what is given to senses, and concepts are the abstractive marks from what is given. Natorp will argue that thought does not require anything external to itself in the production of its object. The laws of thinking do not originate from a process of abstraction from given intuitive contents. On

the contrary, thinking creates the objectivity through its laws. Objectivity consists in this dependency on thinking. Natorp will prove that this dependence is precisely a guarantee of objectivity. More specifically, it will be shown that the only possible way to conceive a relation between the laws of thought and its objects is to base what is objective purely and exclusively on the legality of thinking.

First, we will examine the emergence of the debate on psychologism in the nineteenth century. Our goal is to show the relevance of Natorp's position in the philosophical debate of the time. Second, we will study some of the most representative positions. We will focus on Beneke's thesis, one of the precursors of psychologism. Then, we will study Helmholtz's ideas, as a representative of physiological Neo-Kantianism. Finally, we will focus on Natorp's objections against the subjective method to show how this methodological error of psychologism leads to an incorrect understanding of the relationship between intuitions and concepts.

### 2.1.2 Introduction to the *Psychologismus-Streit*

Natorp's criticism of psychologism is framed by what was known as *Psychologismus-Streit*. The debate on psychologism was one of the most important disputes in German philosophy at the end of the 19th century, and it is concomitant with the emergence of psychology as a scientific discipline independent of philosophy<sup>148</sup>.

By the end of the nineteenth century, philosophy is experiencing a crisis. This is recognized both by numerous philosophers of the time<sup>149</sup> and by contemporary scholars<sup>150</sup>. Philosophy had an "identity crisis."<sup>151</sup> On the one hand, philosophy experiences a strong rejection of post-Hegelian speculative idealism, which is in decline after Hegel's death. There is a generalized rejection of all forms of purely abstract speculation. For the philosophers of nature, the Hegelian philosophy represented a 'complete nonsense'<sup>152</sup>. On the other hand, the evolution of particular sciences led to a reconsideration of the task of philosophy. For many thinkers, the return to Kant was motivated by the loss of credibility suffered by philosophy which started with this fall of speculative idealism<sup>153</sup>. Natorp shares this vision of the state of philosophy. In *The Logical Foundations of the Modern Mathematics*, he considers that the philosophy abandoned the sobriety that for many centuries it had shared with the exact science,

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<sup>148</sup> Windelband considers the separation of psychology from philosophy as one of the paradigmatic scientific facts of the 19th century. Cf. Windelband, W., 1903, p. 519. Külpe, on the contrary, considers that by that time there still had not taken place a total separation between psychology and philosophy. Külpe, O., 1921, pp.76 ss.

<sup>149</sup> Külpe, O., 1907, p.11; Cassirer, E., 1950, p. 3ss. Windelband, W., 1903, p. 511, 513, 519. Heidegger, M., GA1, p. 5; Helmholtz, H., 1950, p.147 ss. Windelband states that the nineteenth century "is far from being a philosophical century". Windelband, W., 1903, p. 511.

<sup>150</sup> Dufour, E., 2003; Kusch, M., 2005, p.2; p.8; Gonzalez Porta, M., A.,2005, pp. 36ss. Beiser, F. 2014, p. 15ss.

<sup>151</sup> This term was first used by Herbart Schnädelbach. Cf. Beiser, F., 2014, p.15.

<sup>152</sup> „Hegels Naturphilosophie erschien den Naturforschern wenigstens absolut sinnlos. Von den vielen ausgezeichneten Naturforschern jener Zeit fand sich nicht ein einziger, der sich mit den Hegel'schen Ideen hatte befreunden können.“ Helmholtz, H., 1950, p.147.

<sup>153</sup> Oswald Külpe argues: „Als dann mit dem Niedergang der Hegelschen Philosophie das Vertrauen zu dieser Wissenschaft überhaupt erlosch und eine gründliche Emanzipation der Einzelwissenschaften von ihrer Führung und Bevormundung einsetzte, da schien den Philosophen keine bessere Hilfe möglich zu sein, als die Rückkehr zu Kant.“ Külpe, O., 1907 p.11. Following the line of Külpe, Martin Heidegger holds in one of his first published works: "When, with the decline of Hegel's philosophy, the particular sciences energetically freed themselves from the tutelage of philosophy and threatened to repress it completely (with positivism the precarious situation and the philosophy-dependent task was noticed), the only salvation was seen in the "return to Kant". Heidegger, M., GA1, p.5.

ending up falling into the empty speculation opposed to the rigorous thinking of the mathematics.<sup>154</sup>

The return to Kant is a reaction to the challenge presented by, on the one hand, the fall of Hegel's speculative idealism, and, on the other, the total emancipation of the sciences with respect to philosophy.

Hence arises the question of the relationship that philosophy has with the emerging scientific disciplines that are now emancipated from it. Philosophy faced two dangers. The first danger is to fall into a speculative metaphysics that cannot give a proper explanation of any fact. As Ernst Cassirer explains, some thinkers argued that philosophy does not contribute to the development of science. Moreover, philosophy could be an obstacle to its progress<sup>155</sup>. The second problem that philosophy has is to be reduced to a particular area of positive science. Philosophy is not only faced with the problem of justifying its method, but it must also give an account of what its proper object of investigation is<sup>156</sup>. Thus, while philosophers must dispute their objects of study to positive science, some scientists of nature consider that philosophy is not only useless but harmful for the progress of knowledge<sup>157</sup>. In this context, empirical psychology emerges as a science, and with it the philosophers who seek in this discipline a kind of refuge from the end of speculative idealism<sup>158</sup>.

Some of these thinkers take psychology as a new fundamental branch of philosophy<sup>159</sup>. Friedrich Beneke is one of the main representatives of this current. Beneke

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<sup>154</sup> Natorp states: „Die alte, nach beiden Seiten fruchtbringende Verbindung zwischen Philosophie und Mathematik schien eine Zeitlang sehr gelockert. Was die Mathematik vielleicht einmal der Philosophie zu danken hatte, die Strenge des Beweisverfahrens, ja den ganzen Begriff des formalen Aufbaus einer Wissenschaft, das hat sie längst aus eigener Kraft und eigenem Trieb so in sich aufgenommen, dass sie mit gutem Grunde glauben darf, darin von der Philosophie nicht viel mehr lernen zu können. Diese dagegen schien die nüchternen Bahnen, in denen sie manches Jahrhundert mit den strengen Wissenschaften Hand in Hand gegangen war, zeitweilig ganz zu verlassen und sich in spekulativen Abenteuern zu gefallen, die das streng geschulte Denken des Mathematikers zu allererst zurückstossen mussten.“ Natorp, P. ZLGNM, p. 177.

<sup>155</sup> Cf. Cassirer, E., 1950, p. 4.

<sup>156</sup> Cf. Beiser, F., 2014, p. 18.

<sup>157</sup> Helmholtz holds: „Die Naturforscher wurden von den Philosophen der Borniertheit geziehen; diese von jenen der Sinnlosigkeit. Die Naturforscher fingen nun an, ein gewisses Gewicht darauf zu legen, dass ihre Arbeiten ganz frei von allen philosophischen Einflüssen gehalten seien, und es kam bald dahin, dass viele von ihnen, darunter Männer von hervorragender Bedeutung, alle Philosophie als unnütz, ja sogar als schädliche Träumerei verdammt.“ Helmholtz, H., 1950, p.147.

<sup>158</sup> As Beiser claims: “The sciences now seemed to cover the entire *globus intellectualis*, so that there seemed no special subject for philosophy.” Beiser, F. 2014, pp. 16ss. Windelband explains that these philosophical schools arise in the broader context of the rise of materialism. Windelband, W., 1903, p. 513 y 519.

<sup>159</sup> Cf. Beiser, F. 2014, p. 16 ss. Anderson, Lanier, 2010, p. 288.

believes that philosophy is the first science, the science on which the rest of the sciences depend. This ‘science of science’ is ultimately called psychology. Psychology is the starting point of all philosophy<sup>160</sup>. Psychology will be the grounding science of philosophy. Logic, ethics, and aesthetics are applications of psychology as a fundamental science. However, the incipient institutionalization of psychology as a science must be distinguished from the accusation of psychologism<sup>161</sup>. The term ‘psychologism’ was first used by Eduard Erdmann in 1866 as an accusation towards Frederick Beneke<sup>162</sup>. His criticism points to the attempt of some thinkers to make psychology the grounding science of philosophy and science in general. This term denotes rather a “philosophical accusation.”<sup>163</sup>

Beneke has been considered the pioneer of the ‘back to Kant’<sup>164</sup>. Paradoxically, he was the first philosopher accused of psychologism. For Beneke, the starting point of

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<sup>160</sup> Beneke, F. 1933, p.2. For Beneke, psychology is: "...the center of all philosophy as a whole: the sun from which all other philosophical sciences receive their light. Only in this way is it possible to achieve true unity and true order, true universal validity for philosophy. All philosophical concepts are certainly a product of the human soul; and only by knowing the way and the way in which they have been generated is how can they receive their supreme clarity. In the same line, in his brief of 1833 (*Philosophy in its relations with experience, speculation and life*), Beneke holds: „Nicht nur als Anfangs oder Mittelpunkt; nicht nur als Grundlage für alle philosophie Erkenntnis haben wir die Selbst Erkenntnis oder psychologische zu betrachten, sondern alle übrige philosophische Erkenntnis. Können wir nur und in dieser gewinnen. In den Begriffen aller übrigen philosophischen Wissenschaften denken wir nichts Anderes als psychischen produkten, welche demnach auch nur als solche in voller Wahrheit und Tiesse gewürdigt werden können.“ Beneke, F., 1833, p. 14.

<sup>161</sup> Martin Kusch compiled various definitions of the concept of psychologism in order to show that although the objection of psychologism was widespread at the time, what these authors understood by psychologism varied greatly. Thus, for example, Oswald Külpe defines psychologism as "the unjustified application of psychological consideration in the field of theory of knowledge." „Man pflegt die unberechtigte Anwendung der psychologischen Betrachtungsweise in der Erkenntnislehre als »Psychologismus« zu bezeichnen. Die Psychologie hat es nämlich nur mit der tatsächlichen Beschaffenheit und Entwicklung der seelischen (und also auch der Erkenntnis-) Vorgänge zu tun, die Erkenntnislehre dagegen untersucht den Erkenntniswert der letzteren, d. h. ihre Bedeutung für die Erfassung von Gegenständen (ihre »objektive Gültigkeit«).“ Külpe, O., 1921, p. 39. However, his differentiation between psychology and theory of knowledge could well be considered as a psychologist from other perspective. The difficulty consists in determining who exactly the ‘enemy’ is.

<sup>162</sup> Cf. Kusch, M., 2005, pp. 98 ss.

<sup>163</sup> Jacquette, D., 2003, p. 43. This controversy on the theoretical level has deep consequences in the institutional sphere. As numerous studies noticed, the problem was not only theoretical but also the university positions in the faculties were at stake. In 1913, a group of 107 philosophers in Germany, Austria and Switzerland wrote a petition claiming that no more positions were given to experimental philosophers. They claimed that all chairs of philosophy were becoming chairs of psychology. Natorp signs this petition and speaks publicly against that university chairs were given to experimental psychologists. According to this request, it is questioned that thinkers who study mental life occupy positions corresponding to philosophy. Natorp - along with other thinkers such as Husserl, Rickert and Riehl - theoretically and institutionally resists the dissolution of philosophy in empirical psychology. This institutional separation can be considered as the translation of theoretical separation. Cf. Kusch, M., 2005, p.186 ss. Beiser, F., 2014, p.18.

<sup>164</sup> Brandt Burke holds: “The historical importance of Beneke as the real pioneer of "the movement back to Kant," has never been sufficiently recognized, or more than that, it has not been recognized at all.” Brandt, B., 1895, p. 29.

philosophical research is the reflective moment of self-awareness. Man is conscious in the reflection of the mental acts that he carries out to obtain knowledge<sup>165</sup>. This awareness of mental acts is the foundation of the possibility of psychology. The psychology that describes the processes found in self-perception is empirical psychology. Empirical psychology is the basis of philosophy<sup>166</sup>. The possibility of obtaining knowledge should be sought in the mental mechanisms that allow the formation of representations. Beneke believes that philosophy must identify the origin of the formation of representations. Being is being represented<sup>167</sup>. The truth is based on mental representations. Then, philosophy must study how representations are generated in the soul of man<sup>168</sup>. Beneke believes that logic is certainly the core of philosophy. However, logic depends on psychology<sup>169</sup>. Psychology will be responsible for explaining the principles that govern the formation of knowledge in mental representations. Mental representation requires two conditions. In the first place, a soul that has the senses as instruments. Second, it requires an affecting object. The sensations are the first elements in the elaboration of the representation and, therefore, the starting point of the investigation<sup>170</sup>. The intuitive moment is required for the explanation of the process of knowing because it is the first required moment in the genesis of the representation. Psychology reveals the conditions that lie in the mind for the formation of these representations that constitute knowledge. Thus, Beneke proposes a foundation of philosophy in psychology. Through the psychological foundation, philosophy is prevented from the two dangers outlined above. On the one hand, philosophy avoids empty speculation. On the other hand, it follows the method of natural science. This path initiated by Beneke, as a continuator of the currents of modern empirical psychology, is deepened in subsequent years<sup>171</sup>. With the growth of the institutionalization of psychology as a science, the theoretical interference that psychology has on the philosophical level also increases. Beneke thought that

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<sup>165</sup> Brandt holds: "And again, only on the basis of inner experience can philosophy, and in particular scientific knowledge of the human soul, be established with certainty and steadfastness." All this is only to give special prominence to inner consciousness as a fundamental datum of individual experience. And with the recognition of this fact, we reach the fundamental starting point of Beneke's psychology and philosophy". Cf. Brandt, B., 1895, p.51 s.

<sup>166</sup> Messer, A., 1920, p. 92.

<sup>167</sup> Beneke, F., 1840, p. 67.

<sup>168</sup> It is interesting to note that the central concept of Beneke's investigation is not the mind but the soul. The soul is the determining element of human life and one of the first conditions in the formation of representations. The senses are defined as instruments of the soul. Cf., Beneke, F., 1871, §2.

<sup>169</sup> Beneke, F., 1842, p.21.

<sup>170</sup> Beneke, F., 1871, §2.

<sup>171</sup> Oswald Külpe sees in Beneke a developer of the studies initiated by Tetens in the eighteenth century. Külpe, O., 1921 p. 82.



psychology, as a grounding science of philosophy, should follow the method of natural sciences. Later, many authors will deepen this conception. Thus, arises physiological psychology. Not only were the foundations of logic sought in the life of consciousness but, more particularly, in the physiological processes that are carried out in the formation of mental representations. Wilhelm Wundt and Hermann von Helmholtz are pioneers in this direction.

### 2.1.3 Psychologism in Neo-Kantianism.

Hermann von Helmholtz, who worked with Wundt<sup>172</sup>, is one of the representatives of physiological psychologism. Helmholtz is one of the first Neo-Kantians and one of the first thinkers of the nineteenth century who seeks to ground the theory of knowledge in the physiology of the senses<sup>173</sup>.

Helmholtz defines himself as a representative of natural philosophy<sup>174</sup>. This place that Helmholtz occupies is of particular relevance considering that Marburg's Neo-Kantianism emerges as one of the first reactions against the psychologization of logic. Helmholtz not only receives the influence of the psychology of Wundt, but he is also influenced by Kantian and Fichtean idealism. His proposal emerges as a peculiar form of association between these two influences. On the one hand, his work as a scientist and his work with Wundt, on the other, his studies in Kantian and post-Kantian idealism. Helmholtz tries to reconcile philosophy with the sciences of nature. From Helmholtz "a new and peculiar relation between empirical sciences and philosophy is created."<sup>175</sup>

According to Helmholtz, the philosophy and science of nature share the same question. They investigate the relationship between representations and reality. For Helmholtz, the core of the problem of knowledge is expressed in the Kantian question: "In what sense do our representations correspond to the reality?", the question that, as we

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<sup>172</sup> Cf. Kusch, M., 2005, p. 197.

