

Paul Natorp's reformulation of the Kantian distinction between intuition and concept

Pelegrin, L.A.

Citation

Pelegrin, L. A. (2023, July 6). *Paul Natorp's reformulation of the Kantian distinction between intuition and concept.* Retrieved from https://hdl.handle.net/1887/3629812

Version: Publisher's Version

License: License agreement concerning inclusion of doctoral thesis in the

Institutional Repository of the University of Leiden

Downloaded from: https://hdl.handle.net/1887/3629812

Note: To cite this publication please use the final published version (if applicable).

Paul Natorp's Reformulation of the Kantian Distinction between Intuition and Concept Proefschrift

ter verkrijging van

de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof.dr.ir. H. Bijl,
volgens besluit van het college voor promoties
te verdedigen op donderdag 6 juli 2023
klokke 13.45 uur

door

Laura Alejandra Pelegrin geboren te Buenos Aires, Argentina op 18 juni 1985 Promotores:

Prof. dr. F.A.J. de Haas

Prof. dr. H. Pringe (Universidad de Buenos Aires-Universidad

Diego Portales)

Copromotor:

Dr. V.A. Gijsbers

Promotiecommissie: Prof. dr.Susanna Lindberg

Prof. dr. Lydia Patton (Virginia Tech)

Dr. Christian Krijnen (VU)

Dr. Marijana Vujos

Title: Paul Natorp's Reformulation of the Kantian Distinction between Intuition and Concept.

Author: Pelegrín, Laura Alejandra



Prof.dr. H. Pringe (Universidad de Buenos Aries-Universidad Diego Portales)

Dr. V.A. Gijsbers

This thesis was funded by the doctorate scholarship of the CONICYT (Chilean National Committee for Scientific and Technological Research). Throughout the years of my research, I have also received funding from Diego Portales University.

Contents

Acronyms. 6

Introduction, 8

Chapter 1. The Kantian Distinction between Intuitions and Concepts. 15

- 1.1. Introduction to the Distinction between Intuitions and Concepts. 16
- 1.2. Brief Sketch on the History of the Distinction between Intuitions and Concepts. 26
- 1.3. The Problem of Incongruent Counterparts. 37
- 1.4. The Reference to the Given as a Consequence of our Finitude. 46
- 1.4.1. Original Intuition and Derivative Intuition in the Fourth Observation to Transcendental Aesthetics. 46
- 1.4.2. The Contrast of *Intellectus Archetypus* and *Intellectus Ectypus* in Kant's Letter to Marcus Herz of February 21, 1772. 50

 Conclusion, 56

Chapter 2. Contemporary Approaches to the Problem of the Distinction between Intuition and Concepts. 59

- 2. 1. Against Psychologism. 61
- 2.1.2. Introduction to the *Psychologismus-Streit*. 63
- 2.1.3. Psychologism in Neo-Kantianism. 67
- 2.1.4. Natorp's Critique of Psychologism. 71
- 2.2. Against Logicism. 83
- 2.2.1. Introduction to the Debate. 84
- 2.2.3. Main Thesis of Logicism. 87
- 2.2.3. Points in Common with Logicism. 98
- 2.2.4. Natorp's Criticism of Logicism. 104

Conclusion, 112

Chapter 3. The Method Required to Overcome the Heterogeneity between Intuition and Concepts. 114

3.1. The Task of Philosophy. 115

- 3.2. The Necessary Starting Point of the Deduction of Categories. 121
- 3.3. The Elements of the Definition of Thought as Correlation. 135.
- 3.3.1. The Point of View of the Unity. 135
- 3.3.2. The Point of View of the Multiplicity. 137
- 3.3.3. The Point of View of the Relationship between Unity and Multiplicity. 140
- 3.3.4. The Preeminence of Correlation. 143
- 3.4. Concept and Judgment. 147

Conclusion. 152

Chapter 4. Overcoming the heterogeneity between Intuition and Concepts. The

Deduction of Categories. 154

- 4.1. Main Features of the Deduction. 155
- 4.2. The Levels of Quantity. 162
- 4.3. The Levels of Quality. 165
- 4.4. The Levels of Relation. 172
- 4.5. The Levels of Modality. 181

Conclusion. 190

Chapter 5. The Reformulation of the Kantian Distinction between Intuitions and Concepts. 191

- 5.1 The Reformulation of the Notion of Intuition. 191
- 5.2. The Reformulation of the Notion of Concept. 197

Conclusion. 210

Bibliography. 220

Acknowledgments. 230

Acronyms

Works by Natorp:

AM: "Aristoteles' Metaphysik K, 1–8, 1065a26", *Archiv für Geschichte der Philosophie*, 1888, 1: 178–193.

DE: Descartes'-Erkenntnistheorie. Eine Studie zur Vorgeschichte des Kriticismus, Elwert, Marburg, 1882.

DED: "Die Entwickelung Descartes' von den "Regeln" bis zu den "Meditationen"". *Archiv für Geschichte der Philosophie*, Vol. 10, Issue 1-4, 1897, pp. 10–28.

EGM: "Die erkenntisteoretischen Grundlagen der Mathematik" in: *Unterrichtsblätter Für Mathematik und Naturwissenschaften*, F. Pietzker, Verlag von Otto Sall, Berlin, 1902.

EP: Einleitung in die Psychologie nach kritischer Methode, Akademische Verlagsbuchhandlung von J. C. B. Mohr (Paul Siebeck), Freiburg i.B., 1888.

KMS: "Kant und die Marburger Schule", Kant-Studien 17, 1912, pp. 193 – 221.

L: Logik. Grundlegung und logischer Aufbau der Mathematik und mathematischen Naturwissenschaften in Leitsätzen zu akademischen Vorlesungen, Marburg: Elwert. 1910.

LGEW: Die logischen Grundlagen der ecxakten Wissenschaften, Berlin, 1910

LM: "Leibniz und der Materialismus", (1881). Aus dem Nachlaß herausgegeben (Helmut Holzhey), Studia Leibnitiana, Bd. 17, H. 1, Franz Steiner Verlag, 1985, pp. 3-14.

NTE: «Nombre, temps, espace, dans leurs rapports avec les fonctions primitives de la pensé», *Bibliothèque du Congrès International de Philosophie*, vol. 1, *Philosophie générale et Métaphysique*., Colin, Paris, 1900, pp.343-389.

PILa: Platos Ideenlehre, Leipzig, Meiner, 1903

PILb: *Platos Ideenlehre*. Zweite, durchsehene und um einen metakritischen Anhang vermehrte Ausgabe, Leipzig, Felix Meiner Verlag, 1921.

PIP: Philosophie. Ihr Problem und ihre Probleme. Göttingen. 1911.

PP: Philosophische Propädeutik, Marburg, 1904.

SP: Sozialpädagogik. Theorie der Willenserziehung auf der Grundlage der Gemeinschaft, Fr. Frommanns Verlag (E. Hauff), Stuttgart, 1899.

QQ: "Quantität und Qualität im Begriff, Urteil und gegenständlicher Erkentnnis", *Philosophische Monatshefte*, Bd XXVII, Heidelberg, 1891, 1–32; 129–160.

TDM: "Thema und Disposition der aristotelischen Metaphysik", *Philosophische Monatshefte*, 24, 1888, p. 37-65; 540-574.

UOSB: "Über objective und subjective Begründung der Erkentnnis I.", *Philosophische Monatshefte*, Bd XXIII, Heidelberg, 1887.

ZLGNM: "Zu den logischen Grundlagen der neueren Mathematik", *Archiv für systematische Philosophie*, Bd. 7, 1901, pp. 177-209/372 -438.

ZLM: "Zur Frage der logischen Methode. Mit Beziehung auf Edm. Husserls 'Prolegomena zur reinen Logik'", *Kant-Studien* 6, 1901, 270–283.

References to Kant

References to the *Critique of Pure Reason* are to the pagination of the first (A) and/or the second (B) editions. Other references to Kant are to the volume and page of the Akademieausgabe (AA), i.e., Kant's gesammelte Schriften, edited by the Preussische Akademie der Wissenschaften (Berlin: Walter de Gruyter, 1900-).