<sup>173</sup> According to Köhnke, Helmholtz belongs to the "programmatically" Neo-Kantian phase. For Alois Riehl, Helmholtz "was the first to declare that Kantian ideas were still alive." He states: „Er war der Erste, der es aussprach, dass Kants Ideen noch leben.“ Riehl, A., 1904, p. 1. Beiser considers Riehl is wrong when he states that Helmholtz is the first Neo-Kantian. Beiser, F., 2014, p. 196.

<sup>174</sup> „Ich habe umso mehr Veranlassung, die Frage nach dem Zusammenhang der verschiedenen Wissenschaften hier zu erörtern, als ich selbst dem Kreise der Naturwissenschaften angehöre...“ Helmholtz, H., 1950, p. 145.

<sup>175</sup> Cassirer, E., 1998, p.12. For Riehl, the merit of Helmholtz's philosophy lies in recovering the relation between philosophy and science, and not in his physiological reading of Kant. Riehl, A., 1904, p. 2.

showed, was reduced to the problem of the relation of intuitions and concepts. Helmholtz holds:

Das Grundproblem, welches jene Zeit an den Anfang aller Wissenschaft stellte, war das der Erkenntnistheorie: „Was ist Wahrheit in unserem Anschauen und Denken? in welchem Sinne entsprechen unsere Vorstellungen der Wirklichkeit? Auf dieses Problem stossen Philosophie und Naturwissenschaft von zwei entgegengesetzten Seiten; es ist eine gemeinsame Aufgabe beider. Die erstere, welche die geistige betrachtet, sucht aus unserem Wissen und Vorstellen auszuscheiden, was aus den Einwirkungen der Körperwelt herrührt, um rein hinzustellen, was der eigenen Thätigkeit des Geistes angehört. Die Naturwissenschaft im Gegentheil sucht abzuscheiden, was Definition, Bezeichnung, Vorstellungsform, Hypothese ist, um rein übrig zu behalten, was der Welt der Wirklichkeit angehört, deren Gesetze sie sucht. Beide suchen dieselbe Scheidung zu vollziehen, wenn auch jede für einen andern Theil des Geschiedenen interessiert ist.<sup>176</sup>

According to Helmholtz, this question of the relation between concepts and intuition is the common point between philosophy and the science of nature. Both science and philosophy try to understand the relation between thinking and reality. They want to explain the relationship between our representations and what is real. Philosophy and natural science are included in the problem of the *Erkenntnistheorie*. The philosophy and science of nature deal with this problem of knowledge in general but each of them from a different perspective. Philosophy studies the problem of the generation of representations in the mind. Its task is to distinguish in the representation the element that corresponds to reality from the element of the cognitive faculty. That is to say, one must separate in the representation that which belongs to the mind from what corresponds to the world. The field of investigation of philosophy is the mental process. The science of nature, on the other hand, deals with the objective side. His field of research is that which corresponds to the world, the reality. However, Helmholtz believes that both philosophy and natural science seek to answer the question of how our representations relate to reality. For both, it must be explained how concepts relate to intuitions. The problem of knowledge arises in terms of the correspondence of the human mind with reality, and reality is that which must be achieved by thinking. Philosophy focuses its study on the

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<sup>176</sup> „in welchem Sinne entsprechen unsere Vorstellungen der Wirklichkeit?“ Helmholtz, H., 1879, p. 6.

subjective pole of knowledge, in the psychic life, establishing the limits, justification, and extension of empirical knowledge. Science focuses on the objective pole, the content of experience. For Helmholtz, philosophy and natural science share the same fundamental basis. They are just two sides of the same problem. According to Helmholtz, Kant expressed this fact clearly. Kantian philosophy has the same basis as natural science<sup>177</sup>.

Helmholtz identifies the structure of our organs that determine the representations with the Kantian forms of knowledge, physiologizing the *a priori* forms of Kant. The subject has certain structures that determine the way in which we experience the world. These structures can be established by a physiology of the senses because it is the structure of the senses that determines the particular form that the human experience takes. Helmholtz believes that his theory of perception will confirm the thesis presented by Kant<sup>178</sup>. Indeed, men have *a priori* forms that determine experience. However, these forms must be sought in the structure of the organs. For this reason, the main task of theoretical philosophy is the physiology of the senses.<sup>179</sup>

In his research on the concept of space, Helmholtz shows that the peculiar form that spatiality acquires is determined by empirical factors. The form of space is built from the relationship between the affection and the peculiar constitution of the senses. Philosophy must explain the generation of the representation of space from its genesis in empirical consciousness. Helmholtz studies the psychological representation of space. This representation of space depends on the possibility of the subject to voluntarily perform movements that vary the perception of objects. Those sensations that are altered by these movements are the spatial sensations. The representation of space results from our subjective construction as we perform certain types of movements<sup>180</sup>. Helmholtz concludes that *a priori* representation of space must be distinguished from space properties. Space is an *a priori* representation but its properties are determined *a posteriori*. That is, the priority of space does not indicate an *a priori* proof of the character

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<sup>177</sup> Helmholtz, H., 1950, p. 146.

<sup>178</sup> Helmholtz, H., 1879, p.8.

<sup>179</sup> Beiser argues: “Helmholtz does not leave the connection between Kant’s epistemology and science simply on the level of physiology, however. He takes it a step further by also considering the psychology of perception, that is, the psychic acts that are necessary for perception. Helmholtz is far from thinking that we perceive the world just by having sensations; he goes on to consider some of the many psychic acts of inference and judgement—most of them automatic and subconscious—necessary to convert sensations into perception”. Cf. Beiser, F., 2014, pp. 198, 200.

<sup>180</sup> He holds : „Und eine gegebene, vor aller Erfahrung mitgebrachte Form der Anschauung würde der Raum sein, insofern seine Wahrnehmung an die Möglichkeit motorischer Willensimpulse geknüpft wäre, für die uns die geistige und körperliche Fähigkeit durch unsere Organisation gegeben sein muss, ehe wir Raumschauung haben können.“ Helmholtz, H., 1879, p. 16.

of its properties. For this reason, we are allowed to affirm the transcendental character of space as a form of intuition, but we are not allowed to affirm the *a priori* character of laws of geometry. The structure of the eye determines, by its own internal constitution, the general features of the visual and, in this sense, it is a form of intuition. However, the particular colours that the eye sees, “the relations among them and the order in which they appear are effects of external causes that are not determined by any law of our organization”<sup>181</sup>. The same occurs with the representation of space. Therefore, even if it is accepted that the representation of the space is *a priori*, the specific axioms of space must be verified *a posteriori*<sup>182</sup>. For Helmholtz, concepts such as space and number should be elucidated by reference to their empirical genesis in the mind<sup>183</sup>. The problem of knowledge must be addressed in its formation in the subjective pole. Thus, philosophy has contact with the science of nature through the theory of perception. The main philosophical concepts are explained in the doctrine of sensible perception. In this way, the results of the empirical sciences validate the results of the Kantian system. Helmholtz shows how the Kantian theory finds a translation in the physiology of the senses. The reference to what is given is required by this physiology of senses. Helmholtz tries to explain the genesis of the representation in the mind. From this point of view, a theory of sensation is required and, concomitantly, the reference to something that is given to the mind. As it was for Beneke, the intuitive moment is represented by what is given to the mind as the first element of the formation of the representation. On the other hand, concepts of experience are abstractions from what is given. For this reason, the answer to the question of the relation between thinking a reality -between concepts and intuitions- is grounded in a theory of correspondence. It can only be satisfied by an *a posteriori* proof. From this perspective, the construction of the object of experience depends on what is given. The object experience cannot be fully constructed by thinking. On the contrary, thinking depends on the object. The standpoint of psychology and physiology led to this result.

We can identify the following common features of psychologism. First, empirical psychology is the basis of all philosophy. Then, logic must be based on a theory of mental

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<sup>181</sup> „Unser Auge sieht alles, was ist sieht, als ein Aggregat farbiger Flächen im Gesichtsfelde; das ist seine Anschauungsform. Welche besonderen Farben bei dieser und jener Gelegenheit erscheinen, in welcher Zusammenstellung und in welcher Folge, ist Ergebniss der äusseren ein Einwirkungen und durch kein Gesetz der Organisation bestimmt. Ebenso wenig folgt daraus, dass der Raum eine Form des Anschauens sei, irgend etwas über die Tatsachen, die in der Axiomen ausgesprochen sind.“ Helmholtz, H., 1879, p.23.

<sup>182</sup> Helmholtz, H., 1879, p.28.

<sup>183</sup> Cf. Cassirer, E., 1950, pp. 57 ss.

acts. Logic depends on psychology. Then, the sciences that depend on logic will also be submitted to psychological laws. Second, logic must focus on the study of the origin of representations. Representations originate from the senses. The sensations are the first elements in the elaboration of the representation and, therefore, the starting point of the investigation. Thought requires an intuitive element in the construction of knowledge. Third, psychologism argues in favour of a subjective foundation of knowledge. The problem of knowledge must be addressed in its formation in the subjective pole. The subject's mind and its processes are what should be investigated. Fourth, the problem of validity is based on the problem of genesis. The legitimation of the act of knowledge must be sought at the origin of the representations. Logic is grounded on the discipline that explains how representations originate in the mind. Fifth, psychologism considers that the foundation of the real requires sensation as a starting point. The intuitive element, considered as a posteriori data is essential for the construction of the object of knowledge. The mind cannot produce the object of experience by itself. There is an element that must be given for the construction of the experience. Knowledge requires sensibility as a passive faculty to receive representations. The passivity of sensibility indicates a reference to an element that the subject cannot construct.

#### 2.1.4. Natorp's Critique of Psychologism

The work of Natorp *On the objective and subjective basis of knowledge* is the first *manifesto* of the Neo-Kantian school against psychologism<sup>184</sup>. Even though Cohen had already raised some objections against the subjective orientation of knowledge, it is Natorp who systematically develops for the first time the problem of the subjective path of the foundation of knowledge. Natorp will incorporate these arguments into his *Introduction to Psychology*, a work published the following year of this research. As noted, this work is influential in the dispute against psychology. In his *Logical Investigations*, Husserl highlights the influence that Natorp's arguments had on his own productions. Husserl expressly refers to *Social Pedagogy*, the *Introduction to Psychology*, and the article published in the *Philosophische Monatshefte*, *On the objective and subjective basis of knowledge*. Husserl emphasizes that it was these last two works that had the greatest impact on his thinking<sup>185</sup>.

Natorp will show that psychologism confuses a particular science, psychology, with a fundamental science: logic. The problem of knowledge should not be studied according to its genesis in the consciousness of the individual. On the contrary, one must seek a fundamental science that proceeds in such a way that it can guarantee the legitimation of knowledge in general. The subjectivist perspective takes as a starting point of the investigation what is immediately given to intuition. The intuitive aspect of the process would involve this relation to something that is immediately given in natural experience as an external element to thinking. This requirement emerges as a consequence of the subjective point of view. Husserl confuses the problem of the genesis of the representation with the problem of the validity. This methodological error, as we shall see, will lead to the loss of any notion of objectivity. Natorp will show that the objectivity can only be guaranteed if it is exhibited how the thinking process can produce its objects. The mind constructs the cases in the creation of laws<sup>186</sup>. An idealistic conception of the

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<sup>184</sup> Cf. Edgard, S., 2008, p. 54.

<sup>185</sup> „Auch in einigen anderen, nicht minder wesentlichen Punkten berühren sich meine Prolegomena mit diesem Werke des scharfsinnigen Forschers, welches mir für die Bildung und Darstellung meiner Gedanken leider nicht mehr hilfreich sein konnte. Dagegen konnten auf mich zwei ältere Schriften Natorps, der oben zitierte Aufsatz aus den *Phil. Monatsh.* XXIII und die Einleitung in die *Psychologie* anregend wirken — wie sehr sie mich auch in anderen Punkten zu Widerspruch reizten.“ Husserl, LU, *Prolegomena*, p. 156. § 41.

<sup>186</sup> This point will be developed in Part III.

law will be defended as opposed to the naturalistic notion of psychology. Finally, Natorp will show that the problem is not resolved by appealing to a transcendental subjectivity.

### Logic and psychology

The problem concerns the foundation of the method of logic. The question is whether the foundation of logic should follow a subjective path or an objective path. The problem is to determine if the starting point of the investigation should be oriented to the subjective pole, to the agent of knowledge, or to the objective pole, to knowledge as a result. As we observed, the defenders of psychology, even with their multiple differences, agree that the foundation of knowledge must be found in the subject. Psychologists agree that the problems posited by logic can be solved by attending the subjective processes that give rise to the act of knowing. The central problem here is whether in the foundation of knowledge the determining factor is the subjective or the objective. The subjective side represents the subject of knowledge, it is the activity or experience of the subject. A subjective study of knowledge will investigate the factual experience of the cognitive agent. The objective side represents what is known, that is, the content of knowledge. The product is the objective side while the agent of the process is the subjective side<sup>187</sup>.

Natorp begins his argument by accepting that knowledge has two dimensions. On the one hand, knowledge is an objective determination. It means what must be known. In this sense, knowing means relating to the mechanisms of conceptual determinations of the object. On the other hand, knowledge is also an activity, an experience of the subject that carries out the knowledge process. Knowledge includes these dimensions that are correlated. These two dimensions cannot be separated. However, the problem is to establish the path for the foundation of knowledge. Knowledge means both: the process of knowing and the result<sup>188</sup>.

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<sup>187</sup> The rational finitude that realizes the activity of knowing is an "abbreviation" of the logical space (absolute subjectivity) in which it is inserted. Natorp claims: „Unser wirkliches Denken begnügt sich sozusagen mit Abbrüvierungen des Denkens...“ Natorp, P., QQ, p. 16.

<sup>188</sup> „Erkenntniss aber stellt sich von vornherein zweiseitig dar: als „Inhalt“ (Erkanntes oder zu Erkennendes) und als „Thätigkeit“ oder Erlebniss des Subjects (als Erkennen). Zwar sind in jeder Erkenntniss beide Beziehungen miteinander gegeben und eng verbunden; es gibt so wenig ein Erkanntes ohne Erkennenden, wie einen Erkennenden ohne Erkanntes. Aber doch muss in abstracto beides unterschieden werden, und offenbar wird eine, die Erkenntniss in ihrem eigenen Gesetze begründende Theorie nur eine von beiden Beziehungen unmittelbar betreffen können. Es fragt sich somit, welche von beiden in der Begründung der Erkenntniss als die erste, zu Grunde liegende, bestimmende anzusehen sei.“ Natorp, P., UOSB, p. 260.