Introduction

In his letter to Marcus Herz on February 21, 1772, Kant introduces the general problem of knowledge by the following question: "... on what foundation rests the relationship of what we call representation in us with the object?" According to Kant, it is necessary to justify how the human mind represents objects. More specifically, the problem is how our thinking can *legitimately* be related to what it represents. In this letter, Kant argues that one of the main tasks of philosophy is to explain how thinking can have a reference to objects. The Kantian answer to this question is that thinking requires two types of representations: intuitions and concepts. The intuition is a singular representation, and the concept is a universal representation. Concepts abstract the marks that many objects have in common while intuitions refer to singular things. Our thinking can relate to objects only through intuition, because singular things are completely determined. There can be complete determinate cognitions only as intuitions (not as concepts). In relation to intuition, the logical determination can be complete but in regard to concepts, it can never be fully achieved. In the conceptual representation, the complete determination can never be accomplished². Concepts are, *per definitionem*, abstractive representations. Concepts contain partial marks of the objects. For this reason, the possible reference of thinking to the thoroughly determined individual requires intuition. According to Kant, intuitions are given while concepts are products of the understanding. Thus, if our understanding produces only universal representations (concepts), how can it *legitimately* be related to singular objects? if our intellect knows things only through concepts (universal representations), how does it know singular things? The Kantian answer to this question rests on his distinction between intuitions and concepts.

In the Transcendental Aesthetic, Kant introduces this fundamental distinction of critical idealism. Sensibility provides intuitions while the understanding is the source of concepts. This differentiation between intuitions and concepts as two different products of heterogeneous faculties introduced in the very beginning of the *Critique* is the pillar upon which the critical philosophy rests³. In the arguments developed in the

⁻

¹ Kant, I., AA X: 130.

² Kant, I , AA IX: 99.

³ As Lorne Falkenstein marks: "This 'two-faculty' account of cognition' lies at the foundation of his theoretical philosophy, and almost everything he has to say in the *Critique of Pure Reason* presupposes it."

Transcendental Aesthetic, Kant uses the definitions of the notions of intuitions and concepts as premises for his arguments. Through intuitions, thinking can have an immediate reference to objects. The intuitions are provided by sensibility, which is the capacity to be affected. The sensibility is the faculty of receptivity by which we receive representations. According to these first lines of the Transcendental Aesthetic, the objects affect our minds and provide us with intuitions. In contrast to intuitions, concepts are a product of the understanding. The understanding by its own means generates concepts. Through intuition, the objects are given; through the understanding, the objects are thought. The mind is affected and receives the matter for the formation of the representation. Thus, the sensibility (as receptivity) provides us with intuitions. Scholars agree that in these first passages of the Aesthetic, Kant is providing a series of definitions. Commentators consider that these statements are not conclusions of any previous argument provided by Kant in this section of the Critique of Pure Reason⁴. In effect, this exegetical tradition may differ in the details of their interpretations. However, for all of them, there is not really an argumentative structure in these first passages. For this reason, the distinction between intuitions and concepts has been considered as one of the weakest points of the Kantian system⁵. The Kantian dichotomies introduced in the first section of the Critique of Pure Reason have been strongly questioned by various contemporary scholars and different philosophical schools developed their own systems in an attempt to overcome this dualism.

Marburg Neo-Kantianism proposes as one of the pillars of their "return to Kant" a new approach to the dualism between intuitions and concepts. One of the main goals of the Neo-Kantian Marburg school is to overcome this Kantian dualism between intuitions and concepts. While for Kant, intuitions and concepts have their origin in different faculties of the human mind, Marburg Neo-Kantianism redefine these notions as modes of thinking. According to Paul Natorp, one of the founders of the school, the conception according to which the object is determined by two factors - intuitions and concepts - is

_

Falkenstein, L., 1991, p.165. Also: Brandt, R. 1998, p. 81. Smyth, D., 2014, p.1. Gloy, K., 1984, p.1. Willaschek, M.; 2015, p.129.

⁴ Cf. Caimi, M., 1996, p.27. Kolb, D., 1992, p. 244. Kemp Smith, N., 1918, p.79n. Vaihinger, H., 1892, p.1. Parsons, C.; 1992, p. 66. Falkenstein, L., 1995, p.28 ss. Strawson, P. F; 1966, p.23. Pippin, R., 1982, p. 32. Willaschek, M., 2015, p.129.

⁵ Kolb, D., 1986, p. 223.

⁶ Cf. Ferrari, M, 1997, p. 118. Dufour, É.; 2003, p.90. Giovanelli, M., 2005, p.116. Munk, R. 2005, p. 8. Holzhey, H., 2010, p.25. Giovanelli, M., 2011, p. 217. Mormann, T.; 2013, p. 241. Malter, R., and Hamilton H. H. Beck, 1981, p. 539. Krijnen, C., 2013, p. 168. Warren, N., 2015, p. 90. This is the core of Cohen's position. Cf. Pringe, H., 2020, pp.137 ss.

an assumption inherited from Aristotelian realism. The position that affirms that there are determinations of the object that thinking cannot provide by its own means is an inheritance from Aristotle's metaphysics. For Natorp, Plato would have been the first to demonstrate that thinking is not dependent on objects. On the contrary, objects are dependent on thinking. In this sense, Plato was the first idealist philosopher⁷. Aristotle gave a realistic interpretation of the theory of ideas, far from the spirit of the theory of Plato⁸. The debate between Plato and Aristotle is the controversy between idealism and realism⁹, and realism is, as Christian Krijnen holds, the "real antagonist" of Neo-Kantianism¹⁰. Natorp considers that the idealism of René Descartes tried to overcome the dualism between intuition and concepts, and Kant followed this line. However, neither Descartes nor Kant were deep enough in their approaches. After all, Descartes fell into a gross dualism too¹¹. Even in the Cartesian idealist philosophy, there is always an element of experience that thinking cannot provide from its own source. The object is a strange element for knowledge. Cartesian idealism recovers the task begun by Plato. However, in the system of Descartes, the naive belief in the existence of the object, which exists in itself before all knowledge, and to be grasped by the mind remains unchanged 12. The overcoming of dualism between intuition and concepts was not carried out either by Descartes or by Kant. Kantian idealism failed to show the way in which thinking is the

7

⁷ Natorp, P., PILb, p.459., Cohen, H., 1902 p.13. Cf. Serón, D., 2012, p.3; Reale, G., 1999, p. xi. Cohen and Natorp agree to see in Plato the founder of idealism. Cf. Serón, D., 2012, p. 3. As Andrea Poma notes: "Plato occupied a place of equal importance to that attributed to Kant, and that the former's influence on Marburg philosophy was in no way inferior to that of the latter". Poma, A., 2006, p. 22.

⁸ Natorp, P., PILb, p.147,445. Cf. Reale, G., 1999, p. xxiv. Aguilar Martínez, E., 2014, p.227. Fronterotta, F., 2000, p.324.

⁹ Natorp, P., USOB, p.276 ss., PILb, pp.147, 388, 445. Cf. Poma, A., 2006, p. 30.

¹⁰ Krijnen claims: "Namentlich der Realismus entpuppt sich im Zuge der Entwicklung ihrer Erkenntnisauffassung als der eigentliche Gegenspieler." Krijnen, C., 2014, p.12. In this line, Geert Edel explains: "Für Cohen gibt es "eigentlich nur zwei Gegensätze in aller Philosophie und Wissenschaft: Idealismus und Eklektizismus". (*LrE* 595) Im Bannkreis der analytischen Philosophie dagegen ist der Idealismus gleichsam nicht satisfaktionsfähig: es gehört hier fast schon zum guten Ton, dass man eine Theorie entwickelt oder verteidigt, die 'empiristisch', 'materialistisch', 'naturalistisch', kurz, die 'realistisch', auf keinen Fall aber 'idealistisch' ist." And then, he poses the following question: "Warum also, wenn die Dinge so klar, die Standpunkte so unvereinbar sind, jenem Zusammenhang überhaupt nachgehen? Kann denn dabei etwas philosophisch Belangvolles herauskommen?" Edel, G. 1993, p.179.

¹¹ "Aber schließlich fällt Descartes in den groben Dualismus zurück." Natorp, P., DED, p. 18.