Natorp explains the reasons why the subjectivist conception is untenable. First, the foundation of subjective knowledge constitutes a *metabasis eis allo genos*, a change to another genus<sup>189</sup>. There is a confusion of the levels of knowledge. The subjectivist position confuses the grounds and what is grounded. The ground is the objective and the subjective is what is grounded. Logic is a fundamental science, psychology is derivative. Psychology is a special science. Logic is the science of science. These two levels cannot be mixed. There is a gap between logic and psychology. It can be conceded that knowledge is made up of a subjective and an objective side. However, logic deals with the objective laws of knowing. It does not study the individual subjectivity. The problem of the ideality of the law cannot depend on the effectiveness of the psychic process<sup>190</sup>. Second, after all, the subjective foundation leads to the abandonment of any idea of objectivity<sup>191</sup>. Grounding logic in psychology also implies abandoning the very idea of objectivity of knowledge. Objectivity would not be properly objectivity if it were grounded on the process of each factual subject. The choice of a subjective path of knowledge foundation makes all objective validity a mere subjective validity. Universal and objectively valid knowledge depends on a process that is valid only from the point of view of the subject. Then, the very concept of objective validity is abandoned if the science that should give the conditions of universal validity can only provide the subjective mechanisms of the formation of representations. Thirdly, subjective foundation falls into a vicious circle. Logic must explain the problem of the objective validity of knowledge. If logic depends on psychology, this science of consciousness lacks the necessary parameters to establish whether its arguments are valid or not. The task of finding the ultimate foundation of logic implies the grounding of objective knowledge that psychology itself cannot offer. To ground the logic in psychology, psychological legality should have a foundation that guarantees the objectivity of its propositions, even the thesis that the parameter of truth depends on the psychic

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<sup>189</sup> Originally, the expression comes from Aristotle (in Posterior Analytics I.7., 75a 38). John K. O'Connor makes an investigation of the sources of the use of this expression. He analyzes the use of the expression in Aristotle and argues that to gain a better understanding of the problem in Husserl one should go back to Brentano. Cf. O'Connor, J., 2008.

<sup>190</sup> „Zwar räumten wir bereitwillig ein, dass kein Erkanntes sei ohne Erkennenden; dass Erkenntnis allein gegeben sei im Erlebnisse des Subjects, im Bewusstsein des Erkennenden; aber, so wie diese Beziehung zum Subject hier nicht den Fragepunkt bildet, so finden wir uns auch bei der Beantwortung der Frage nicht genöthigt, auf sie zu recurriren. Jeder Recurs auf das Subject des Erkennens, auf die Art der Betheiligung des Bewusstseins dabei, muss uns vielmehr von vornherein als *metabasis eis allo genos* erscheinen.“ Natorp. P., UOSB, p. 262.

<sup>191</sup> As Scott clearly explains: “As Natorp sees it, accepting a psychologistic or subjective method for logic entails giving up the very idea of objective knowledge”. Scott, E., 2008, p. 57.



processes<sup>192</sup>. Psychology aims to state true propositions. The claim that the truth is based on psychic processes must be true as well. However, psychology depends on a certain conception of the truth that validates this statement. Natorp acknowledges that this argument is insufficient since logic must also prove the truth of its propositions 'logically'.<sup>193</sup> Similarly, the dependence of logic on psychology implies the abandonment of the possibility of logic in general<sup>194</sup>. Logic must be grounded on itself. It must have an immanent foundation. All other sciences must be based on it because logic is the science of sciences. If logic is grounded on psychology, it is not logic anymore<sup>195</sup>. The objective truth of the principles of knowledge cannot be based on the subjective experience of the cognitive subject; since if we make logic depend on psychology, the very claim to find the legal foundation of knowledge is suppressed. Therefore, "logic has nothing to say about thinking as a fact, or as a psychological process"<sup>196</sup> because what must be found are the fundamental concepts and principles that give this first science an autonomous validity. In this sense, logic is the opposite of psychology. The latter deals with the empirical aspect of subjective process while the former seeks the principles of the general validity of objective knowledge. Only then, "the autonomous and purely objective foundation of truth"<sup>197</sup> can be guaranteed. The subjective path would lose the very meaning of the concept of truth since objective validity cannot depend on empirical subjectivity. Then, the possibility of determining the objective validity of knowledge depends ultimately on the possibility of establishing an objectivity at some point independent of subjectivity. Natorp argues that this demand led to the mistaken belief that there are data that the subject apprehends that are independent of objectivity.

### The dogmatism of the given

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<sup>192</sup> „Handelt Logik von dem Kriterium der Wahrheit, von dem, was allgemein, weil auf gesetzmässige Art, die Wahrheit einer Erkenntnis bestimmt, so darf doch die Gültigkeit dieses Kriteriums nicht abhängen von einer Erkenntnis, die nur nach diesem Kriterium als wahr zu behaupten wäre.“ Natorp, P., UOSB, p. 264.

<sup>193</sup> Natorp, P., ZLM, p. 270.

<sup>194</sup> „Entweder also, es gibt keine Logik, oder sie muss mit dem Anspruch auftreten, ganz auf eigenem Grunde zu bauen, nicht von irgendeiner andern Wissenschaft ihre Fundamente borgen zu sollen.“ Natorp, P., UOSB, p. 264.

<sup>195</sup> „Allem voraus, die subjectivistische Ansicht unannehmbar macht, ist die Erwägung, dass der ganze Sinn der Logik, als einer allgemeinen, die Wahrheit der Erkenntnis begründenden Theorie, aufgehoben wird, wenn man, wie die Konsequenz jener Ansicht es fordert, Logik von einer besonderen Wissenschaft, Psychologie, ihrem Princip nach abhängen lässt.“ Natorp, P., UOSB, p. 264.

<sup>196</sup> Natorp, P., LGEW, p. 99.

<sup>197</sup> Natorp, P., UOSB, p. 148.

Natorp identifies the type of independence required. There are two possible senses of such independence. One is that objects are completely exterior to the mind. This response would invalidate the very concept of objectivity since being an object – as will explain in Chapter 3- is to be a term of an act of thinking. Thinking is a discursive process. It implies establishing relationships. The terms required by the concept of relationship are nothing outside it. The terms do not precede the relationship, but they are established by it as requirements<sup>198</sup>. The object, as a term of the relationship that represents knowledge, is nothing outside of this relationship. The object is placed in front of knowledge and, nevertheless, is grounded by it. Certainly, one could ask how the object can be independent of the act of knowledge and, at the same time, be grounded by it. Natorp answers that this independence is generated by virtue of the process of establishing laws.

The establishment of laws involves a process of abstraction. However, the abstraction does not depart from a given *sensa data*. From his perspective, the process of abstraction consists in disregarding certain marks of the objects that are given to senses and taking into consideration only certain determinations in order to form a concept. The abstraction depends on the object that is given to the senses. This was the perspective of psychologism, which defines the process of abstraction explaining the genesis of the representation. According to Natorp, on the contrary, abstraction must be defined entirely positively as the choice of a point of view that guarantees the unity of determinations. This is the only legitimate way to interpret the concept of abstraction. The process of the concept formation involves neither disregarding marks nor the removal of marks of a given object to intuition. The negative definition of abstraction is misleading<sup>199</sup>. The positive definition of the notion of concept must show the parameter that allows articulating the multiple determinations of the object, that is, the unity of the determination. This unity of determination allows us to establish in advance what elements will be considered in the object of knowledge. The marks that are not considered as part of the object will be a corollary of the chosen point of unity. The choice of the point of view provides which marks are part of the object. The choice of this point of

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<sup>198</sup> „Dem Inhalte nach aber ist Denken: Setzen von Beziehung, nicht anderes. Beziehung fordert Termini; aber auch nicht diese gehen der Beziehung voran, sondern die Beziehung setzt auch erst die Termini.“ Natorp, P., LGEW, p. 99.

<sup>199</sup> „Ich glaube, man erklärt die Abstraction, wo nicht überhaupt untriftig, so doch unzulänglich und nicht von dem richtigen Anfang her, wenn man sie bloss negativ versteht : logisch, als den Abzug eines Merkmals ; psychologisch, als Ausserachtlassen, Absehen, Abwenden oder Abziehen des geistigen Blicks von einem bestimmten, thatsächlich doch in der Vorstellung enthaltenen Moment.“ Natorp, P., UOSB, p. 270.

view establishes the selection of determinations and how to establish the relationship among them. The object is nothing but this complex of relationships that are determined by the choice of the point of view. Abstraction is not a process in which a mark belonging to the object is eliminated but the choice of a determining unity that defines which marks constitute the object under consideration and their forms of relationship. This articulating unity of multiplicity is the law<sup>200</sup>. The multiplicity of the marks that define the object is only the correlation of the unity that articulates it. The required abstraction is found in the concept of law. In this way, the law can guarantee the independence of the object of knowledge. Likewise, the law can be related to the singular case without losing its universality. The law produces its instances, and the object of knowledge is produced by the law. For idealism, the meaning of the case for knowledge is only to be an instance of legality. The case is not only the subjective appearance but precisely, the particular with respect to the universal that is the law<sup>201</sup>. The correlate in the subjective pole is the appearance. The appearance of the phenomenon is always changing, it changes according to the changes in the state of the subject. On the contrary, the law forms its case in such a way that the object constructed by it is a unity completely independent of any subjective state. This abstraction of the law guarantees its validity regardless of any modification in the state of the subject. The objectively valid is, precisely, what was articulated by the unity of the law<sup>202</sup>.

According to Natorp, this mistake of psychologism consists in a misunderstanding of the meaning of the law. The psychologist's account cannot trace a distinction between law and states of facts. A law can be considered a fact only if by facts it is understood 'being the case', something that could be verified. In this general sense, the law can be considered a fact. However, the problem is to identify the law with a temporarily determined event. The expression:  $2 \times 2 = 4$  is a fact in the sense that it is the case. However, in no way does this imply that the operation entails a temporary character. The

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<sup>200</sup> „Der Gegenstand bedeutet positiv das Gesetz; er bedeutet die beharrende Einheit, worin die wechselnde Mannigfaltigkeit der Erscheinung gedanklich geeint und festgestellt wird.“ Natorp, P., UOSB, p. 271.

<sup>201</sup> „Für den Idealisten hat im Gegentheil das Einzelne in der Erkenntniss nur Bedeutung kraft des Allgemeinen, dessen Einzelnes es ist; es entlehnt somit alle Geltung, die es in der Erkenntniss beanspruchen kann, vom Allgemeinen, darf dagegen ursprünglich und von sich selbst nichts gelten wollen.“ Natorp, P., UOSB, p. 278.

<sup>202</sup> „Sowie aber die gesetzmässige Auffassung den Gegenstand, das objective Gültige vertritt, so ist die Erscheinung, vor der Reduction aufs Gesetz und damit auf den Gegenstand, der concreteste Ausdruck der Subjectivität. Erscheinung ist die noch nicht im Gesetz objectivirte, mithin noch subjective Vorstellung, sowie die durch die Erhebung zum Gesetz, zum Standpunkte des Allgemeingültigen, zur Einheit gebrachte Vorstellung die gegenständliche ist.“ Natorp, P., UOSB, p. 273.

law is not a general expression for facts if we define facts as temporarily determined phenomena. Natorp holds:

Gewiß, jedes Gesetz sagt aus, was allgemein stattfindet; sofern man also jedes Stattfinden ohne Unterschied Tatsache nennt, ist jedes Gesetz eine allgemeine Aussage über Tatsachen. Es ist in diesem Sinne Tatsache, daß  $2 \times 2 = 4$ , und Tatsache, daß Widersprechendes nicht gleichermaßen wahr ist u. s. f.; aber zu dem Schluß: also sind alle Gesetze Ursachgesetze, gelangt man nicht durch diesen allgemeinsten Sinn der Tatsache, sondern durch das stillschweigend mitgedachte spezifische Merkmal zeitlicher Bestimmtheit. Ursachgesetze sind Zeitgesetze des Geschehens, und nur sofern man unter Tatsache, im auch zulässigen engeren Sinn des Worts, Geschehen versteht, deckt sich „Gesetz von Tatsachen“ und „ursächliches Gesetz“. Aber daß  $2 \times 2 = 4$ , ist kein Geschehen in der Zeit, weder ein einzelnes noch ein allgemeines, sondern ein Stattfinden, das an gar keine Zeitbedingung gebunden ist oder sie irgendwie einschließt. Dasselbe gilt von den logischen Gesetzen; sie sind nicht Zeitgesetze, folglich nicht ursächliche Gesetze, weder physische noch psychische, oder in solchen begründet, sondern von einer fundamentalen Ordnung; denn das ursächliche Gesetz ist vielmehr dem logischen, ebenso wie dem mathematischen, unterworfen, nicht das logische, das mathematische dem ursächlichen.<sup>203</sup>

The laws of nature depend on the laws of logic, but logic does not depend on any other science. The laws of logic are constructed without being events in time, that is, events determined by the law of causality. This does not mean that the laws of logic do not apply to temporary events but that temporary events presuppose the laws of logic. The determination of events in time presupposes the laws that regulate all determination in general. For example, any temporary determination implies the possibility of determining the event as identical to itself, that is,  $A = A$ . This logical law, the law of identity, grounds the event that takes place. However, no fact of nature can ground this fundamental logical law. This does not imply denying the temporal nature of the thinking process. Indeed, the process of thinking can be considered a phenomenon in nature. Thinking can also be studied as long as it is conditioned by causal laws. It is not denied that there is a process that takes place. It is affirmed that the validity of the laws of logic does not depend on the legality of the generation of representations. The establishment of the laws that regulate how effective thinking is consummated is a problem of a different field. Logical laws

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<sup>203</sup> Natorp. P., SP, p. 18.

have universal validity while the legality of the succession of representations is limited. The legality of the thinking process is conditioned while the laws of logic have an unconditional validity<sup>204</sup>.

The methodological error of psychology leads to conceiving the data of immediate experience as the first in the order of knowledge. It considers that what is given to perception is the most objective since it is what subsists regardless of the act of knowledge. As we showed, the data is the ultimate goal, and its independence is only guaranteed by its reduction to the law. Objectivities are nothing but the products of laws-construction. It is only the unity of the determination of the law that determines the reality of phenomena<sup>205</sup>.

### Against transcendental psychology

Transcendental psychology does not escape the aforementioned problems. Natorp argues against one of the rival schools of the time: phenomenology. Husserl tried to solve the problems of psychologism by appealing to a transcendental consciousness. He claimed that the transcendental approach to subjectivity would avoid the problems involved in psychologism. However, according to Natorp, this resolution of the problem does not escape the critique made against the subjective foundation of knowledge. In the first place, transcendental psychology also confuses the problem of the gestation of knowledge with the question of the problem of its sources of validity. The contents of knowledge are not subjective representations, but ideal elements that can be verified independently of any subject. This distinguishes a scientific phenomenon from a mere fact. The ultimate goal of the process of thinking is to find the legality of the appearance. It seeks to turn the mere appearance into an object. For this reason, the point of view of thinking must be the objectivity of the law. The knowledge cannot in any way obtain legitimation in the subjective processes of the mind. Thinking as a subjective process can only have appearances as its objects. The subjective path that is rejected is not only the one that

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<sup>204</sup> „Der Inhalt eines logischen Satzes ist nicht, dass unter solchen und solchen Bedingungen Gedanken sich so, unter andern anders verbinden, sondern dass, ohne jede einschränkende Bedingung, gewisse Gedankenverbindungen wahr, davon abweichende falsch sind. Diese Unbedingtheit der logischen Gesetze würde fraglich werden, wenn die überaus bedingte zeitliche Gesetzmäßigkeit des Vorstellungslaufs für die logischen Gesetze eintreten sollte.“ Natorp. P., SP, p.19.

<sup>205</sup> Natorp. P, LM, p. 13.

seeks the foundation of knowledge in individual consciousness<sup>206</sup>. Natorp not only rejects the foundation in the individual consciousness but also every attempt to ground knowledge in its genesis. The subjective path affirms that knowledge can be grounded by explaining the processes that are its origin. The rejection of the subjective path of foundation denies both. On one hand, Natorp argues that knowledge cannot be grounded in processes of an individual mind. On the other hand, it is generally denied that knowledge is legitimized according to the activity of consciousness in general, whether it is individual or supra-individual. In the second place, transcendental psychology incurs in the same mistake of psychologism by taking immediate experience as the first *factum* of knowledge. The point of view of subjectivity leads to assume as a first *factum* what is given to sensible intuition. The subjectivity standpoint led to this result. Natorp recognizes the peculiarities that phenomenology gives to ordinary experience. Natural knowledge is immediately related to objects. Immediacy is one of the fundamental conditions that phenomenology uses to take the natural experience as an original *factum*<sup>207</sup>. Natorp, even recognizing the immediacy of that kind of cognitive relationship, will show why this is not a sufficient condition to grant the privilege that phenomenology gave it. The problem with this type of knowledge is its union with the present perceptual experience. The spontaneous knowledge, which starts from perception, “always takes place in the presence of objects”, or, phenomenologically expressed, “has the object in flesh and blood”. However, this peculiarity of natural knowledge prevents characterizing this immediate experience as that first *factum*. Natural experience is anchored in immediate perception, while the prosecution of the explanatory system requires turning on these perceptions to constitute them as representations. For Natorp, there is no possibility of immediate access to the experiences. Even the path for a psychological analysis, which aims to move away from the realm of the merely empirical experience,

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<sup>206</sup> In this sense, we disagree with the reading of Edgar Scott. According to Scott's interpretation, Natorp only rejects that knowledge can be grounded in relation to the factual awareness of a particular cognitive subject. The problem would be to ground knowledge in the mechanism of a particular consciousness. However, according to Scott, Natorp would not reject the foundation of knowledge in a general consciousness. On the contrary, intersubjective consciousness would be the external parameter that guarantees the independence of knowledge with respect to the consciousness of the individual. The author holds: “At the very least, this suggests that subjectivity consists in being particular to the representations of individuals. That is, in order for objective knowledge to be independent of ‘the subjectivity of knowledge’, it need not be independent of all consciousness. Rather, being independent of ‘the subjectivity of knowledge’ means being independent of only ‘this or that’ consciousness. It means being independent of any particular, individual consciousness. [...] The intersubjective or shared body of knowledge provides an independent standard against which any individual knower must measure her representations” Cf. Edgar, S. 2008, p.58.

<sup>207</sup> Husserl, *Ideas I*, §24.

can only return from the objectification of the human spirit. Thus, the methodology that starts from ordinary experience turns into an “absolute idealism”, since it stops the flow of objectification of living in the rigid conceptual elements that cannot run parallel to the progress of the spirit. On the contrary, the critical method does not face the danger of absolutism. Faced with this absolutism of the regressive path, it is pointed out that the transcendental regressive method hides a progressive element while its reduction occurs from the fact of the development of the spirit<sup>208</sup>.

We can summarize Natorp’s criticisms of psychologism as follows:

First, Natorp’s criticism is based on the distinction between the logical and psychological aspects of cognition. To be a grounding science of knowledge, logic must be an objective science. The logic does not deal with the process of knowing. Its task is to find the laws that govern knowledge as a result. Psychologism confuses the laws of knowledge with the rules that regulate the psychic life of empirical subjects. Consequently, it aims to legitimize a fundamental science (logic) in a special science (psychology). This conception is circular. Psychology cannot provide by itself a definition of truth, but it requires a concept of truth that legitimizes its own propositions. In its rejection of the definition of truth offered by logic, psychology becomes circular. This error leads to confusing the problem of genesis with the problem of validity. The foundation of knowledge does not require an explanation of the way in which knowledge is generated but should explain the way of legitimization of cognitions.

Second, we showed that this methodological mistake leads psychology to take the object of knowledge as something given. This mistake is the product of “naive thinking” that considers objectivity as something that is given to the mind. From this perspective, the completely determined object is given to sensible intuition, and the mind generates its concept by a process of abstraction. The object is what is given to intuition and the concepts are constructed abstracting certain marks from the objects. Natorp shows that thinking does not require anything external to itself in the construction of its object. On the contrary, the objects of cognition are generated, they are produced and not given. As we explained, this process of production of objects is the generation of laws. The law is not an abstraction of concrete cases but produces its instances. Knowledge seeks to

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<sup>208</sup> „Die transzendente Methode, als immanente, ist in solcher Gefahr nicht; sie ist selbst fortschreitend, entwickelnd, auch unendlicher Entwicklung gewachsen.” Natorp, P., KMS, p. 199.

establish the case as an instance of the law. There is no element given to thinking. Having started from the subjective point of view, psychologism considers the immediate data as the paradigmatically real, as the first for the act of knowledge. According to Natorp, the conception of psychology leads to the loss of the concept of objectivity as it makes logic dependent on psychology. For Natorp, the validity of knowledge is precisely independent of the mental processes of factual subjects. Thus, on the one hand, the object is in some sense the most dependent, since it is nothing more than a construction of thinking expressed in the law. However, as opposed to mere appearances, the fact is also independent. The law guarantees its independence from all subjective consciousness. In fact, the only guarantee of independence, required by the object of knowledge, is its foundation in the law. The laws of logic are not facts conditioned by time. On the contrary, temporary events involve the laws of logic. The facts, the temporarily determined events, suppose the laws of thinking.

Psychologism starts from the methodological error of taking the subject as a starting point and the object as what is contrasted as part of psychic life. The object given to intuition is the first element. Concepts are generated through a process of abstraction. As we saw, this approach cancels every possibility of the foundation of knowledge and, more specifically, every possibility of the foundation of objectivity of facts. Therefore, an accurate approach to the relation between conceptual and intuitive representations necessarily demands the abandonment of this point of view. Natorp explains how considering the construction of concepts as laws solves the problems raised above. We will further develop this relationship between concepts and law in chapters 3 and 4. In what follows, we will study why the point of view of logicism is also insufficient.



## 2.2. Against Logicism.

Natorp argued that the perspective of psychologism confused the problem of the genesis with the problem of the validity of knowledge. Logic, Natorp showed, should not start from any initial data given to thinking, as psychologism considered. The intuitive given content cannot be a point of departure. Rather, logic must show how thinking constructs its objects by its own means. To establish his position, Natorp will argue against the representatives of logicism. As Frege explains, the logicism program aims to show that “arithmetic is a branch of logic and need not borrow any ground of proof whatever from either experience or intuition”<sup>209</sup>. Thus defined, it could seem as if the logicist program has only to do with the problem of the foundation of mathematics. However, it must be borne in mind that by the nineteenth century, ‘logic’ was almost a synonym of a theory of knowledge<sup>210</sup>. The problem was not only to ground mathematics but also to clarify the role that the mathematical determinations play in the constitution of objectivity<sup>211</sup>. Then, to show how mathematics is grounded in pure thinking is also the first step to exhibit how objectivity, in general, can be constructed by conceptual determinations. Natorp also aims to show that logic is built on the basis of pure thinking, without any reference to anything given to intuition, in the Kantian sense of the word. He argues that thinking can build the foundations of experience. This perspective was shared by that time with the representatives of logicism. This philosophical current also held that the process of cognition could not depart from something given to intuitions. They wanted to exhibit that logic could be grounded by thinking. Thus, logicism challenged the core of the Kantian program by denying any reference to any intuition in the construction of objectivity. There is no place for a distinction between sensibility and understanding, nor between intuition and concepts because thinking alone can produce the form and the content of objectivity<sup>212</sup>.

Natorp agrees on the need of a logical foundation and of a revision of the distinction between intuitive and conceptual representations. However, Natorp did not

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<sup>209</sup> “Logicism is the thesis that mathematics is reducible to logic, hence nothing but a part of logic”. Carnap, R. 1931, p. 91. According to Frege, arithmetic is a branch of logic. For this reason, it does not depend on intuition. Frege claims to have proved this point in his *Grundlagen der Arithmetik*. Cf. Frege, G., 1893, p.1.

<sup>210</sup> Cf. Dufour, E., 2010, p. 19.

<sup>211</sup> Dewalque, A., 2009, p.45.

<sup>212</sup> Frege holds: „...das von jedem durch die Sinne oder selbst durch eine Anschauung apriori gegebenen Inhalte absehende reine Denken allein ans dem Inhalte, welcher seiner eigenen Beschaffenheit entspringt, Urtheile hervorbringen vermag, die auf den ersten Blick nur auf Grund irgendeiner Anschauung möglich zu sein scheinen.“ Frege, G., 1879, p. 55. Later, we will highlight the role of intuition for geometry.

share with the logicians all the points of their program. For Natorp, logicians also misunderstood the relationship between intuitive and conceptual representations on the grounds of a methodological mistake. If the mistake of psychologism was to depart from the perspective of what is given, logicism will absolutize the perspective of the concept. Natorp showed that investigation cannot depart from the perspective of the genesis of the representation in the mind. Psychology cannot be the ground of knowledge. Natorp will argue that formal logic is insufficient for a foundation of knowledge. The foundation of knowledge in formal logic assumes a separation between the principles of knowledge and the object that is known. He will argue that the foundation of knowledge requires a transcendental logic. The aim of this section is to analyse Natorp's arguments in relation to the inadequacy of the position of logicism. Formal logic is insufficient for a logical foundation of the sciences. This methodological error will lead to logicism to a misunderstanding of the relationship between intuition and concepts. As we shall see, the position of logicism in relation to the formation of logic will lead to a separation between the universal and the particular. Arguing against this direction, Natorp introduces the conception that thinking can produce the object of knowledge by itself. However, the process of the formation of concepts will be considered in a very different way from that proposed by the logicist program. Natorp will maintain a synthetic grounding of knowledge. In this conception, the creation of the instances will be explained by reference to the formation of law. The conception of logicism will lead to an analytical conception of the foundation of knowledge. First, we will briefly focus on the debate with logicism to put Natorp's arguments in context. Second, we will study Frege's positions, considering that Natorp argues primarily against this conception<sup>213</sup>. Third, we will analyze Natorp's common points with this position and the main objections. We will show that the methodological error of logicism leads to an incorrect formulation of the relationship between intuitions and concepts.

### 2.2.1. Introduction to the Debate.

As we mentioned, at the beginning of the nineteenth century, philosophy was experiencing a crisis. One of the reactions to this crisis was psychologism. However, another line of philosophers argued that philosophy must follow the mathematical

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<sup>213</sup> According to Charles Parsons, Natorp seems to be only familiar with *The Foundation of Arithmetic*. Parsons, C., 2014, p.13.

method. The debate can be divided into two closely related problems. The first problem concerns the relationship between philosophy and mathematics. According to Natorp, the relationship between these two disciplines underwent a profound modification with the decline of Kantian philosophy. On the side of mathematics, the progress achieved by this discipline at the beginning of the 19th century led to the loss of the close relation it had with philosophy. Mathematicians believed they can dispense with the instruments provided by philosophical disciplines and intended to ground their procedures with purely mathematical methods. The philosophical analysis of the exact sciences was still largely based on the assumptions inherited from Aristotelian logic, which is increasingly insufficient to ground the course of the new mathematics<sup>214</sup>. Besides, the development of non-Euclidean geometries contributes to the discredit suffered by philosophy. Philosophical presuppositions, this time more related to the Kantian paradigm, cannot account for the new developments in geometry. Then, new systems emerge that try to overcome the logical foundation of the exact sciences based on the Aristotelian-Kantian assumptions, and to generate a more fruitful and consistent logical system with the new mathematical model. Philosophy moves away from the rigor that formerly the mathematical method had provided. According to Natorp:

Die alte, nach beiden Seiten fruchtbringende Verbindung zwischen Philosophie und Mathematik schien eine Zeitlang sehr gelockert. Was die Mathematik vielleicht einmal der Philosophie zu danken hatte, die Strenge des Beweisverfahrens, ja den ganzen Begriff des formalen Aufbaus einer Wissenschaft, das hat sie längst aus eigener Kraft und eigenem Trieb so in sich aufgenommen, dass sie mit gutem Grunde glauben darf, darin von der Philosophie nicht viel mehr lernen zu können. Diese dagegen schien die nüchternen Bahnen, in denen sie manches Jahrhundert mit den strengen Wissenschaften Hand in Hand gegangen war, zeitweilig ganz zu verlassen und sich in spekulativen Abenteuern zu gefallen, die das streng geschulte Denken des Mathematikers zu allererst zurückstossen mussten.<sup>215</sup>

The metaphysical tendencies of post-Kantianism increasingly led philosophy to depart from the rigor to which it originally aspired. The mathematical method was a way to find

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<sup>214</sup> Cf. Jacquette, D., 2006, p.11 ss; Detlefsen, M., 2004, p.55.

<sup>215</sup> Natorp, P., LGNM, p.177.

that rigor of philosophical thinking that seemed to be lost after Kant. Mathematics was its refuge to ground conceptually the procedure of logic. The rigor of mathematics and its formal character provided the philosophy with a safe method of analysis. This rigor that philosophy had lost could be recovered based on the firm ground of the mathematical method. Just as Spinoza and Leibniz had done in modernity, the crisis of philosophy could be faced by adopting the model that science, especially mathematics, applied successfully. Rigorous reasoning could be guaranteed if they could adopt the mathematical method. The possibility of defining symbols and creating a system of rules that defined how these signs relate to each other would avoid the vagueness of natural language. Philosophy could have the rigor of the mathematical method by following procedure in combinatorial art and calculation. Philosophical reasoning could guarantee its validity by following these calculation rules. The logicians to which Natorp refers are continuators of this current of reasoning initiated by Leibniz<sup>216</sup>.

Secondly, at the end of the 19th century and the beginning of the 20th century, a revolution of the logical method arises and, with it, a debate regarding the relationship between logic and mathematics. The question is whether mathematical relationships can be deduced from logic or if they constitute a peculiar domain of relationships. According to some thinkers of the time, mathematics can be based on a series of fundamental logical concepts. Mathematical laws could be derived from a logic of thinking. Mathematics would be a branch of formal logic. The axioms of mathematics could be derived from the laws of logic. If arithmetic is an extension of logic, a study of the fundamental logical principles would suffice to provide this discipline with a solid foundation. Therefore, in general, the project of arithmetic logic is a characteristic problem of the time. It seeks to find the legality that determines the mathematical object. As Natorp explains there are two separate closely related problems. On the one hand, logic receives the influence of mathematics. Logic itself starts receiving a mathematical treatment. On the other hand, mathematics aspires for a logical foundation. Mathematicians want to show that the objects of mathematics can be built without any reference to intuition but purely

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<sup>216</sup> Cf. Placencia, L., Espinosa, R., 2017, p. XI.

conceptually<sup>217</sup>. This problem occupies Natorp's thinking from his early writings<sup>218</sup>. The question is, on the one hand, to establish the role that those mathematical determinations play in the construction of the object in general and, on the other, to investigate whether the determinations of the mathematical object could be established from logical laws. This problem about the status of the mathematical object has a peculiar relevance to Natorp's thinking. The problem is not only on the status of mathematics but the more general problem of the nature of the object of cognition. The problem is whether the object of knowledge, which is grounded primarily on mathematical determinations can be reduced to pure logical relationships. Lastly, the question is whether the object can be purely conceptually determined. Natorp will agree with the logicist program on the need to ground knowledge in logic, which is one of the main points of the logicist program. Indeed, the object of experience is firstly defined by mathematical determination, and mathematics requires a logical foundation. While their ideas differ internally, representatives of nineteenth-century logic converge on some fundamental ideas. The question can be put into two separate problems. On the one hand, logic aspires to proceed mathematically. It aims to proceed purely conceptually without any reference to intuition. On the other hand, mathematicians were searching for such a method to avoid any reference to intuition, to an intuitive given content. Within this problem, the current of logicism emerges. In what follows, we will study the main thesis of this project, focusing on the problem of the relation between intuition and concepts.

### 2.2.3. Main Thesis of Logicism.

#### Rejection of mathematical psychology

One of the common points of this current is its adversary. Logicists reject the foundation of logic in psychology and, more specifically, mathematical psychologism<sup>219</sup>.