¹² "Vielleicht wendet man ein, dass selbst bei Kant dieser Dualismus keineswegs ganz überwunden sei, dass neben dem in der Erkenntnis rein erzeugten doch noch etwas wie ein "gegebener" Gegenstand übrig bleibe. Allein das ist bei ihm blos ein rudimentärer Rest einer in der Hauptsache verlassenen Stufe des Philosophirens; ein Rest, der auf der Höhe des Systems verschwindet. Bei Descartes im Gegenteil ist der erste Ansatz rein und folgerecht, aber daneben wuchert das naive Vorurteil des an sich vor aller Erkenntnis vorhandenen und nun zu erfassenden Gegenstandes ungestört weiter, um endlich auch jenen richtigeren Ansatz zu überwuchern und sich auf der Höhe der Entwickelung des Philosophen, in seiner Metaphysik, zum System zu verhärten." Natorp, P., DED, p.19.

producer of objectivity. The task of developing a coherent idealism remains. At the very beginning of the *Logical Foundations of the Exact Sciences*, Natorp argues:

Die nachfolgende, von Kant ausgegangene Philosophie, auch die gegenwärtige, nichts weniger als orthodoxe neukantische Richtung hat an den Dualismus von reiner Anschauung und reinem Denken mehr und mehr Anstoß genommen und endlich entschlossen mit ihm gebrochen. Vielleicht schon etwas zu entschlossen; denn daß in Kants Begriff der Anschauung sich ein keinesfalls zu vernachlässigendes Problem barg, davon werden wir uns bald überzeugen. Aber vorerst war es durch das eigene Prinzip der Kantischen Transzendentalphilosophie gefordert, daß man, was bei Kant zum wenigsten mißverständlich in die zwei Faktoren: reine Anschauung und reines Denken zerlegt wird, in strenger Einheit wieder zusammennahm und als ein Einziges, für das man den Namen des "reinen Denkens" unbedenklich festhalten kann, zu verstehen suchte. ¹³

The position of Natorp is two-fold. On the one hand, he claims that it is necessary to overcome the heterogeneity between intuitions and concepts introduced by Kant. On the other hand, he considers that it must be shown the role that intuition plays in the process of thinking. According to him, philosophers failed in accomplishing these tasks. The philosophical efforts of his contemporaries to overcome the dualism were infertile, since they were grounded on the basis of this Kantian error, which - ultimately - was rooted on Aristotelian naive realism. The attempts of his contemporaries to overcome the dichotomy were fruitless. Thus, the problem of how a cognition may refer to objects is introduced in concomitance with the possibility of accomplishing a genuine idealism. A coherent idealism can only be achieved by exhibiting the way in which thinking can be the source of objectivity. The Kantian approach of how thinking can refer to objects must be reformulated in terms of how thinking can produce the object in its singularity. The problem of the determination of the singular object is introduced with the problem of the possibility of the prosecution of a genuine idealism.

In this context of the problem, the goal of our research is to analyze Natorp's redefinition of the distinction between intuitions and concepts in relation to his conception

¹³ Natorp, P., LGEW, p. 2. Also: KMS, p.202.

of the method of philosophy. We will exhibit the connection between the way in which Natorp redefines the Kantian dichotomy and his conception of the method of philosophy. We will show how a new position regarding the problem of the method leads to a new conception of the way in which thinking relates to the object in its singularity. We will investigate how Natorp introduces his position in dialogue with Kant's position and with contemporary approaches to the problem, psychologism and logicism. We will argue that the question of how a cognition may refer to the object requires rethinking the method of philosophy. It will be shown that the Kantian question of how thinking can have reference to objects was badly posed. The question is how the understanding can be the source of objectivity and not how it relates to the objects. We will study the arguments that led Natorp to argue that the object is the result of a construction of the understanding. This is the only approach to the problem consistent with critical idealism¹⁴. The hypothesis of this investigation is that Natorp's redefinition of the distinction between intuitions and concepts is based on a new conception of the philosophical method. Our aim is to show that the way in which Paul Natorp understands the notions of intuition and concept is grounded on his conception of the problem of method. We expect to exhibit that the criticism of psychologism and logicism are the pillars on which the new conception of the distinction between intuitions and concepts is based.

Our research will be divided into five main chapters. In the first chapter, we will make a brief study of the distinction between intuitions and concepts in the philosophy of Kant. We will investigate how Kant inherited these notions from rationalism and reinterpreted their meaning. We will present an overview of the meanings of these terms before Kant. Second, we will study the Kantian argument of incongruent counterparts. We will show that the intuition is introduced in order to satisfy the requirement of the complete determination of the object. Finally, we will study the reference of intuition to affection as a result of the consideration of the imperfection of our thinking. We will conclude that the reference to the intuition is one of the marks of our finitude. According to Kant, a finite being may have legitimate representations of objects only through

.

¹⁴ In this sense, as Christian Krijnen notes, the question of how the mind can have reference to objects assumes, even in its contemporary formulations, the assumption of naive realism. He states: "Die erkenntnistheoretische Grundfrage lautet für sie ihrem geltungsidealistischen Ansatz gemäss nicht wie im älteren Empirismus und in dessen neueren Varianten: Wie kann das 'Subjekt' ('Ich', 'Bewusstsein', 'mind', usw.) ein von ihm unabhängiges 'Objekt' ('Aussenwelt', 'äusseren Gegenstand', 'world' usw.) erkennen? Sie lehnen nämlich die Voraussetzung dieser Frage ab: die ursprüngliche Unverbundenheit von (erkennendem) Subjekt und Objekt. Dieser Getrenntheit von Subjekt und Objekt geht den Neukantianern zufolge eine ursprüngliche Verbundenheit beider in der Erkenntnisrelation voraus." Krijnen, C., 2014, p. 16.

intuition. In the second chapter, we will study Natorp's arguments that highlight the deficiencies of the proposals of psychologism and logicism. We will exhibit that these considerations led Natorp to reformulate the Kantian distinction between intuitions and concepts. We will study how Natorp presents his proposal in dialogue with these philosophical tendencies. These schools misunderstood the problem of the relation of concepts and intuitions. We will exhibit that both currents have a wrong conception of the relationship between intuitions and concepts due to methodological errors. Natorp will show that both positions are unsuccessful to explain the issues raised by Kant. We will conclude that it is necessary to reformulate the notions of intuition and concept. In the third chapter, we will study Natorp's position on the object and method of philosophy. We will focus on his definition of thinking as correlation. We will argue that this new starting point will be the clue to solve the problems raised by the Kantian distinction between intuitions and concepts. In Chapter 4, we will analyze the way in which Natorp defines the notions of intuition and concept starting from his deduction of categories. In this chapter, we will find the clue to understanding how Natorp conceives the relationship between intuitive and conceptual representations. We will analyze the specific development of the process by which the object is constructed in and by thinking. We will exhibit the place that the concepts of space and time play as forms of thinking and, more specifically, as modes of the category of relation. The general goal of this section is to highlight how Natorp's proposal overcomes the heterogeneity between intuitions and concepts, by proving that the object is a construction and not something given. In the deduction of categories, it must be shown that the understanding does not need any reference to intuition in the process of knowledge. In chapter five, we will study Natorp's new definition of the distinction between intuitions and concepts. According to this new definition, intuition will be considered as the always distant goal to which knowledge aims. The singular completely determined object is the goal that can never be reached. It will be shown that the expression of the imperfection of knowledge is exhibited at this point. Second, we will analyze Natorp's definition of the notion of concept. We will argue that the Kantian definition of the notion of concept is insufficient to explain the productive nature of concepts. Finally, we expect to exhibit the new answer to the Kantian problem of the foundation of the relation of the representation with the object. We will show that the answer to the Kantian question relies on the exhibition of the functional character of concepts.

Chapter 1. The Kantian Distinction between Intuitions and Concepts.

Introduction

The aim of this chapter is to study the way in which Kant understands the distinction between intuitions and concepts. We will show the relation between the Kantian conception and how it was understood by the tradition. We will highlight some of the central problems of the Kantian proposal. This aspect will be taken up in chapter four. We will exhibit that the problems faced by Kant depend on the peculiar way in which he defined these notions.