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<sup>217</sup> Natorp explains: „Ich hätte nicht den Mut, mich, als Nichtfachmann, an Mathematiker zu wenden, wenn ich nicht sachliche Gründe dafür zu erkennen glaubte, dass die Logik, die Erkenntniskritikenge Föhlung mit der Mathematik zu suchen hat; nicht um sie zu belehren, mehr, von ihr zu lernen, genauer, ihre Mitarbeit an einigen ihrer schwersten Aufgaben zu erbitten, die ohne die Mithilfe der Mathematik nicht zu bewältigen sind. Ich denke dabei nicht so sehr an einen besonderen Zweig unserer Wissenschaft, dem, nachdem er lange in aristotelischer Tradition erstarrt war, durch die mathematische Behandlung neues Leben zugeführt worden ist: die Syllogistik, sondern ich denke an die ganz allgemeine Tendenz der neueren Mathematik, sich zu einer rein logischen Gestaltung durchzu arbeiten, so dass die Berufung auf „Anschauung“ mehr und mehr überflüssig wird.“ Natorp, P., EGM, p. 2. See: Porta, González, M. A., 2011, p. 205 ss.

<sup>218</sup> Cf.: Natorp, P., QQ, ZLGNM, EGM, NTE, LGEW.

<sup>219</sup> Cf. Natorp, P., LGEW, p. 3.

For the logicians, the study of the formation of mental representations has no relation to the formal sciences. In fact, the intrusion of psychology into logic has hindered the task of a philosophical foundation of mathematics. For logicism in general, logical laws cannot be considered abstractions of the thinking process. The process of formation of the subjective representation must be distinguished from the conceptual definition of legality. The logical law is not obtained from a determination of the thinking process. The logic consists neither in the study of the processes of forming representations nor, consequently, in the investigation of the possibility of the convergence of mental representation with things. Subjective representations cannot have the force of law and, therefore, cannot be the foundation of mathematical statements. Psychologism confuses the logical law that governs mathematical statements with the natural laws that rule mental processes. The knowledge of the subjective formation of a representation does not allow us to know any property of the legality of thinking. The law that governs objectivity cannot be obtained from an intuitive fact given to sensation. The objectivity of a factum presupposes its being independent from the point of view of the subject. If the fact were dependent on the subjectivity, it would be a subjective fact. The objectivity presupposes the independence of the fact from our sensations. While psychology analyzes the problem of subjectivity, logic studies objective thinking, and “there is nothing more objective than arithmetic.”<sup>220</sup> The domain of the objective is heterogeneous with respect to the scope of the merely subjective.

The logicians agree on the need to radically separate the logic from empirical psychology. They oppose the possibility of founding logical concepts such as validity or truth in psychic mechanisms. Psychology cannot determine the conditions of truth, since being true and taking something for true are completely different phenomena. The laws of the genesis of representation and its correspondence with a represented object have no inherent connection with the truth. The same applies to the study of historical genesis. The study of the historical conditions in which a mathematical discovery takes place has no relation to the conditions of validity of mathematical truth, a problem that constitutes the object of logic. Psychology confuses the question of the fact with the question of validity. The problem of logical justification must be separated from the problem of how

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<sup>220</sup> Frege holds: „Nein, mit Gefühlen hat die Arithmetik gar nichts zu schaffen. Ebenso wenig mit innern Bildern, die aus Spuren früherer Sinneseindrücke zusammengeflossen sind. Das Schwankende und Unbestimmte, welches alle diese Gestaltungen haben, steht im starken Gegensatz zu der Bestimmtheit und Festigkeit der mathematischen Begriffe und Gegenstände.“ Frege, G., 1884, p.v.

the process of thinking takes place. The problem of logic is restricted to the domain of pure thinking. There should be no intrusion of intuitive data taken from the experience. The problem of the logical law is restricted to the purely conceptual. For the logicians, the study concerning the problem of the legality of thinking cannot have any reference to intuition, be it pure or empirical. The logical law concerns the realm of pure thinking, the realm of pure concepts. To ground the legality of thinking, one cannot appeal to pure intuition either. While they agree that the laws of logic are necessary and sufficient foundations of arithmetic, logicians also reject the idea that pure intuition is necessary for the foundation of the science of numbers. Against Kant, logicism believes that the idea of number is independent of both empirical intuition and pure intuition. The logical law on which mathematics is based is a product of pure thinking<sup>221</sup>. Thinking is particularly free in arithmetic because it has freed itself from its restriction to intuition, in this respect the representatives of logicism agree. For Frege, for example, this point differentiates arithmetic from geometry. Geometric laws “govern the domain of what can be intuited spatially” while arithmetic laws govern the domain of everything thinkable<sup>222</sup>. Arithmetic is in this sense an analysis of the laws of thinking itself. Frege is one of the authors who develops extensive arguments against psychologism and devotes much of his work to prove that arithmetic is based on logic and does not depend on any kind of intuition.<sup>223</sup>

#### The definition of logic

The representatives of logicism agree on the need to base mathematics on logic. Logic must provide the basis of thinking, and with it the fundamental legality of everything that falls under its scope. Logic is the science of thinking<sup>224</sup>. However, as we saw, the concept of thought must be separated from that of the act of subjective thinking<sup>225</sup>.

The goal of logic is to investigate the foundations of true thinking and has no relation to subjective psychic acts. The problem of knowledge is independent of the act of thinking of a factual subject. Logic does not study the genesis of representations - how

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<sup>221</sup> Natorp, P., LGEW. p.4.

<sup>222</sup> Cf. Frege, G., 1884, p. 20 ss.

<sup>223</sup> „In meinen *Grundlagen der Arithmetik* habe ich wahrscheinlich zu machen gesucht, dass die Arithmetik ein Zweig der Logik sei und weder der Erfahrung noch der Anschauung irgendeinen Beweisgrund zu entnehmen brauche.“ Frege, G. 1893, p.1.

<sup>224</sup> Frege, G., 1979, p.5.

<sup>225</sup> For Frege, this is the definition of the science of logic. However, this same definition is what has led to the error of psychology. Frege, G., 1979, p. 4.

they are empirically obtained. The problem of logic is the problem of the justification of truth. We legitimize our statements through the laws of inference. For this reason, the laws of valid inferences are the object of the study of logic. We must distinguish the way in which inferences are effectively made from their legitimation. Logic is the science of inferences in this last sense<sup>226</sup>. Logic includes the entire domain of thinking, providing the laws that determine every being. This science is not restricted to any particular field<sup>227</sup>. In this sense, logic is the ground of every object of cognition, because it bases the legality of every being. Everything that falls into this area is governed by this legality. According to this conception, logic provides the most general truths of thinking, and then the logical laws that govern all constructions of thinking.<sup>228</sup> Concepts are expressions of thinking functions. They are expressions of how thinking proceeds in the construction of its objects. Logic can be defined as a science of concepts. Mathematics is based on the laws of thought and arithmetic purely and exclusively in this area. Logic is the science of the purely conceptual field and mathematics is based on this logical space. The fundamental idea on which logicism is based affirms that mathematics rests on a series of fundamental logical concepts that are a pure product of thinking. The logical realm is that of pure thinking. Arithmetic, as it is based on logic, is also a pure a priori science. Therefore, a demonstration of the logical derivation of arithmetic is also proof of its analytical and a priori character. According to this conception, arithmetic is “a more widely developed logic, and each arithmetic statement would be a logical law, although a derived law.”<sup>229</sup> Therefore, the fundamental objective of logicism is to reduce the fundamental concepts and principles of mathematics to purely logical concepts and principles. The logic thus conceived is self-sufficient, it does not require a subsequent logic that legitimates it. For this conception, logic requires neither a metaphysical foundation nor a theory of knowledge. On the contrary, logic is the founding science that provides the foundations of mathematics and does not require further legitimation.

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<sup>226</sup> Frege gives the following definition of logic: Logic is concerned only with those grounds of judgments which are truths. To make a judgment because we are cognizant of other truths as providing justification for it is known as inferring. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic. Frege, G., 1979, p. 3.

<sup>227</sup> For Frege, the task of logic: “... is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter. We must assume that the rules for our thinking and for our holding something to be true are prescribed by the laws of truth. The former is given along with the latter. Consequently, we can also say: logic is the science of the most general laws of truth. Frege, G., 1979, p. 128.

<sup>228</sup> “[t]o say that the laws of logic are the most general laws of truth is to say that they are the most general truths” [Ricketts, 1986, p. 80]. Quoted in: Suillivan, P, 2004, p. 727.

<sup>229</sup> Frege, G., 1884, p. 99. Frege does not include geometry because the latter requires an intuitive element.



## Validity and truth

The verification of the validity of mathematical reasoning is a logical problem, a matter of calculation.

The validity of logical propositions does not depend at all on the mental mechanisms that generate the representation of the law. The genesis in empirical consciousness is the problem of psychology. Logic, on the contrary, focuses on the study of valid inferences. Validity does not refer to how we actually think but to how we should think. Therefore, arithmetic propositions cannot be justified through the explanation of their empirical genesis in the mind. One of the consequences of mathematical psychology is that the truths of mathematics would be limited to the contingency of the peculiar constitution of the nature of the human mind. The validity of arithmetic judgments would be restricted to the field of human knowledge. On the contrary, mathematical truths are valid for the whole scope of the rational and not only for this or that particular rationality. Logical truths are universally valid, independent of all time and space. Logicism considers that arithmetic judgments have universal validity. Frege argues that the conception of psychology culminates in an elimination of the very concept of truth. The conception of psychologism necessarily leads to relativism. If logic were founded on the mental act, mathematics - which relies on logic - would lose all possibility of holding any objective validity. For psychologism, thinking proceeds according to natural laws or laws whose essence does not differ from natural laws. Then, the concept of truth loses all its meaning. Psychologism confuses the laws of thought with the natural law. The psychological analysis is precisely opposite to a rational procedure. The validity of logical reasoning cannot be based on the contingency of the subjective act of thinking. Logic governs every true being (*Wahrseins*) and not the holding it as true (*Fürwahrhaltens*) of a particular subject<sup>230</sup>.

The pursuit of truth is the characteristic feature of all science. All science aspires to the truth<sup>231</sup>. However, they do not have the truth as an object of study<sup>232</sup>. On the contrary, truth is for logic its most proper object. Just as ethics is the science that studies

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<sup>230</sup> Frege, G., 1893, p. XVII.

<sup>231</sup> In general, "the objective of scientific work is the truth". Frege, G., 1979, p.2.

<sup>232</sup> Frege, G., 1979, p. 3. In relation to this point, Peter Suillivam comments: "Frege's commitment to the a priority of logic is intelligible only if he recognized a distinction of kind, and not merely of degree, between the most general laws of truth and laws of special sciences". Sullivan, P., 2004, p. 727.

the concept of good, and aesthetics the concept of beauty, logic is the science of truth. The laws of logic are the laws of true thinking<sup>233</sup>. This science exhibits the laws of thinking and the laws that thinking should follow in the search for truth in general. As the laws of logic are the most general truths, this science can be defined as the “display of the content of the concept of truth.”<sup>234</sup> The way to display the content of this concept is by displaying the laws of inference<sup>235</sup>. The laws of truth are the laws of inference. The truth of each logical law can only be justified using another logical law<sup>236</sup>. For logicism, the validity of mathematical reasoning is based on its subjection to the laws of logic. Meanwhile, mathematics relies on logic, the possibility of mathematics to arrive at true propositions is based on the subjection to the laws prescribed by this fundamental science. The verification of the validity of mathematical reasoning is a logical problem. The legitimacy of a mathematical judgment is based on the logical law. Valid judgments are those that are based on the laws that logic prescribes. Logicians consider that it is necessary to generate a symbolic system and define calculation rules that allow them to operate with these symbols. Logic would be the discipline that determines the correct formation of symbols and defines the legitimate modes of relationships. The need to generate a formal vocabulary - free from natural language ambiguities - and syntax rules is recognized; that is, definitions of terms and laws that regulate the relations among the defined elements. Definitions generated through logical language should not reproduce the structure of natural language but the language of pure thinking. The symbolism must be generated so that it can be an expression of this legality of pure thinking. The logical definition must be based on the structure of thinking and not on natural language. Furthermore, the definitions that can be extracted from natural language are taken from the experience. On the contrary, the logical vocabulary is creative. Logic creates concepts and gives the rules, also purely conceptual ones, that allow operating with concepts as terms. These concepts are creations of thinking and not results of an abstraction from intuitive content, as concepts of natural language. The symbols of logic are not abstracted from any element outside thinking but are created by thinking itself. The syntax of logical language is not an abstraction of the actual use of natural language but the expression of

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<sup>233</sup> “The word ‘true’ can be used to indicate such a goal for logic, just as can ‘good’ for ethics and ‘beautiful’ for aesthetics”. Frege, G., 1979, p. 128. Also, p.4.

<sup>234</sup> “It would not perhaps be beside the mark to say that the laws of logic are nothing other than an unfolding of the content of the word true.” Frege, G., 1979, p.3.

<sup>235</sup> For this reason, Frege defines logic both as the science of valid inferences or as the science of truth. Frege, G., 1979, p.88

<sup>236</sup> CF. Frege, G., 1893, p. XVII.

the way in which thinking operates in the formation of the concepts. The conceptual system is complete, no further grounding of the definitions or of the rules of operation is required. The logical system has intrinsic legitimacy. Logical language must be separated from natural language<sup>237</sup>. However, considering the differences mentioned, an analogy can be drawn, logic is to thinking what grammar is to language. The generation of this logical language is one of the greatest contributions of logicism and, particularly of Frege<sup>238</sup>. As long as the justification of arithmetic judgments rests on the possibility of their derivation from logical laws, the calculation can be reduced to derivation functions. The mathematical calculation is legitimized in the logical deduction. For Frege, for example, the fertility of a definition is determined by the possible use that can be made of it in the deductions. A legitimate definition, being free of contradiction, can always be an element of a demonstration. However, the absence of contradiction is not a firm probative foundation. Therefore, it is necessary to examine the general logical principles that govern the chains of reasoning. Frege identifies the calculation operation with the logical derivation, “to calculate would be to deduce.”<sup>239</sup>

#### Pre-eminence of the analysis over synthesis

According to logicism, all the statements of arithmetic are analytical since they can be derived from logical laws. The distinction between analytical and synthetic judgments must be understood in relation to the legitimacy of the judgment and not the problem of its genesis. The problem of its formation is not relevant to mathematics. Empirical psychology studies the genesis of judgment. For the problem of arithmetic statements, it must be considered the way in which they are legitimized and not how they are formed. The problem is the validity of the judgment and not its genesis in the empirical consciousness.<sup>240</sup>—The judgments of arithmetic will be considered analytical if its foundation can be obtained purely from logical laws. Arithmetic judgment will be considered synthetic if to ground its legitimacy, it is necessary something outside the primitive laws of thinking. Frege explains in his *Foundations* that arithmetic truths are

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<sup>237</sup> Frege, G., 1979, p. 6.

<sup>238</sup> Imbert, C., 1972, p. 139

<sup>239</sup> Frege, G., 1884, p. 99.

<sup>240</sup> Cf. Frege, G., 1972, p. 26 ss. §3.

the result of a logical derivation and that, according to the definition he has provided, arithmetic judgments should be considered analytical and non-synthetic judgments.

The Kantian definition of analytical and synthetic must be reformulated, both for concepts and judgments. The Kantian confusion is based on an overly narrow definition of synthetic judgments. Thus, in the first place, as we mentioned, Kant would have confused the problem of genesis with the problem of justification. The Kantian distinction between analytical judgments and synthetic judgments would fall into the same error as a psychologism, in a confusion of the problem of genesis with the problem of validity.

Kant's second mistake is to define the concept as a sum of properties. This error originates in a prejudice inherited from Aristotelian logic. The definition of the concept as a sum of properties and the attributive conception of judgment derives from this fundamental error: taking elements of natural language to express formal language. Kant's mistake when considering arithmetic judgments as synthetic judgments is based on this incorrect definition of the notion of concept. The understanding of the concept as abstraction of common marks led Kant to consider that arithmetic judgments are non-analytical judgments. The definition of concepts as the sum of properties and the definition of judgments as the attribution of a predicate to a subject must be rejected. These definitions are fruitful for an understanding of natural language, but they do not express the way in which pure thinking operates. The concept must be understood as a function. The function defines the extent of its content, and the content is limited to what is expressed in the function<sup>241</sup>. The rejection of the synthetic character of the judgments of arithmetic is based, first, on the rejection that knowledge requires at any point to resort to intuition. For Frege, the judgments of arithmetic are built on the basis of pure thought. Second, Kant inherits the prejudices of Aristotelian logic in his definition of the notion of concept. This conception of concepts explains the process of formation of concepts following the model of Aristotle metaphysics, based on the relation of substance and accident. Frege considers that it is necessary to reformulate the notion of concept. Third, Kant distinguishes the analytic from the synthetic by confusing the problem of genesis with the problem of validity. It is necessary a redefinition of what is understood by the distinction between analytical and synthetic. On this basis, Frege states that the analysis can be amplifying<sup>242</sup>. Frege considers that analytical judgments allow us to increase our

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<sup>241</sup> For Imbert, "the core of Frege's work is the identification of the concept and function." Imbert, C., 1972, p.208n.

<sup>242</sup> Frege and Couturat argue in the same direction. Cf. Natorp, P., LGEW, p.19.

knowledge in some way. Through these judgments, certain aspects are revealed that, although contained in other concepts, had not been put in evidence. According to the Kantian conception, judgments of this kind should be considered synthetic. For Frege, the legitimation of the judgments of arithmetic can be obtained purely from logical laws. While they can be derived purely from logical laws, these judgments should be considered analytical. However, new elements are extracted in the conclusions of the arithmetic reasoning that were not contained in the previous laws. In this sense, the judgments of arithmetic are analytical and amplifying. That is, analytical judgments allow us to increase our knowledge. The demonstration of an arithmetic judgment may require a variety of definitions. The grounded judgment was contained in the definitions but required a peculiar act so that its legitimacy could be revealed<sup>243</sup>. The application of the laws of logic allows us in this sense to expand our knowledge without implying that its propositions are synthetic<sup>244</sup>. The laws of logic have an intrinsic foundation. A logical law, as we have already observed, can only be grounded if it can be legitimized by another logical law<sup>245</sup>. The application of a law is valid if this law can be reduced to another subsequent logical law. The propositions of logic are analytical. To prove that the laws of arithmetic are all analytical, it must be shown how they are deduced from the laws of logic. This task would require demonstrating that every arithmetic statement can be deduced from a logical law<sup>246</sup>.

In summary, Frege's rejection of the synthetic character of arithmetic is based, first, on a redefinition of the concepts of analysis and synthesis. Second, for Frege, arithmetic judgments can be amplifying without any reference to intuition. Third, Frege reformulates the notion of concept. The concept should not be understood as a sum of

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<sup>243</sup> Several mathematicians of the time share the conviction that it is necessary to separate the geometry of arithmetic at this point. Cf. Detlefsen, M., 2004, p.54.

<sup>244</sup> For Michael Detlefsen, Frege fails to prove that mathematical inferences can be analytical and "epistemically productive." Detlefsen observes: "Frege's conception of mathematical inference was thus faced with two apparently competing demands: on the one hand, the need to endow analytic judgments with tacit content so as to enable analytic inference to be epistemically productive; and, on the other, the need to restrict the mechanisms producing tacit content in such a way as to guarantee that synthetic content can never be tacitly contained in what passes for analytic content. In the end, I believe, he failed to meet these two demands adequately. He did not succeed in providing a set of basic laws and a criterion of tacit content the pair of which were guaranteed to permit only the production of analytic truths as tacit contents of the basic laws. Nor did he manage to ensure that the epistemic productivity sustainable by means of his mechanisms of tacit content production are capable of matching those which may be observed to hold in arithmetic." Detlefsen, M., 2004, p. 64.

<sup>245</sup> „Die Frage nun, warum und mit welchem Rechte wir ein logisches Gesetz als wahr anerkennen, kann die Logik nur dadurch beantworten, dass sie es auf andere logische Gesetze zurückführt.“ Frege, G., 1893, p. XVII.

<sup>246</sup> Cf. Frege, G., 1884, pp.101 ss.

properties of things. The Fregean conception is based on a redefinition of what is understood by the distinction between analytical and synthetic. Positively, the judgments of arithmetic are analytical because their propositions can be justified from the laws of logic.

### Application of mathematics

As we mentioned, the laws of arithmetic are based on pure thinking. They have no reference to any intuition, neither pure nor empirical. Therefore, the arithmetic applied to intuition loses its distinctive feature and incorporates a foreign element to it. The laws of number apply to objects of thought. Arithmetic laws determine the relations of pure thought as opposed to the natural laws that regulate the order of empirical phenomena. Natural law is the term that mediates between arithmetic and its application to phenomena. The laws of arithmetic may govern the domain of natural law. Therefore, it can be said that the laws of numbers are laws of laws.<sup>247</sup> The arithmetic law, subject to the logical law, governs the domain of concepts. Arithmetic can rule in the order of intuition only because it regulates the judgments of physics, which establish the laws of the natural world. Arithmetic is based on pure concepts. The laws of logic are necessary and sufficient conditions for the legitimation of the judgments of arithmetic.

For Frege, the laws of arithmetic differ from the laws of geometry. The distinction between the mode of legitimization of geometry and arithmetic is one of the features of Frege's logicism<sup>248</sup>. Frege's position on geometry is much closer to Kant's conception. According to Natorp, this is the breaking point of logicism in two directions. Dedekind and Cantor follow Frege as they appeal to intuition. They consider that intuition is necessary to ground geometry. Russell and Couturat oppose this idea and propose to ground mathematics in general on pure thought. As we will see later, Russell and Couturat are closer to Natorp than to Frege at this point<sup>249</sup>. In relation to its origin, the laws of

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<sup>247</sup> Cf. Frege, G., 1884, p. 99.

<sup>248</sup> Cf. Detlefsen, M., 2004, p. 64.

<sup>249</sup> Natorp holds: „Unter den Mathematikern etwa seit Kants Zeit findet man denselben Zwiespalt der Ansichten: eine ältere, deutlich von Kant beeinflusste Richtung, die aber nur noch wenig Anhänger zu zählen scheint, hält an einem Sonderanteil der Anschauung neben dem reinen Denken bei der Begründung der Mathematik, wenn nicht der ganzen, dann doch der Geometrie, noch immer fest; gerade die vorwärts strebenden aber, an der Spitze Frege, Dedekind, Cantor und schon früher Graßmann, im Ausland, um nur die jüngsten und eifrigsten zu nennen, Russell und Couturat, verwerfen diesen Dualismus ganz und arbeiten mit Anstrengung daran, den Bau der Mathematik rein auf logischem Fundament zu errichten.“ Natorp. P., LGEW, p. 3.

number are a pure product of thought. Arithmetic originates only in thinking and does not require any reference to intuition. On the contrary, geometry is based on thinking but needs intuition for the construction of its objects. The geometric law is not just an extension of the logical law, as in the case of arithmetic. Therefore, the judgments of geometry are synthetic, because to legitimize their statements the laws of geometry cannot be based exclusively on the logical law as in the case of arithmetic. Arithmetic, as we mentioned, can always be justified by resorting to the logical law. Therefore, all the arithmetical judgments are analytical. Arithmetic, as it is a pure construction of thought, is identified with logic. The laws of number apply to the entire field of thought. Arithmetic, unlike geometry, regulates everything conceivable and not only the objects of possible experience. The universality of the arithmetic law is based on the universality of the logical law. The scope of the laws of arithmetic is universal. Arithmetic, as an extension of logic, valid for every object of thinking. Conversely, the whole field of thinking is governed by the laws of number. The whole field of thought is countable, and it is subject to the laws of arithmetic. Geometry is applied to what can be intuited spatially, which does not cover everything conceivable. The laws of geometry do not have the universality of the laws of arithmetic.

In sum, we could identify the following main features of logicism. In this characterization, we have attended to the central points on which Natorp focuses. From the above, we can highlight Natorp shares with logicism the following thesis: Logicism rejects any attempt to ground logic on intuition, be it pure or empirical. The laws of logic owe nothing to any intuition. The domain of logic is purely conceptual. The laws of thought are not founded on intuitions, neither empirical nor pure. Logic is the science of thought. Concepts are the pure functions of thinking. Arithmetic is grounded on logic. Then, arithmetic has a foundation in pure concepts of thought. Arithmetic, as long as it has a foundation in logic, is legitimized in the laws of thinking. Some logicists, such as Frege, consider that there is a necessary reference to intuition in geometry. The logicists agree that the conceptual foundation is a necessary and sufficient condition for arithmetic. In the case of geometry, some consider it a necessary but not sufficient condition. Logic is the science of valid inferences. Thus defined, the logic must generate a symbolic system and define calculation rules that allow operating with these symbols. Logic would be the discipline that determines the correct formation of symbols and defines the legitimate modes of relationships. The verification of the validity of mathematical reasoning is a logical problem, a matter of calculation. The validity of mathematical reasoning is based

on its subjection to the laws of logic. A judgment is admitted if it is derived from a valid inference. For this reason, the rules of calculation can be reduced to rules of inference. The mathematical calculation is legitimized in the logical deduction. The Kantian distinction between analytical judgments and synthetic judgments must be reformulated. Analytical judgments are those that do not require anything external to thinking for its legitimization. The problem is in relation to validity and not in relation to the genesis of knowledge. The laws of logic are analytical as long as they have immanent legitimacy. The laws of arithmetic, while they can be derived from logical laws, are analytical as well. The judgments of arithmetic are analytical because their foundation can be obtained purely from logical laws. Kant confused the distinction because he inherited the prejudices of Aristotelian realism. Kant confused the problem of genesis with the problem of validity. The Kantian error is based on a realistic definition of the notion of concept. The concepts of logic are creative and not the results of an abstraction process, as in natural language. There is no necessary reference to intuition so that the judgment can be amplifying. Thought is amplified by virtue of its own creations without reference to intuition. Therefore, the judgments of arithmetic are analytical and amplifying. There is an identification between logic and arithmetic. Arithmetic is “a more widely developed logic, and each arithmetic statement would be a logical law, although a derived law,” “calculate would be to deduce.”<sup>250</sup> The laws of arithmetic can only be applied to the phenomena of experience only mediately. Arithmetic governs the natural world by regulating the laws of the science of nature. The universality of the law is applied only indirectly to objects of knowledge. The laws of logic apply to objects indirectly. The objects of nature are, in relation to the laws of logic - and, consequently, in relation to the laws of arithmetic-given. For some representatives of logicism, the laws of geometry, unlike those of arithmetic, have an extrinsic foundation. The construction of the geometric object requires appealing to the intuition of space. Therefore, the judgments of geometry are synthetic.

### 2.2.3. Points in Common with Logicism.

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<sup>250</sup> Frege, G., 1984, p. 99.



Natorp's first common point with logicism is the rejection of mathematical psychology<sup>251</sup>. Natorp shares with the logicists the conviction that it is necessary to separate the problem of genesis from the problem of validity. The laws of thinking, the object of study of logic, must be separated from the laws that govern subjective consciousness<sup>252</sup>. Logic must be independent of any other science, including psychology. Logic and psychology are sciences with different objects of study. Logic can be defined as the science of thinking. However, this conception can lead to a confusion between the legality of pure thinking and the laws that govern the psychic life of the individual. Logic is not an empirical science. In his arguments against psychologism, as we exhibited, Natorp showed the need to distinguish between the mental fact and the principles of cognition, not determined by empirical legality; that is, by natural causality<sup>253</sup>. The logical laws do not determine a temporary event. In the same way, the sciences that are based on logic, such as mathematics, are completely independent of the mental processes that the subjects carry out. Therefore, the legitimation of the statements of mathematics cannot be obtained from a study of the genesis of representation. The recognition of the validity of a statement of mathematics does not require the possibility of representing this truth as a mental content. The problem of the validity of mathematical judgments cannot be solved through an analysis of the formation of representation in the subject. Frege and Natorp argue in the same direction.<sup>254</sup> The logical law, on which the legality of mathematics is based, is not a fact. The law is precisely what opposes the subjective point of view. The law grounds the possibility of objectivity<sup>255</sup>. Natural law regulates events. The logical law is the expression of the relations of thinking. Logic, as a universal science, cannot depend on a particular science, i.e., psychology. The method of logic cannot follow the same

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<sup>251</sup> Cf. González Porta, M., 2006, p.166. As Gonzales Porta rightly observes, there are two currents of anti-psychologism, that of Neo-Kantianism and the position of Frege and Husserl, which he calls "logical realism". González Porta explica: "existen dos variantes diferenciables en la crítica al psicologismo, una, la neokantiana, y otra, la representada por Frege y Husserl, y a la cual en el actual contexto me referiré como "realismo lógico"". González Porta, M. A., 2021, p. 166.

<sup>252</sup> Natorp, NTE, p. 343.

<sup>253</sup> Natorp, P., L, p.10.

<sup>254</sup> Cf. Frege, G., 1979, p.2. Natorp, P., SP, p. 18.

<sup>255</sup> „ein Gesetz besagt überhaupt ein allgemeines Stattfinden; oder allgemeinen Bestand einer Relation. Darin muss aber nichts von Zeitbedingungen enthalten sein, d. h. es gibt Gesetze, die nicht Zeitgesetze eines Geschehens sind. Und zwar sind diese was von Zeitbedingungen unabhängigen Gesetze, nämlich die der Logik und Mathematik, fundamentaler als alle Gesetze, welche Zeitbedingungen einschließen, denn Zeitbestimmung setzt selbst erstens die allgemeinen Gesetze der Bestimmung, d. h. die logischen Grundgesetze, und zweitens Größenbestimmung (Zählung und Messung), mithin die Gesetze der Mathematik voraus.“ Natorp, P., L, p.10.

delineations as the psychological method because the problem of logic is the relations of the contents of thinking and not the genesis of the mental life<sup>256</sup>.

The conditions for the possibility of a judgment are not conditions for the formation of the representation. Frege and Natorp differ with respect to the solution on how to establish the validity of the judgment. However, both agree on the need to make this distinction between the conditions that allow the formation of a representation and the conditions for the possibility of judgment. Both also claim that the introduction of something given to intuition comes along with the standpoint of psychologism. The genesis of the representations requires an element given to an intuitive faculty. The account of the formation of the representation demands that something is given. Both agree that this problem of the genesis has nothing to do with the logical problem, the problem of the theory of knowledge. The latter do not deal with the problem of empirical origin. Frege and Natorp also agree on the need to ground mathematics in logic. Then, the validity of the mathematical judgments depends on the way in which this foundation is carried out. This foundation cannot be established by the analysis of a study of the mind.

#### The necessity of a logical foundation

The second point in common with the logicist consists in the admission of a logical foundation of mathematics. Both for Natorp and for logicism, logic is the fundamental science of thinking<sup>257</sup>. Logic as a universal science of knowledge must provide the foundations to the remaining sciences<sup>258</sup>. Natorp recognizes the merit of logicism at this point. Logicism has correctly undertaken the task of a purely logical foundation of the exact sciences<sup>259</sup>.

Logic will be the fundamental science of thinking, and it will exhibit the laws that regulate it. Each particular science rules over a certain field of objects, the logic is universal. The whole field of thinking is regulated by the laws of logic. Neo-Kantianism

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<sup>256</sup> „Die Methode der Logik is also weder kausal (psychologisch oder biologisch) noch teleologisch, sondern im gleichen Sinne rein objektiv wie die der Mathematik.“ Natorp, P., L, p. 11.