In the first place, we will study some of the central problems of the Kantian proposal. We will show that Kant starts with a series of unclear definitions. The distinction between intuition and concepts is introduced as if it were completely evident. The distinction between intuition and concepts is merely assumed without any grounding. We will show the complexity of determining the specific features of intuitions and concepts. Then, we will exhibit that the Kantian definition of intuition leads to the problem of affection. Second, we will briefly study the history of the definition of intuitions and concepts. We will show that this distinction was first drawn by Duns Scoto and then continued by modern rationalism. We will study the relation between the Kantian proposal and the tradition. We will exhibit how Kant built his own definition of these notions in dialogue with the tradition. Third, we will show that Kant's reference to intuition arises from the requirement of a complete determination of objects. We will analyze the problem of incongruent counterparts. We will explain how this problem leads Kant to formulate the distinction between intuitions and concepts. The argument is used to prove the insufficiency of conceptual determination for a complete determination of objects. As concepts are insufficient, it is necessary another sort of determination, i.e., intuitive determination. Finally, we will study Kant's letter to Herz of February 22, 1772. It will become clear that Kant's reference to affection arises as a peculiar way of understanding the limits of human knowledge. We will argue that the reference of intuition to affection is a consequence of the imperfection of our knowledge.

1.1. Introduction to the Distinction between Intuitions and Concepts

In Transcendental Aesthetics, Kant introduces one of the central distinctions of critical idealism. Intuitions are singular and immediate representations, and concepts are general and mediated representations. Intuitions have their origin in sensibility while concepts are products of the understanding. Kant explains:

Auf welche Art und durch welche Mittel sich auch immer eine Erkenntniß auf Gegenstände beziehen mag, so ist doch diejenige, wodurch sie sich auf dieselbe unmittelbar bezieht, und worauf alles Denken als Mittel abzweckt, die Anschauung. Diese findet aber nur statt, sofern uns der Gegenstand gegeben wird; dieses aber ist wiederum uns Menschen wenigstens nur dadurch möglich, daß er das Gemüth auf gewisse Weise afficire. Die Fähigkeit (Receptivität), Vorstellungen durch die Art, wie wir von Gegenständen afficirt werden, zu bekommen, heißt Sinnlichkeit. Vermittelst der Sinnlichkeit also werden uns Gegenstände gegeben, und sie allein liefert uns Anschauungen; durch den Verstand aber werden sie gedacht, und von ihm entspringen Begriffe. Alles Denken aber muß sich, es sei geradezu (directe), oder im Umschweife (indirecte), vermittelst gewisser Merkmale zuletzt auf Anschauungen, mithin bei uns auf Sinnlichkeit beziehen, weil uns auf andere Weise kein Gegenstand gegeben werden kann.

This dichotomy between intuition and concept works as a foundation upon which is built not only the Aesthetics but all the Kantian critical system. There is a general agreement that the peculiar critical conception of the distinction between intuitions and concepts is "the pillar of Kant's theoretical philosophy." However, unfortunately, there is a second general agreement among the Kantian studies. Scholars agree that in the first passages of the Aesthetics, Kant is providing a series of definitions¹⁶. Commentators consider that

-

¹⁵ Cf. Falkenstein, L., 1991, p. 165. Also: Brandt, Reinhard, 1998, p. 81. Smit, Houston; 2000, p. 235. Smyth, D., 2014, p. 1.

¹⁶ We use the concept of definition in a broad sense and not in technical sense. Kant does not give a definition of the concepts of space and time, but he makes an exposition because, as Luciana Martínez explains: "los conceptos de la metafísica no pueden ser definidos en el sentido matemático de la definición, y ii) la elucidación de tales conceptos no puede darse, como en matemática, al inicio de la investigación". Cf. Martínez, L., 2019, p. 683. See: Kant, I., *KrV*, A727/B 755.

these Kantian statements at the beginning of the Aesthetics are not conclusions of any previous argument provided by Kant¹⁷. Kant introduces the distinction as if it were completely evident, and it did not require any further explanation 18. For this reason, as Daniel Kolb notes, the distinction between intuitions and concepts has been considered one of the weakest points of the Kantian system¹⁹. Mario Caimi highlights: "it is generally assumed that Kant begins by setting forth some definitions, to immediately committing himself to the study of what is considered to be the central subject of the Aesthetic, namely space and time, in §2."²⁰ Indeed, as Caimi claims, most of the studies of this section go in the same direction. Kemp Smith considers that "the Aesthetics opens with a series of definitions" that must be understood in relation to later results."21 Hans Vaihinger stresses in the first volume of his Commentar this point too. In this first paragraph of the Transcendental Aesthetic, Kant provides definitions and basic assumptions of the system. For him, "this introductory paragraph gives a series of important definitions and fundamental assumptions."22 Henry Paton supports this reading²³. Charles Parsons, in the same direction, remarks that this passage cannot be considered as a conclusion of any argument but, rather, it is an initial premise assumed in the system²⁴. The analysis of Reinhard Brandt shares this reading. For this scholar, the dichotomies of these passages are introduced by Kant without any support. Kant introduces the differentiation between intuitions and concepts as if it were completely evident, and it did not require any further clarification. The differentiation is introduced without any grounding²⁵. Likewise, Lorne Falkenstein affirms that Kant's critical

_

¹⁷ Daniel Smyth holds against this canonical interpretation: "Indeed, it can seem that the Critique and the Prolegomena begin by presupposing, stipulating or otherwise hypothesizing certain robust conceptions of judgement, intuition, conceptual representation, mathematical cognition, etc. and then proceed to demonstrate (with more or less success) the fruitfulness of these conceptions indirectly, by showing how they (alone?) serve to resolve various philosophical difficulties. In what follows, I will resist this impression and suggest that Kant does, in fact, provide the materials for an extended argument in favour of his nuanced conceptions of conceptual and intuitive representation over the course of the Aesthetic and Analytic." Smyth, D., 2014, p. 1.

¹⁸ Cf. Brandt, R., 1998, p. 82.

¹⁹ "Given its centrality to the entire Kantian system, it is surprising that Kant nowhere undertakes a sustained, rigorous defense of the distinction." Kolb, Daniel, 1992, p. 244.

²⁰ Caimi, M., 1996, p. 27.

²¹ According to him, "Kant is here defining his terms in light of his subsequent results. Kemp Smith, N., 1918, p. 79n.

²² Vaihinger, H.; 1892, p. 1.

²³ "At the beginning of the Aesthetic, Kant gives us a rather complicated explanation of the terms which he employs. There is a considerable element of ambiguity in what he says, and the full meaning of his terms can be grasped only from their use as the argument develops." Paton, H. J., 19, p. 93.

²⁴ Parsons, C., 1992, p. 66.

²⁵ Brandt, R., 1998, pp. 81 ss.

philosophy begins with the postulate of these dichotomies: intuition-concept, sensibility-understanding. In this sense, he points out:

Kant's critical philosophy begins with the postulate that man has two distinct cognitive faculties, sensibility and intellect. Critical philosophy begins with this postulate in every sense of the word.²⁶

For this commentator, the reference of the intuition to the affection, which Kant alleges in this passage, exhibits that "empiricism is an assumption of Kant." In his *The Bounds of Sense*, Strawson follows this reading. He considers, taking the argument even further, that the distinctions presented, those with which the *Critique* opens, are a necessary assumption of any philosophical system dealing with the problem of knowledge²⁸. In the same way, Robert Pippin considers that in these first passages of Aesthetics, Kant offers a series of definitions. Specifically, regarding the distinction between intuition and concept, he says:

His first attempt to define these terms occurs at the beginning of Transcendental Aesthetics. In fact, he starts with a flurry of definitions." (B33 = A19 ff.) $(...)^{29}$

Moreover, Kant not only assumes the definition of these concepts but the exhaustiveness of the distinction. The arguments of the Aesthetic require us to assume that all our representations are divided into intuitions and concepts. The distinction between intuitions and concepts is presented as an exhaustive one.³⁰ Precisely for this reason, in the arguments of Transcendental Aesthetics, it is enough to show that space and time are not concepts, to legitimately conclude that they are intuitions. This is particularly evident

²⁶ Falkenstein, L., 1995, p. 28 ss.

²⁷ Falkenstein, L., 1995., p. 367n. In this line, Andrew Stephenson argues "Kant repeatedly affirms his commitment to empirical realism.". Stephenson, A., 2015, p. 509.

²⁸ Strawson claims: "The duality of intuitions and concepts is not in fact but a form or aspect of a duality that must be recognized in any philosophy that seriously deals with human knowledge, its objects or its expressions and communication." Strawson, P. F; 1966, p. 23.

²⁹ Pippin, R., 1982, p. 32.