<sup>257</sup> For Éric Dufour, the conception of logic as the center of philosophy is a common element of German philosophy of the 19th century. The discrepancy is generated in relation to what each current means by logic. Dufour, E., 2010, p.20.

<sup>258</sup> Pulkinm Jarmo: “The neo-Kantians, too, supported the idea that mathematics should be based on a logical foundation. However, their conception of the logical foundation differs greatly from that of Russell and Frege”. Pulkinm J., 1986. p.20

<sup>259</sup> Natorp, P., KMS, p.196.

in general converges with logicism at this point. The sciences in general - and, in particular the exact sciences - must be legitimized from their foundation in the laws of logic. Therefore, the logical foundation allows the scope of thought to remain as a unified whole. The foundation of science in logic prevents the separation of thought in heterogeneous domains irreconcilable to each other. Each particular area may have special laws that regulate it. However, all of them will be subject to the legality of thought.<sup>260</sup> Natorp considers that the fundamental law of consciousness demands the unity of all manifold in thinking<sup>261</sup>. Logic is the unifying science of thinking. Logic, as an expression of the legality of thinking, exhibits the action of thinking in its unity. The logic is then, the fundamental science, which provides the general laws for all particular cognitions.

Mathematics, as a special science, also requires a foundation. This foundation will be carried out by that science that studies the laws of thinking in general: logic. The science of nature derives its foundation from mathematics and mathematics is grounded on logic. The definition of the concept of thinking as a whole allows this unification and, consequently, logic as a science of the expression of the laws of this unifying unity. Therefore, both the laws of mathematics and those of natural science are regulated by the laws of logic; that is to say, they are grounded on the logical law.

Natorp agrees with logicism and with Frege in particular, on the necessity of a logical foundation of the exact sciences. Both authors argue in the same direction. Frege and Natorp propose a logical foundation of mathematics<sup>262</sup>. Logic expresses the laws of thinking and with it, the laws that regulate thinking in general. For both, logic is the science of thought. The discrepancy consists in the way in which this foundation is carried out and what is understood by logic in each case.

#### Need to reformulate the concepts of analysis and synthesis

Both Natorp and Frege, and other representatives of logicism, share the conviction that it is necessary to reformulate Kantian concepts of analysis and synthesis. In particular, the

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<sup>260</sup> „Das Denken ist im Wesentlichen überall dasselbe: es kommen nicht je nach dem Gegenstände verschiedene Arten von Denkgesetzen in Betracht. Die Unterschiede bestehen nur in der grösseren oder geringeren Reinheit und Unabhängigkeit von psychologischen Einflüssen.“ Frege, G., 1884, p. iii.

<sup>261</sup> „Durch das Grundgesetz des Bewußtseins ist Einheit alles Mannigfaltigen oder Gesetzlichkeit bedingungslos gefordert.“ Natorp, P., PS, p. 34.

<sup>262</sup> Natorp, P., LGEW. p.1.

analytical and synthetic nature of the judgments must be redefined. Frege and Natorp consider that Kant was not deep enough in this distinction. This is the third point of convergence of Natorp with logicism. As Frege did, Natorp also argued that the distinction between analytical and synthetic judgment must be reformulated.

The judgment coordinates two concepts, establishing a peculiar relation among them. In the affirmative judgment, “S is P”, S is the subject and P is the predicate. The judgment is analytical if the predicate is contained in the subject. The judgment is synthetic if the predicate introduces an element that is not contained in the subject. Kant considered that the judgments of mathematics are synthetic because they require a reference to intuition. Thought by itself can construct neither the object of arithmetic (the number) nor the object of geometry, objects in space. Arithmetic judgments require a reference to intuition in determining the number in time. A priori synthetic judgments of arithmetic, such as “ $5 + 7 = 12$ ”, require the pure intuition of time. A priori synthetic judgments of geometry, such as “the line is the shortest distance between two points,” require the pure intuition of space.

Natorp, as Frege, is critical of the way in which Kant understood the distinction between analytical and synthetic and considers that this distinction must be reformulated. The Kantian distinction is based on a conception inherited from Aristotelian logic that conceives judgment as a relationship between subject and predicate. Frege and Natorp share the idea that judgment as an expression of the structure of thinking does not take the form of natural language. The grammatical form of judgment is insufficient to establish the relational character of thinking. Therefore, the logical study of judgment does not converge with grammatical analysis. The way in which thought establishes relationships overcomes the restricted mode of the expressions of natural language<sup>263</sup>. Natorp shares this idea with Frege and considers that the most proper expression of thinking is the function. It would be more precise to relate the action of thinking to functions than to judgments. The judgments are ways of bringing the manifold to a unity but the way in which the manifold is reunited in various modalities does not reflect the structure of the Aristotelian form of judgment. Rather, this type of judgement is possible based on an original way of establishing relationships whose most precise expression is the function. More precisely, the judgments should be considered as a function. Only then, judgments could be considered as an expression of the action of thinking. In this

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<sup>263</sup> Frege, G., 1979, p.6.

sense, judgment is the expression of the functional character of thinking<sup>264</sup>. Frege would fully share this idea. Frege states:

Kant scheint den Begriff durch beigeordnete Merkmale bestimmt zu denken; das ist aber eine der am wenigsten fruchtbaren Begriffsbildungen. Wenn man die oben gegebenen Definitionen überblickt, so wird man kaum eine von der Art finden. Dasselbe gilt auch von den wirklich fruchtbaren Definitionen in der Mathematik z. B. der Stetigkeit einer Function. Wir haben da nicht eine Reihe beigeordneter Merkmale, sondern eine innigere, ich möchte sagen organischere Verbindung der Bestimmungen.<sup>265</sup>

Natorp, in this same direction, maintains:

Also aus keinen voraus gegebenen, gleichsam feststehenden Denkpunkten und mit diesen zugleich gegebenen, ebenso festen Lagen solcher Punkte gegeneinander, sondern aus dem Quell einer unerschöpflichen Denkbewegung, aus dem Quell der Methode allein kann das synthetische Urteil, das eigentliche Urteil überhaupt als synthetisches, sich erzeugen. Allerdings stumpft Kant selbst die Schärfe dieser radikal idealistischen Einsicht wieder ab, wenn er den Urakt der Synthesis beschreibt als die „Handlung, verschiedene Vorstellungen zueinander hinzuzutun“ und „ihr Mannigfaltiges“ zu einer Erkenntnis zu begreifen. Danach scheinen die letzten Elemente, in der fragwürdigen Gestalt von „Vorstellungen“, doch wieder vor der Erkenntnis, selbst vor dem Urakt des Erkennens, dem Akte der Synthesis, voraus gegeben sein zu sollen.<sup>266</sup>

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<sup>264</sup> „Zwar folgeweise läßt jedes von diesen sich auch in Form eines Urteils aussprechen, aber nur hinterher; primär ist von Faktoren, oder besser noch (mit Kant von Funktionen (Einzelleistungen) des Urteils zu reden.“ Natorp. P., LGEW, p. 28. We will study later Natorp's definition of concept and judgment.

<sup>265</sup> Frege, G., 1884; §88.

<sup>266</sup> And he follows: „Aber hier ist nun Kant sehr leicht aus seinen eigenen Voraussetzungen zu korrigieren. Man braucht nur zu fragen: sollen diese Elemente vor dem Grundakt der Synthesis voraus einen „gewissen Inhalt“ schon haben oder nicht? Aber die Synthesis soll ja vielmehr das sein, was sie „zu einem gewissen Inhalte erst vereinigt. Also waren sie vordem — Vorstellungen zwar, aber ohne gewissen Inhalt? Vorstellungen, in denen — nichts Bestimmtes vorgestellt war? In der Tat darin liegfs: nichts Bestimmtes. Die Bestimmtheit des „Was“, das ist genau, was der Urakt der Erkenntnis als Akt des Bestimmens erst zu erbringen hat.“ Natorp. P., LGEW, p. 46.

As we shall see later in detail in chapter 4, Natorp agrees with Kant that the mathematical judgments are synthetic. However, he rejects the idea that the synthetic nature of the mathematical judgment is related to some need to appeal to intuition. For Natorp, the judgment of mathematics is synthetic, but it is, in turn, a pure product of thinking without any reference to intuition. Synthesis is the expression of the possibility of thinking progression. Natorp's definition, as we will see later in detail, diverges from Frege's account. However, both authors agree on the need to reformulate the distinction established by Kant.

In short, Natorp shares with Frege the need for a logical foundation of the exact sciences. Exact sciences require a logical function, and in this foundation, there should be no reference to intuition. The principles of mathematics are based on logic, and logic is a purely conceptual science that owes nothing to pure or empirical intuition. Mathematics is based on logic and then, on pure concepts. For Frege, arithmetic has a mediate application. Arithmetic governs the natural world by regulating the laws of the science of nature. Natorp also shares with Frege that it is necessary to develop a better understanding of the analytical and synthetic nature of the judgments. For both, it is necessary to reformulate the notion of concept, rooted in the ancient Aristotelian logic. The realistic conception of the notion of concept must be replaced by an idealistic notion. Likewise, they converge on the idea that an extension of knowledge can occur without any reference to intuition. There is no necessary reference to intuition so that the judgment contributes knowledge and can thus be amplifying.

#### 2.2.4. Natorp's Criticism of Logicism.

Frege and Natorp share the idea that it is necessary to establish a new logical foundation of mathematics. Both thinkers also believe that knowledge requires a new logical legitimation, which is not purely speculative or psychological. However, they disagree on the way in which this task should be carried out. Natorp considers that formal logic is insufficient to ground the procedure of mathematics. Formal logic is insufficient to legitimize both the truths of mathematics and those of natural science. The purpose of the logical groundings of the exact sciences is, as the proposal of logicism, to ground mathematics as *a priori* science. However, this foundation will be carried out in a very

different sense from that defended by authors such as Russell and Frege<sup>267</sup>. Natorp thinks that the task of a genuine logical foundation of the exact sciences had not been developed yet<sup>268</sup>. Contemporary logicism does not overcome the ancient conception of logic. Modern logistics continues the tradition of Aristotle. The reformulation of the sense of 'the logical' is necessary because this concept was particularly misunderstood by the logicians of that time. Logicism did not go much further than Aristotle at this point. They have not understood the fruitfulness of the logical. The purpose of logic is to make understandable the construction of the object of knowledge. Logic cannot start from that object as a mere fact. The logic thus conceived is a productive science. One cannot start the logical investigation under the unjustified assumption of the separation of knowledge, as if we had the act of knowing on the one hand, and the object of knowledge, on the other. A foundation of the exact sciences demands a logic that shows how the objects of these sciences are constructed in and by thinking, thus exhibiting the inseparable relationship between thinking and object. A genuine logical foundation requires a transcendental logic, which studies the unfolding of the process of thinking in the production of its object of cognition. Formal logic is insufficient to provide this foundation. A transcendental logic is required to show the legitimacy of knowledge in the explanation of its conditions of possibility. These conditions will make possible the foundation of knowledge, which is expressed in the physical-mathematical sciences. Natorp rejects that formal logic can be the ground of science. The conception of the mathematical foundation in formal logic implies an identification between logic and mathematics. Mathematics is based on the logic for the legitimization of its procedure. However, the logic to be the foundation of mathematics operates according to the laws of calculation. Logic is turned into a branch of mathematics. There is no proper foundation. Mathematics follows the methods of logic, but logic has a mathematical formulation. In conclusion, there would be no real difference between logic and mathematics. Rather, there would be an identification between the two sciences<sup>269</sup>.

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<sup>267</sup> „Auch das gegenwärtige Buch unternimmt eine rein logische Begründung und behauptet damit den Apriori-Charakter der Mathematik, aber in einem anderen Sinne als die Vorgenannten“. Natorp. P., LGEW, p.3.

<sup>268</sup> „Einer solchen Vorbereitung bedarf es, weil schon die Aufgabe selbst, so wie sie hierverstanden wird, nicht allgemein als solche anerkannt ist.“ Natorp. P., LGEW, p.2.

<sup>269</sup> „Conturat (31, S. 230), der vielleicht am weitesten nach dieser Seite geht, äußert sich darüber immerhin zögernd. Auf der einen Seite sieht er in der Mathematik einen Teil der Logik: sie sei ganz logisch der Form nach, aber beschreibe in ihrem Inhalt nur einen Teil des Umfangs der Logik; auf der anderen Seite will er die Logik rein rechnerisch gestalten, macht sie also unleugbar zu einem Zweig der Mathematik. Wäre es dann nicht folgerichtig, die Verschiedenheit von Logik und Mathematik überhaupt zu verneinen? Denn

## Vicious circularity

The procedure of logicism is circular. Logicians claim that the legitimacy of mathematical propositions can be obtained from their deduction of laws of logic. However, logic itself is a science that operates by deductions. In fact, the justification of a logical law is made, according to logicism, exhibiting its derivation from another logical law. Then, logic as a deductive science must provide the basics of deduction, but the legitimization of its laws can only be done by deductive means. There is a circularity in the foundation. The logicians, on the one hand, want logic to be a purely deductive and calculative science and, on the other, that it is capable of legitimizing that calculation procedure only on its own. Thus conceived, logic can neither ground the mathematical procedure nor ground itself. According to Natorp's characterization, as we observed in the last section, the procedure of logicism would be the following: definitions of the symbols that will be admitted into the system are formulated, the rules that express the way in which it will be legitimate to connect these symbols are defined and, hence, a mechanical process is performed<sup>270</sup>. The only restriction to the way of linking symbols is the principle of non-contradiction. A genuine understanding of this process is not only not necessary, but the introduction of elements outside logic can disrupt the procedure that is purely calculative. The clarification of the meaning of these symbols is not necessary at all to carry out the derivation. Formal logic does not provide the ultimate foundation that makes knowledge possible. The logic thus defined cannot provide a real understanding of the process of knowledge. For the conception of logicism, the logical analysis "is limited to a mere composition of arbitrary symbols carried out with equally arbitrary rules."<sup>271</sup> For Natorp, unlike logicism, the question of the groundings is unavoidable, and formal logic is not enough to provide such a foundation. It is necessary to introduce a more fundamental science than formal logic that exhibits the principles of the process of knowledge. Indeed,

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wenn zugleich A in B und B in A ganz enthalten ist, so sind nach einem bekannten Satze der rechnerischen Logik beide notwendig identisch." Natorp, P., LGEW, p.5.

<sup>270</sup> For Philip Jourdain, Natorp pays little attention to the work of mathematical logicians he criticizes severely. In particular, he tries to show that Natorp misunderstood the procedure of Couturat. Jourdain points out: "But mathematical logicians do not think that they can justify the principles of logic deductively, and do not, of course, attempt the task of beginning with definitions. They begin with primitive ideas as such primitive proposition as are necessary to make deductions". Philip. J., 1911, p. 554. J. J. Maxwell, in his note to this review, tries to refute Jourdain's critique by showing that Natorp did not misunderstand the logical symbolists but, rather, Philip Jourdain misunderstood Natorp. Maxwell, J., 1912.

<sup>271</sup> Natorp, P., ZLGNM, p.182.



the analysis of the scope of the logical derivation is necessary, but the task of a genuine understanding of the procedure, which can only be found in the study of the logical foundations, cannot be skipped. Recognizing these grounding is the task of philosophy<sup>272</sup>. Natorp rejects taking these principles as if they were simply given, and they required no further explanation. This type of circularity, unlike the circularity of the critical method, is vicious. It does not constitute an elucidation or an explanatory basis. The impossibility of finding these last principles means that logical knowledge has a restricted domain. The logical principles hold a purely relative validity as long as they are always valid “in relation to a certain system of definitions and a certain sequence of demonstrations, never absolutely.”<sup>273</sup> The validity of the laws of logic becomes relative. In this conception, the universality that logic must have as a grounding and objective science is lost.

The error of this conception has its origin in the uncritical acceptance of the prejudices inherited from Aristotelian logic. Logicians consider that the fundamental principles are found in the immediate evidence or that they are simply given to the understanding. Although modern logic is enriched, like classical (i.e., Aristotelian) logic, it simply declares its principles and concepts. The simple declaration of principles is a sufficient condition to accept them. Ultimately, the error of logicism is the same as that of psychologism, which is grounded on naive realism. They both rely on the acceptance of data as given to thinking. The only necessary task is analysis. Logicism is thus closely related to naive realism, also of Aristotelian roots. The error of the dogmatism of Aristotelian logic is to depart from certain assumed and unjustified definitions, as if they were simply declared, given to thinking. This is the fundamental error of naive realism that takes things as given to intuition and considers that the task of thought is reduced to operating on the given content. Therefore, for this conception, logical development can take place purely by means of analytical judgments, which are limited to expressing

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<sup>272</sup> „Die Konsequenz dieses Bestrebens muss dahin führen, dass man nicht zufrieden ist, in der Mathematik überhaupt, wie in jeder Wissenschaft, logisch zu verfahren, d. h. Widerspruch zu meiden und, was man behauptet, zu beweisen, sondern dass man sich die weitergehende Aufgabe stellt, auch als Voraussetzung nichts zuzulassen, was irgend noch aus fundamentalen Voraussetzungen herleitbar, also noch nicht schlechthin einfach ist. Die Frage aber nach den letzten Voraussetzungen einer so fundamentalen Wissenschaft, wie die Mathematik, führt unmittelbar in das Herz der Philosophie als Erkenntniskritik.“ (...) „Aber, neben der Aufgabe der Entwicklung der Konsequenzen aus gegebenen Voraussetzungen besteht jedenfalls noch die andere, des Zurückgehens auf die letzten erreichbaren Grundlagen.“ Natorp, P., EGM, p.2. Also. Natorp, P., ZLGNM, p.182.

<sup>273</sup> The position that the fundamental principles have a merely relative validity is held, according to Natorp, by Couturat. Natorp affirms: „Ausdrücklich sollen (nach Couturat, S. 39) die Grundbegriffe und Grundsätze als solche (d. h. undefinierbar und unbeweisbar) allein gelten allemal in bezug auf ein bestimmtes System von Definitionen und eine bestimmte Beweisfolge; nie schlechthin.“ Natorp, P., LGEW, p. 8.

implicit contents in the data, be it intuitive data, as in the case of psychologism, or abstract data, as in the case of logicism. For both, naive realism and logicism, understanding only operates by analysis of the given contents. However, the analysis could not provide clarification of its fundamental principles. In sum, formal logic is insufficient both to ground itself and to give a foundation to mathematics. This kind of grounding is circular and does not constitute an explanatory basis. This conception ignores the essentially productive character of thinking, to which the object is not given as data but produced. Logicism confronts the universality of the concept to the concretion of the object instead of showing how the object is a construction of the conceptual procedure of thinking. In the next chapter, we will exhibit how this is accomplished by transcendental logic.

### Preeminence of the synthesis over analysis

This analytical perspective of the task of knowledge must be revised. Thought, as we will see in detail later, is essentially productive. The preeminence given to the analysis starts from the assumption that there are certain data given to thinking as a piece of information that must be decomposed. For Natorp, as for Kant, nothing can be decomposed unless it was previously gathered<sup>274</sup>. The spontaneity of thinking consists in the ability to produce its own object, without the reference to anything outside of itself. The conclusion of the Transcendental Analytic of the Kantian *Critique* is that the understanding is “the generative and regulatory source of nature (that is, of the nature of natural sciences) and not only its interpreter”<sup>275</sup>. Even if logicism does not make reference to a perceptual

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<sup>274</sup> Natorp, P., LG, p. 9.

<sup>275</sup> Natorp, P., UOSB, p. 140. Cohen’s position is grounded on the same thesis. As Hernan Pringe summarize: “In his *Logic of Pure Knowledge*, Hermann Cohen aims to carry out the Copernican turn which, according to him, Kant fails to achieve. On Cohen’s reading, if objects must conform to our cognition because knowledge produces the object (Cohen 1907, 4), then this cannot just amount to the determination of the mere form of objectivity in general. On the contrary, the spontaneity of thought must also generate the matter of cognition.” For this reason and in opposition to Kant, Cohen claims that in cognition thought does not face any given matter, not even a pure one (Cohen 1922, 26–27). What Kant calls given is nothing but a product: a product of thought. The Kantian distinction between thinking and cognizing an object (CPR B 146), which relies precisely on the consideration of intuition as a non-conceptual representation, is thus abandoned in favor of a doctrine of thought that is at the same time a doctrine of knowledge. Though, in Cohen’s sense, thought does not depend on any receptivity that would provide it with a sensible content. According to Cohen, only in this way can the Copernican turn that Kant prescribed to metaphysics be finally executed successfully.”, Pringe, H., 2020, p.137. In this sense, the Neo-Kantian method should be called a transcendental method. Christian Krijnen explains: „Das Transzendente ist der Sache nach ein Inbegriff von Geltungsgründen, der nicht durch den Rückgang auf ein Seiendes außerhalb der Erkenntnisrelation begriffen werden kann, sondern nur durch einen Rückgang auf das Denken selbst als Grund aller Geltung. Die objektive Gültigkeit konkreter Erkenntnisleistungen des Subjekts findet ihren Grund in einem Inbegriff von Geltungsprinzipien („Bedingungen der Möglichkeit“); die objektive Gültigkeit dieser

element, however, it considers the contents of thought as given. Therefore, for Frege, for example, the rules of analysis are sufficient to legitimize the process of thinking. The rules of the analysis, indeed, are useful to provide intelligibility to the synthetic process but are always grounded by it. According to Natorp, the analysis represents only the ‘reverse’ of the synthesis. The affirmation of the preeminence of the synthesis is introduced primarily as a rejection of the idea of a purely analytical foundation of knowledge. The fundamental problem of the attempt to establish an analytical foundation is the homologous nature of the analysis. The analytical foundation of knowledge cannot express the expansion of thinking content, it cannot exhibit the progression of thinking. The analytical foundation transforms knowledge into tautology under the expression: A is A. The synthetic foundation, on the other hand, is the expression of the heterology of thinking, which no longer expresses that A is A but that A is B<sup>276</sup>. Affirmatively, it is established that thinking is precisely the possibility of setting the differentiated elements. Formal logic, as it is analytical, does not aim to extend our knowledge as long as the principle of non-contradiction is valid only for its clarification and has no function for its extension<sup>277</sup>. According to Natorp, Frege’s arguments are not convincing. Even when analysis reveals new implicit content, it does not create anything new. The function of thinking remains tautological. This conception considers that mathematics, and therefore all sciences, is a closed sum of finished truths, which one day could be completely known. The analytical conception cannot ground the expansion of knowledge. For Natorp, when Frege claims that the analysis is amplificative, he is making reference to synthesis. Natorp explains:

Der Widerspruch kann aber unmöglich ein Prinzip der Fortschreitung sein, sondern allenfalls nur ein Prinzip der Auslese, wodurch sinnwidrig versuchte Fortschreitungen ausgeschaltet werden. Dessen bedürfte es gar nicht, wenn die Fortschreitung streng ihrem Gesetz gemäß geschähe. Der Widerspruch schafft also nichts, erhält auch nicht das Geschaffene. Auch vernichtet er nicht logisch Geschaffenes, sondern

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Geltungsprinzipien wird dadurch legitimiert, daß sie sich geltungsfunktional als Bedingungen der Erkenntnis (Ietzt-)begründen lassen.“ Krijnen, C., 2006, p. 288.

<sup>276</sup> „Also was ist Synthesis? Zunächst nur ein Ausdruck der Abwehr einer bloss analytischen Begründung der Erkenntnis. Der Fehler der Analysis ist, dass sie Erkenntnis bestenfalls in Tautologie verwandelt. Also scheint Synthesis vielmehr Heterologie bedeuten zu müssen: Nicht A is A sondern A ist B “. Natorp, P., LGEW, p. 11.

<sup>277</sup> Natorp, P., LGEW, p.20.

entlarvt nur den falschen Schein einer logischen Schöpfung, wo wirklich keine vollbracht ist; einen Schein, der beim logischen Schaffen als unlogisches Tun vielfach nebenher geht und sich mit- einschleicht. Der Satz des Widerspruchs ist also wirklich, wie Kant es aufgestellt hat, allenfalls ein Prinzip der Verdeutlichung, nicht aber der Erweiterung der Erkenntnis<sup>278</sup>.

Analytical judgments are based on the principle of non-contradiction. The principle of non-contradiction is useful only to verify the legitimacy of a logical creation, not to create it. This principle cannot legitimize the expansion of thinking. The principle of non-contradiction cannot generate the logical content itself. Then, the principle of non-contradiction, upon which the analytical conception of knowledge rests, cannot be the foundation of the progressive character of thinking. The analysis grounded on the principle of non-contradiction must be based on a more original act that allows logical creation and thus the expansion of thinking. Indeed, the principle of non-contradiction can be used to verify the creation of thought, but it cannot be the foundation of creation itself. Rather, it requires a creative act of thought in the first instance. This is the act of synthesis. Synthesis is the expression of this possibility of thinking to create its content and not only operate over given content. Synthesis is the purest expression of the spontaneity of thinking and the only act that can ensure its amplification.

Natorp argues that the conception of logicism of the logical form leads to a separation between intuitive and conceptual content. For this conception, the universal and the particular are separated. There is a gap between the law and its object. The form is conceived here as separate from its objects. Logicism does not recognize that “the general must in all cases be conceived only as general of the particular”<sup>279</sup>. The law as a unifying moment cannot be thought independently of the multiplicity it contains. The concept does not subsume the manifold as something alien to itself. The unification of the multiplicity is not generated departing from certain given data, neither empirical nor abstract. Taking the given as a starting point, logicism, does not overcome the dogmatic realism that, as we saw, is rooted in the Aristotelian tradition. Logicism does not overcome the dogmatism of the given because it takes the knowledge and the object as if they were two dissociated elements. First, it considers two separated elements and then

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<sup>278</sup> Natorp, P., LGEW, p.19.

<sup>279</sup> Natorp, P., ZLGNM, p.180.

tries to explain the relationships among them. The concept must be defined, as Frege correctly notes, in its functional character, as the unity of the multiplicity of instances. The cases, however, are determined a priori by the concept as their instances and not as something that exists independently of the concepts. Thought creates the case in concomitance with the position of the law. The object of knowledge does not exist independently of the act of thinking. The construction of the case in the law exhibits the way in which thought constructs its object and does not start from this object as a mere fact. The universal expressed in the law is productive. Logicism is as dogmatic as psychologism when it accepts the case as given to thinking. A foundation of the exact sciences requires displaying the way in which thought constructs its object. Logic must show the concomitant construction of the universal and of the particular. This is the only way in which the inseparable character between thinking and object can be shown. Logicism mistakenly conceives the relation between the universal and the particular and, therefore, affirms the preeminence of analysis.

In sum, Natorp shows the insufficiency of formal logic to be the grounding of knowledge. Formal logic takes the object of knowledge as if it were given and considers the analysis as the fundamental operation of thinking. For Natorp, knowledge demands a synthetic foundation that shows the construction of the object in and by thinking. Logicism correctly addresses the purely ideal foundation of legality. However, it does not take the creative nature of the concept to the last consequences. The amplifying nature of the judgments does not demand a reference to intuition. Judgment can be synthetic without reference to intuition. Frege evidenced the insufficiency of the principle of non-contradiction but he was not deep enough, he did not see the insufficiency of an analytical foundation of knowledge. All science must, indeed, proceed logically in the sense of avoiding contradictions. However, in addition to the task of developing the consequences from certain given conditions, the need of going back to the last achievable foundations persists. Formal logic must be grounded on the transcendental logic that exhibits the creative principles of thinking. For Frege, the logical law has an indirect application to the objects of knowledge. Arithmetic is based on logic and rules over the natural world by regulating the laws of the science of nature. The universal laws of logic are applied to objects indirectly. For Natorp, on the contrary, the foundation of knowledge requires an original act of synthesis that shows the creation of the object in thinking. There is no data to be decomposed but a creation of the object of knowledge. And according to the conception of Natorp, there is no indirect application of

the law to the object of knowledge. The object of knowledge is not given facing the universality of the law. On the contrary, it must be shown how the law in its universality builds the object. The synthesis is the expression of this possibility of thinking of creating content and not only operating with given contents.

## Conclusion

According to Natorp, psychologism and logicism were incapable of explaining the relation between concepts and intuitions. From an incorrect understanding of the problem of the method, psychologism and logicism are unable to explain the possible reference of thought to reality. For psychologism and logicism, the fact of experience always remains as an extrinsic moment to the legality of thinking. Psychologism confuses logic, the science of the laws of knowledge, with psychology, the science of laws that regulate the psychic life of individuals. Psychologism carries out a subjective foundation of knowledge. From this methodological error, psychology considers what is given to the perceptual intuition as the initial data of the investigation. A given data is conceived as the starting point in the formation of knowledge. The conception of psychology is based on the prejudices inherited from the Aristotelian realistic conception. The starting point from a given data would be the way to guarantee that thought can achieve objectivity. The object is conceived as fully determined, and thought must be able to display the determinations of the object. Psychologism starts from the unfounded assumption of an external element. Thinking would build its concepts departing from this first given *factum*. According to his position, the concepts are constructed from what is given to intuition. Against this current, Natorp argued that this definition of the notions of concepts and intuition must be reformulated. The concept cannot be conceived as a sum of marks that are abstracted from the factum given to sensation. Thought is spontaneous. This means that it is creative. Concepts must be conceived as functions. We will return to this point in the next chapter. Natorp exhibited that taking the data given to intuition as the starting point of the investigation, far from guaranteeing the reference of thought to reality, the core of the Kantian problem, cancels the concept of knowledge itself. The investigation must show how thinking creates the object. The logical foundation of knowledge has the task to show the creative power of thinking in the process of concept formation. The concept is not a sum of marks nor intuition is the element given to thought. The universality of the concept and the concreteness of the intuition do not

oppose. It must be shown how the universal is universal of a particular and the particular is only particular in relation to the universal. According to Natorp, logicism does not escape this misunderstanding. On the basis of a methodological error, logicism is unable to give an accurate account of the relation between intuition and concepts. Logicism grounds the possibility of knowledge on formal logic. The principles of cognition are conceived as given. For them, the main task of thought is analysis. The task of thinking is reduced to the analysis of the given contents. Faced with this primacy of the analytical moment of knowledge, Natorp points out the need for a synthetic foundation. As we shall see in Chapters 3 and 4, this foundation will be carried out exhibiting the general legality of knowledge, its internal law. The exposition of this procedure will display how thinking constructs objectivity. These fundamental procedures will be the categories, or, as Natorp will call them, the levels of thinking<sup>280</sup>. This exhibition will be the way to overcome the dualism between the intuitive and the conceptual representations. In Chapters 3 and 4, we will study how Natorp undertakes this task.

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<sup>280</sup> „Der Mathematiker, auch der logisch interessierte Mathematiker mag sich dabei beruhigen, solche letzten Prämissen zu „postulieren“; die Logik fordert für sie, als synthetische Sätze, wie Kant sagt, „wo nicht einen Beweis“ „Voraussetzungslose“, dh auf solche letzte Voraussetzungen, von denen es möglich ist, sich zu überzeugen, dass sie nicht wiederum andere, fundamentalere voraussetzen, nämlich auf die schlechthin fundamentalen Verfahrensweisen des „Dendenkens. gesetzmässigen Vorstellens der Gegenstände überhaupt, die sie in einer begrenzten Zahl reiner Grundfunktionen des Denkens (Kategorien) festzulegen sucht.“ Natorp, P., ZLGNM, p. 383.