³⁰ As Lorne Falkenstein explains: "Either a representation is a discursive or universal concept, or it must have been originally given in intuition." Falkenstein, L., 1995, p. 218. Also: Chenet, X., 1994, p. 76. Falkenstein's statements have a critical tone here. He considers this premise as "highly controversial and completely unexplicated and undefended." Falkenstein, L., 1995, p. 222.

in the third argument of the metaphysical exposition of space where Kant aims to demonstrate that space is a pure intuition. The argument is formulated in a negative way. It must be shown that space *is not* a discursive concept. It will be enough for Kant's argumentative purposes to show that space is not a concept so that the immediate conclusion that it is an intuition can be inferred. To demonstrate that the representations of space are not conceptual is to show in concomitance that they are intuitive.³¹

This exegetic tradition has, in effect, interpretative nuances. As we saw, some scholars consider that the Transcendental Aesthetic contains certain arguments to ground the distinction between intuitions and concepts that can be reconstructed within the Transcendental Aesthetic or in the light of later results to be found in the Transcendental Logic, while others just hold that there is not such an argument. However, for all of them, there is not really an argumentative structure in these first passages where Kant introduces the distinction. Therefore, the first problem that we are facing is that the core upon which the whole critical system is built seems to be resting on a series of unjustified assumptions.

The second problem is to determine the specificity of these two modes of representation. Kant establishes that immediacy and singularity are the differential marks of intuition. Intuition is an immediate and singular representation. On the contrary, concepts are universal and mediate representations. However, it is still not so clear what he meant by the claim that immediacy and singularity are the main marks of intuition while universality and mediation are those of concepts. For some scholars, immediacy is the feature of intuition that makes it different from the concept³². Lorne Falkenstein considers that when Kant claims that the singularity of the intuitive representation is the distinctive mark of intuition, he is using scholastic terminology which is not truly proper to the critical system³³. He thinks that there is an "old" use of the notion of intuition – which is present in the Dissertation of 1770 (as a singular representation) - and another use of the notion of intuition that is present in the *Critique of Pure Reason*, where it is

-

³¹ Cf. Kant, I., AA 9: 91. Allison correctly marks: "this argument assumes the exhaustiveness of the concept-intuition distinction." François-Xavier Chenet also considers that: "The argument can only be understood in the light of partition of all our representations into intuitions and concepts and on the definition of one and the other." Previously, this had already been noticed by Kemp Smith. In his *Commentary*, he writes: "Kant's proof rests on the assumption that there are only two kinds of representation, intuitions and concepts and also in equal degree upon the further assumption that all concepts are of one and the same type." Allison, H., 2004, p. 104. Chenet, X., 1994, p. 76. Smith, K. 1918, p. 106.

³² Caimi, M., 1996, p.30, Falkenstein L., 1995, p. 28ss.

³³ Falkenstein, L., 1991, p. 165.

defined as a given immediate representation. Houston Smit also agrees on the centrality of the immediacy criterion. However, he considers that it has been traditionally misunderstood. Intuitions are also related to objects by means of their marks. The difference is that concepts relate to objects by general marks whilst intuitions relate to objects by singular marks. Intuition is an immediate representation because it relates to objects only through itself³⁴. In the opposite direction, Jaakko Hintikka argues that intuition is a representation of a particular. Intuitions can be defined as singular representations. He considers that the immediacy criterion is a reformulation of the feature of singularity. Concepts represent the object abstracting its general marks which mediate between the object and the concept. The concept is a mediated representation because it is an abstractive representation. On the contrary, as intuitions refer to singular representations, they do not need this mediation³⁵. Henry Paton had also stressed the importance of the singularity criterion³⁶. Against Hintikka, Charles Parsons argues that intuition cannot be defined merely as a singular representation. The intuition is directly present to the mind and concepts are not. While it is possible to admit the possibility of singular concepts, it is not possible to have an immediate conceptual representation. Thus, a representation is an intuition if it satisfies both requirements: singularity and immediacy, but the singularity criterion is broader than the immediacy criterion³⁷. Daniel Kolb shares this point of view. Intuitions cannot be distinguished from concepts just by the singularity criterion since there are concepts that also refer to singular objects, such as the concept of God³⁸. Manley Thompson argues that Hintikka and Parsons make the mistake of considering intuition in relation to its mathematical aspect and not in relation

⁻

³⁴ "What distinguishes sensible intuitions from concepts is that they are objective perceptions that relate to objects through singular, as against general, marks. It is neither a part, nor a logical consequence, of Kant's notion of intuition that an intuition does not relate to its object through marks." Houston Smit, 2000, esp. p.239. A mark, he claims, is not a mere determination of the thing but "a property through which we can cognize, not just any subject matter, but things." Smit, H., 2000, p. 245.

³⁵ Jaakko Hintikka argues: "There is not the slightest doubt that the idea of immediacy (direct reference to objects) was associated by Kant with the notion of intuition. There is not much more doubt in my mind that it was not an independent aspect of the Kantian concept of intuition but simply a corollary of the individuality criterion." (...) "A general term or its counterpart in the mind does not refer to its object immediately, but only through the mediation of a characteristic which several objects may share. These characteristics, so to speak, 'intervene' between concepts and their objects" Hintikka, J., 1972, pp. 341, 343. ³⁶ Paton, J., 1970, Vol I., pp. 97, 115.

 $^{^{37}}$ "But it evidently means that the object of an intuition is in some way directly present to the mind, as in perception, and that intuition is thus a source, ultimately the only source, of immediate knowledge of objects. Thus, the fact that mathematics is based on intuition implies that it is immediate knowledge and thus, even though synthetic a priori, does not require the elaborate justificatory argument which the Principles do (A 87 = B 120)." Parsons, C., 1992b, p. 44.

³⁸ He concludes: "the concept of a singular representation is, then, too broad to serve as a criterion for distinguishing intuitions from concepts, since both intuitions and concepts may be characterized as singular representations." Kolb, D., 1986, p. 227.

to the integral aspect in which it is defined. While both concepts and intuitions can be defined as singular representations, the distinction should be read in relation to the role of empirical judgments, and therefore, it must be contemplated that Kant here has in mind not only pure intuition but also empirical intuition³⁹. Kirk Dallas Wilson argues that the two criteria, although they are extensionally identical, they are intentionally different. Against Hintikka's reading, he maintains that the trait of immediacy cannot be reduced to that of singularity. The immediacy of intuition is not a mere logical feature. Against Parsons, he argues that the singularity of intuition must be distinguished from the singularity of singular concepts⁴⁰. For others, what properly distinguishes the intuition from the concept is the relation of wholes and parts. In the concept, the parts precede the whole, while in the intuition the whole precedes the parts⁴¹. Thus, we observe how the tradition of Kantian scholars does not agree on what is the distinctive feature of intuition, whether singularity or immediacy⁴². Moreover, there is no agreement on how we should understand these notions. As we shall see, Kant took the notion of intuition as it was established by Duns Scotus. The criteria of singularity and immediacy should be understood in this light.

The third problem implied in the distinction between intuition and concepts is the relation of intuition to sensibility and, therefore, the relation of intuition to affection⁴³. The problem that the concept of affection implied for the Kantian system was already recognized by his contemporaries. Salamon Maimon, in one of his letters to Kant, argues that intuition cannot be related to anything but itself⁴⁴. Jakob Sigismund Beck also rejects the conception of intuition as an object-oriented representation. The object that affects the mind cannot be considered as something external to it. On the contrary, the object of intuition must be regarded as a product of the understanding. It could be claimed that the object affects the mind, just when we consider the objectivity-product from the sensibility

_

³⁹ Thompson, M., 1972, esp. p. 314.

⁴⁰ Wilson, K, D., 1975.

⁴¹ Pippin, R.,1982, p.65. Mario Caimi argues that this is a mistake of Pippin's reading. Caimi, M., 1996, p. 37 n.25.

 $^{^{42}}$ James Conant exhibits that for some commentators there are two possible definitions of the concept of intuition. Cf. Conant, J., 2016, esp. 99ss.

⁴³ Hernán Pringe showed that the cognition of the individual object requires not only perception- and thus, affection, but also the homogeneity of sensible dada and, therefore, regulative principles. Cf. Pringe, H., 2015.

⁴⁴Maimon claimed: "An intuition, in my opinion, is not related to anything other than itself. It becomes a *representation* only by being united with other intuitions in a synthetic unity, and it is as an element of the synthesis that the intuition relates itself to that representation, that is, to its object." AA 12: 286.

point of view⁴⁵. Johann Heinrich Tieftrunk also argues in this direction. According to him, the only reasonable explanation of the problem of affection is that the mind affects itself⁴⁶. Fichte argues against Kant in this direction too. He claims that "a finite rational being has nothing beyond experience; it is this that comprises the entire staple of his thought". In the framework of a genuine idealism, there is no place for anything like affection. The concept of objectivity only makes sense when it is referred to the 'I think'. There is nothing like an object *in itself*. The object of experience is the object *in itself* when it is considered independently from the 'I think' pole. However, this is only a point of view. There is no affecting object because there are no objects independently of the experience. Hegel built his system as an attempt to overcome this dualism between what is given and what is thought. According to Hegel, Kant showed in the deduction of categories that the original synthetic unity of apperception is the principle of sensibility. The receptivity is nothing but a product of the unity of apperception. Both sensibility and intellect depend on the unity of apperception⁴⁸. As we will show, the conception of

15

⁴⁵ Beck writes to Kant: "Allow me to ask whether in what follows I have understood you correctly.... The *Critique* calls "intuition" a representation that relates immediately to an object. But in fact, a representation does not become objective until it is subsumed under the categories. Since intuition similarly acquires its objective character only by means of die application of categories to it, I am in favor of leaving out that definition of "intuition" that refers to it as a representation relating to objects. I find in intuition nothing more than a manifold accompanied by consciousness (or by the *unique "I think"*), and determined by consciousness, a manifold in which there is as such no relation to an object." AA 12: 311.

⁴⁶ In his letter to Kant on November 5, 1797, Tieftrunk writes: "But whence comes the manifold of sensation, *the merely empirical* aspect of sensation? (...) Whence the material? Out of sensibility. But whence did sensibility obtain it? From the objects that affect it? But what are these objects that affect sensibility? Are they things in themselves or -? (...) example, those in which the mind regards itself as spontaneous. If I ask further, What is it that affects the mind? I must answer, it affects itself since it is both receptivity and spontaneity." AA 12: 214.

⁴⁷ "Das endliche Vernunftwesen hat nichts ausser der Erfahrung; diese ist es, die den ganzen Stoff seines Denkens enthält." Fichte, G., GA I, 425. In relation to Beck, Fichte and Maimon, Arnulf Zweig claims: "Although each of these philosophers found his own views to be either subtly or dramatically different from those of the others (Beck, for example, tried to convince Kant that he was radically opposed to Fichte), they agreed that Kant's theory of affection must be reconsidered or reformulated." Zweig, A., 1999, p. 33. The overcoming of dualisms is an inherent element of German idealism as a whole. Lucas Amaral correctly highlights: "...a number of dualisms, deriving from the old Cartesian scheme, which the author of the Critique of Pure Reason had accepted largely in the context of his doctrine, would have been dissolved by idealism". As González Porta affirms, the rejection of dualisms is an inherent element of German idealism as a whole. He explains: "La superación de los dualismos, la eliminación de la cosa en sí, la tarea de la totalidad y la concepción de la filosofía como "Sistema", son elementos inherentes al idealismo alemán en su conjunto." Amaral, L., 2015, p. 250. González Porta, M., A., 2005, p. 44. Marco Giovanelli, correctly concludes: "La fondamentale funzione sistematica che la distinzione tra concetto e intuizione riveste nella filosofia di Kant trova d'altra parte conferma nel fatto che la filosofia post-kantiana, dall'idealismo classico al neokantismo, fece proprio il programma di superare tale opposizione, nell'intento di dare unità a un pensiero che, a causa di essa, sembrava spezzarsi in una serie di dualismi insanabili." Giovanelli, M., 2005, p.116.

⁴⁸ In *Glauben und Wissen*, Hegel holds: "the original synthetic unity of apperception is recognized also as the principle of the figurative synthesis, i.e., of the forms of intuition; space and time are themselves conceived as synthetic unities, and spontaneity, the absolute synthetic activity of the productive imagination, is conceived as the principle of the very sensibility which was previously characterized as

Natorp will follow this line⁴⁹. This difficulty remains one of the hardest issues for Kantian studies. Mario Caimi has called the fact of affection "an unexplained fist moment", a "zero moment" which "defies all explanations." According to Caimi, this reference to something outside experience could make the Kantian building stumble and fall⁵¹. Marcus Willaschek holds that "Kant gives no explicit argument" to explain the relation of intuition to sensibility. According to him, Kant takes this relationship for granted⁵². Some authors explained the role of affection by emphasizing the finite nature of human knowledge. Martin Heidegger, following Hegel, developed this interpretation.⁵³ Heidegger considers that the receptive character of human intuition is grounded on the finite essence of human beings. As we are finite beings, we cannot create the object of intuition, but it must be given in some way⁵⁴. Heidegger is followed – directly or indirectly- by many other contemporary scholars who also claim that the finitude of the human essence constitutes an explanation for the relation of intuition to affection⁵⁵. Alberto Rosales, criticizing Heidegger, deepens his reading. According to him, it must be taken into account that not only intuition is affected by finitude but thinking too. The

_

receptivity." (...) "The Kantian forms of intuitions and the forms of thought cannot be kept apart at all as the particular, isolated faculties which they are usually presented as. One and the same synthetic unity- we have just determined what this means here- is the principle of intuition and of the intellect." Hegel, G. W., 1986, p. 16ss. Hegel claims that the Kantian idealisms reduced knowledge to finite knowledge. Then, the Kantian conception is constrained within the boundaries of the finite cognitive subject. His theory is not truly a theory of knowledge but merely a theory based on the perspective of the finite thinking subject. Hegel, G.W., 1986, p. 10.

⁴⁹ We will develop this point in Chapter 3.

⁵⁰ "The whole development of the Transcendental Aesthetic may be said to originate at a sort of Big Bang, at a zero moment, a starting point beyond which it is not possible to reach. This point- that is, the affection-defies all explanation. It is recognized in the first paragraph of the Aesthetic, and thereafter no revert to it is made. The whole sequence of thoughts stems from this unexplained first moment onwards..." Caimi, M., 1996, p. 29.

⁵¹ Caimi, M., 1983, p. 109.

⁵² "The claim that human intuition is sensible is an integral part of Kant's distinction between sensibility and the understanding, of which he briefly "reminds" us at the end of the Introduction to the first *Critique* (A 15, B 29) and from then on takes for granted without any argument." Willaschek, M., 2015, p. 129.

⁵³ Cf. Heidegger, GA 3, esp. §5. GA 21, p.115ss., and §23. GA 25, esp. §§5 - 6. GA 41, §24d.

⁵⁴ "In the first place, we can say negatively: finite knowledge is noncreative intuition. What has to be presented immediately in its particularity must already have been "at hand" in advance. Finite intuition sees that it is dependent upon the intuitable as a being which exists in its own right. The intuited is derived from such beings; thus, this intuition is also called *intuitus derivativus*, "deduced" ["abgeleitete"], that is, intuition which conduces [sich herleitende Anschauung].33a Finite intuition of the being cannot give the object from out of itself. It must allow the object to be given. Not every intuition as such, but rather only the finite, is intuition that "takes things in stride." Hence, the character of the finitude of intuition is found in its receptivity. Finite intuition, however, cannot take something in stride unless that which is to be taken in stride announces itself. According to its essence, finite intuition must be solicited or affected by that which is intuitable in it." Heidegger, GA3, p. 25.

⁵⁵ Stadler, A., 1897, esp. pp. 101, 103. Allison, H., 2004, esp. p. 14. Kolb, D.C., 1992, p. 215. Chenet, F., 1994, p. 43. Cazeaux, C., 1995, p. 348. p.43ss. Heidemann, D., 2019.

reference of sensibility to affection is just one of the expressions of the limitations of finite human cognition⁵⁶.

However, even when the problem of affection could be overcome, the problem of the relation between two heterogeneous faculties still remains. It must be shown how concepts relate to intuition and thus get content. The problem of endowing the concept of content is a result of the pure origin of a priori concepts because pure concepts, independently of their relation to intuition, are empty. In the Kantian conception, there can be concepts without any content. The problem of giving content to the concepts arises as a result of this novelty of the Kantian system: the possibility of empty concepts⁵⁷. As we shall exhibit, this will be one of the main points of discussion with Leibnizian rationalism which will lead Kant to reformulate the notion of intuition. For the Kantian conception, that a concept does not contain any contradiction does not guarantee that it has a possible content. It must be proved that the concept has a possible relation to the form of sensibility. To have a reference to objects, concepts must have a reference to intuition. To know something at all, concepts must have a relation to intuition⁵⁸. Concepts have their origin in the understanding and intuitions in sensibility. Then, it must be explained the relationship between these two heterogeneous faculties. It must be shown how these two different sources of knowledge, which are interdependent, cooperate so as to get cognitions⁵⁹. Moreover, the relation of concepts to existent objects requires the reference to sensibility. The problem is not only to explain the relationship between two heterogeneous faculties but also the relation of concepts to existent objects, and then it is necessary to explain not only the relation of concept to the pure form of sensibility but also the relation of thinking to sensibility, i.e., to given objects⁶⁰. Furthermore, singular objects must be subsumed under categories. Then, even if the task of the Transcendental Deduction is fully accomplished, it still subsists the problem of subsuming the particular object under the categories. This is explained by Kant in the so discussed chapter of

⁵⁶ Rosales A., 2000, esp. pp. 46, 58ss, 350ss.

⁵⁷ Cf. Caimi, M., 2005, esp.142ss.

⁵⁸ As Mario Caimi explains: "Ein Begriff kann also leer sein, wenn er auch nach formallogischen Kriterien tadellos ist. Er kann nämlich auf diese neue Weise leer sein, indem er keine ihm entsprechende Anschauung aufweisen kann. Kants Leistung, seine Neuerung der leibniz- wolffschen Philosophie, tritt hier hervor. Sie besteht in der Anerkennung der Anschauung als notwendige Bedingung der Erkenntnis. Das bringt die Anerkennung der Unzulänglichkeit des Verstandes als alleinige Erkenntnisquelle mit sich." Caimi, M., 2005, p. 145.

⁵⁹ Cf. Caimi, M., 2007, p. 68ss.

⁶⁰ Cf. Caimi, M., 2007, p. 66ss.

schematism, where he introduces the third faculty of imagination⁶¹. All in all, regardless of the different conceptions of the respective task of each section of the *Critique*, the problem of reconciling these two different sources of the mind is acknowledged by every reader of the critical system.

That Kant does not give an accurate definition of these concepts can be explained by the fact that these notions were far understood by philosophers at that time. As we will see, the general problem of the relation of universals and particulars was already widespread as one of the philosophical central issues, and the notions of concepts and intuitions were commonly used. A brief sketch on the history of the distinction will reveal how Kant built his own definition of these notions in dialogue with the tradition. Furthermore, we will see that the peculiar way in which Kant defined the relation of intuition and concept is intimately related to a) his rupture with rationalism and b) a new conception of the limits of human knowledge. This brief sketch on the history of the distinction between intuitions and concepts will show, not so much how Kant solved the problems we have just presented but rather how they arose at first.

_

⁶¹ Cf. Moledo, F., 2011. Henry Allison believes that the function of schematism is to explain how the pure concepts of the understanding are expressed in sensible terms. Allison, H. E., 1992, p. 274. Paton tries to show that this chapter will indicate the universal and necessary characteristics of sensible objects without which the pure categories would not refer to any assignable object. Paton, H.J., 1965, p. 23. For Roberto Torretti, the schematism of concepts is the procedure by which pure categories organize time (that is: the universal form of sensibility) and thus apply to the empirical multiplicity given in that form. Torretti, R., 1980, p. 406. All in all, some interpreters have chosen to take schematism as a continuation of the deduction of the categories, others have considered the possibility of giving the deduction a new foundation, and others have considered it superfluous and unnecessary.

1.2. Brief Sketch on the History of the Distinction between Intuitions and Concepts

The distinction between intuition and concepts has a long history that can be traced back to the Aristotelian tradition⁶². The specific distinction among these notions appeared for the first time in medieval philosophy, and it has been shown that the Kantian use of the word *intuitus* (*Anschauung*) dates back to Middle Ages terminology⁶³. The medieval theory of intuitive cognition is mainly grounded in the philosophy of Duns Scotus and William Ockham⁶⁴. Even when the debates on the status of universals and particulars were widely developed⁶⁵, it was Scotus who introduced for the first time this specific distinction. Duns Scotus was the first who systematically developed a theory of intuition⁶⁶, and William Ockham inherited the distinction from him⁶⁷.

In his *Questions on the Metaphysic of Aristotle*, Scotus distinguishes a kind of cognition proper of senses from another type of cognition proper of the intellect:

Note that in the sense there is one cognition primarily proper, [viz.,] intuitive cognition; another sort of cognition is proper primarily and *per se* and that is knowledge through a species, but it is not intuitive (...) An example of the first: the visual sense sees color; an example of the second, the sense imagination or phantasy imagines color.

In the intellect, intuitive cognition or vision, which is primarily knowledge, is not possible in this life, because no potency reserving the species or the formal principle of knowledge in the absence of the object, could know in this fashion. For such a potency has the same

⁶² Cf. Falkenstein, L., 1995, p. 29.

Norbert Hinske explains that the German word *Anschauung* was rather infrequent in early modern philosophers. Kant uses for the first time the noun *Anschauung* around 1762, and he uses it just eight times in the pre-critical writings. According to Giorgio Tonelli, the Scholastic is one of the main sources of the Kantian new terminology, mainly from 1770 on. Lorne Falkenstein maintains that "the meaning of the term *Anschauung* is to be determined by looking at traditional uses of Latin term *intuitus*, not the German *ascouuen*." Falkenstein considers that Kant's definition of the term 'intuition' is, in fact, in accord with the scholastic distinction. Hinske, N., 1983, p.VI ss. Tonelli, G., 1964, p. 233. Falkenstein, L. 1995, p. 18.

⁶⁴ John Boler holds: "Especially notable among those landmarks are the theories of intuitive cognition in Duns Scotus and William Ockham. Nearly all the medieval discussions of intuition that follow them are an attack on or defence of one or the other." Boler, John F., 1982, p. 460.

⁶⁵ Katherine Tachau holds in this regard: "the history of medieval theories of knowledge from ca. 1310 can be traced as a development of this dichotomy." Quoted in Pasnau, R., 2002, p. 296.

⁶⁶ Cf. Boler, John F., 1982, p. 463. Day, S., 1947., p. xiii. Pasnau, R., 2002, p. 297. For Robert Pasnau, "this would prove to be, by far, Scotus's most influential contribution to the theory of cognition." According to Camille Bérubé, Scotus was the first to use the term intuition to make reference to the cognition of individual material objects. Bérubé, C., 1964, p. 179. Pasnau, R., 2002, p. 297

⁶⁷ Cf. Scott, Kermit; 1969, p. 431. Gilson, E., 1952, p. 426.

principle [of knowing] whether the thing is present or not present, and that knowledge [i.e., intuitive] is only of a thing present under the aspect of its being present.⁶⁸

The concept of intuition was opposite to the notion of abstractive concepts. The abstractive knowing was the process of the intellect to know things by means of their common features. It is a discursive process. The formation of concepts takes place through this process of abstraction. The conception of the object by concepts disregards the problem of its existence⁶⁹. On the contrary, intuitive cognition gives the object in its singularity, i.e., "in its proper nature." Intuitive cognition involves a relation to what exists right here, right now. We apprehend something as existing by intuition. While by the process of abstraction we can get the concept, the intuition gets in touch with what is real. Intuitive cognition is "an intuition of something as existing and present in its own existence." For Scotus, the impossibility of grasping the object in its individuality was an expression of the imperfection of the human mind. We, human beings, do not have an intuition of the object in its singularity. We do not grasp the object *in individuo* by the process of thinking but just by sensation. Scotus holds:

... the intellect does not know the object as here-and-now because it grasps it in its absolute quidditative form, whereas the senses cannot know the object in this fashion because the power of each is limited to knowing it under the aspect of existing...⁷²

Intuition "must include in itself real and actual relation to the object itself" as:

⁶⁹As Gilson explains: «Il est en effet remarquable que, pour définir la connaissance abstractive (*cognitio abstractiva*), Duns Scot la présente comme faisant abstraction de toute existence actuelle: *cognitio objecti secundum quod abstrahit ab omni existenitia actuali*. Prenons cette formule au pied de la lettre: être «abstraite», pour une connaissance, c'est ne pas inclure l'existence de son objet. Inversement, la connaissance intuitive est celle qui saisit l'objet en tant qu'existant et que présent dans une existence actuelle…» Gilson, E., 1952, p. 425.

⁶⁸ Scotus, D., 1997, p. 193.

⁷⁰ "The first is that of intuitive cognition which is of a thing present, and not just through a species, nor only under a knowable aspect, but in its proper nature." Scotus, D.,1997, p. 197.

⁷¹ *Quad.* 6:19. Scotus, D., 1975, p. 137. Etienne Gilson remarks: «L'intuition seule saisit le réel comme existant.» Gilson, E., 1952, p. 109. As Bérubé explains: «L'intuition nécessairement comporte une relation réelle et actuelle à son objet.» Bérubé, C., 1964, p. 181.

⁷² Quad. 13:32, Scotus, D., 1975, p. 292.

.... there can be no knowledge of this sort unless the knower has to the object an actual relationship that is such that the relata actually exist and are really distinct, and given the nature of the relata the relationship arises necessarily.⁷³

Moreover, what we apprehend in sensation is just the existence. The existence does not belong to the concept of the thing. The thing can be fully determined without existing. To exist is not a property of the thing. However, to know something about the thing, we need a process of abstraction⁷⁴. The abstraction allows the intellect to get the common marks of the thing. For this reason, the sciences deal only with abstractive concepts and not with intuitions of the objects, as sciences do not attend to the existence of the things in their particularity. The existence does not concern the reality of the concept. The concept is built by abstraction, the existence is apprehended by intuition. Within this theory, the concept is always, per definitionem, an abstractive concept. The intuition is immediate and of what is singular and gives the existence of the thing. Thus, the problem of the distinction between abstractive and intuitive cognition came along with the issue of the possibility (or impossibility) of the human intellect of grasping the object in its individuality, and the problem of the existence of particular objects. The problem is that if our mind knows things only through concepts – which are per definitionem abstractivehow does it know singular things? Gilson puts the problem as follows: « l'intellect ne connaît, pro statu isto, que les quiddités abstraites du sensible ; enfin, que les êtres sensibles connus de nous sont des existants singuliers: il est donc inévitable de se demander si et comment l'intellect humain, pro statu isto, connaît le singulier.» 75

Kant inherited this problem from the medieval tradition through the glass of modern thinkers. The Kantian distinction between intuitions and concepts is constructed

⁷³ *Quad.* 13:33, Scotus, D., 1975, p. 292

⁷⁴ "... in the case intuitive knowledge, it is the thing in its own existence that is the per se motive factor objectively, whereas in the case of abstractive knowledge what moves the intellect per se is something in which the thing has "knowable being", whether this be an effect such as the [intelligible] species or likeness that contained the thing of which it is the likeness representationally" *Quod.* 13:33, Scotus, D., 1975, p. 292. Gilson explains: « Seulement, c'est dans la sensation que notre intellect atteint le singulier, et puisque le sens même ne le perçoit pas dans sa différence individuante, mais comme « nature » il ne révèle à l'intellect, du singulier existant, que son existence. Percevant la « nature » indifférente de cet être, le sens permet à l'intellect de connaître abstractivement la nature de ce singulier, et intuitivement son existence. » (...) « Puisqu'elle ne va pas au-delà de la nature indéterminée, la connaissance qu'en prend l'intellect est nécessairement abstraite." Gilson., E., 1952, p. 546.

⁷⁵ Gilson, E., 1952, p. 543.

mainly with and in opposition to the modern use of the terminology⁷⁶. To establish the distinction between intuition and concepts, Kant had to contend mainly with the Cartesian⁷⁷ and Leibnizian⁷⁸ tradition. According to Descartes, intuition is one of the sources of knowledge. Descartes claimed that intuition is an immediate and direct apprehension of simple ideas. Intuition is the faculty of the mind capable of direct and immediate cognition. In the *Rules for the Direction of the Mind*, Descartes defines intuition as follows:

By 'intuition' I understand, neither the fluctuating testimony of the senses nor the deceptive judgment of an imagination which composes things badly, but rather the conceptual act of the pure and attentive mind, a conceptual act so easy and so distinct that no doubt whatsoever can remain about what we are understanding. Alternatively, it amounts to the same thing to say by intuition I understand the conceptual act of the pure attentive mind, which conceptual act springs from the light of reason alone. Because this act is simpler, it is more certain than deduction, which, however, as we have noted above, a human being also cannot perform wrongly⁷⁹.

Intuition does not require any process in order to acquire knowledge. The mind has access to objects directly and immediately by means of intuition. There is no process involved. The mind grasps all at once its object. Intuition is the product of the understanding by means of which the mind is able to form doubtless ideas⁸⁰.

⁷⁶According to Giorgio Tonelli, Kant started introducing new terminology in 1769, mainly, due to his reading on Locke and Leibniz. Particularly, he holds, that the concept of *Anschauung* was rather infrequent in the eighteenth century "Anschauung" (intuitio) wird im 18. Jahrhundert sehr wenig gebraucht. Zwar spricht man gelegentlich von der intellektuellen Anschauung Gottes, aber im Bereich der Psychologia Empirica ist von intuitio sehr selten die Rede. Allein bei Resewitz erhält dieser Terminus einen gewissen Nachdruck. Bei Locke und Leibniz ist er dagegen sehr geläufig als "intuition". Tonelli, G., 1964, p. 233.

⁷⁷ It has been shown that Kant read, at least, the following Cartesian works: *Geometry, Metaphysical Meditations*, and the *Principles of Philosophy*. Cf. Gatto, A., 2017, p. 141.

⁷⁸ Anja Jauernig noted that "with the exception of God, Leibniz is the most mentioned individual in the Kantian corpus overall." Jauernig, A., 2008, p. 41. Manuel Sánchez Rodríguez makes a brief and accurate analysis of how the notion of intuition was appropriated by Wolff and Baumgarten. Cf. Sánchez Rodríguez, M., 2013.

⁷⁹ Descartes, R., Regulae III, AT X p. 368.

⁸⁰ According to the canonical reading, the simple ideas apprehended by intuition are purely intellectual. Cf. Lewis Beck, 1969, p. 192. Caimi, M., 2009, p. VIII. Against this reading, Frederick van de Pitte holds that intuition does not exclude sensory awareness. He holds that the object of intuition is not necessarily purely intellectual. van de Pitte, F., 1988, p. 457.

The intuition grasps what is singular, i.e., what cannot be divided into simpler parts. Intuition provides the simplest elements upon which the intellect operates, making relations among them. Thus, intuitions are the first step in the path of knowledge as the process of knowledge begins with these simple ideas. Having analyzed the idea up to the point when no further distinction can be made, the method prescribes to unite those simple elements into one. We must pass from a simple idea to another simple idea to form a new unity. That is the task of synthesis. The synthesis comes after the analysis, and it operates on the basis of what the intuition provided. The process is secure as long as it retains these simplest elements grasped in the first step.

Starting at the simplest elements, intuition provides clear and distinct knowledge⁸¹. An idea is distinct when it is completely separated from any other, and it is clear when it manifests directly to the spirit. The criterion of clarity ensures that we have a direct and immediate access to the idea. The idea is directly presented to our minds. An idea is distinct when it is completely separated from any other idea. The intuition can be clear even when it is not distinct, but a distinct idea is always clear as we have the possibility to separate every element in the idea just when it is patent to our understanding. Descartes gives the example of the sensation of pain⁸². We have a present and immediate access to the sensation of pain without truly distinguishing it properly. In this case, the sensation is confused. The idea is present but is not precise. There is no intuitive access to the representation.

The distinct and clear ideas grasped by intuition are necessarily true. Thus, when we apprehend intuitively, there is no possibility of error⁸³. When we apprehend by means

-

⁸¹ "I call that clear which is present and manifest to the mind giving attention to it, just as we are said clearly to see objects when, being present to the eye looking on, they stimulate it with sufficient force. and it is disposed to regard them; but the distinct is that which is so precise and different from all other objects as to comprehend in itself only what is clear." Descartes, R., *Principia* I, XLV, AT VIII p. 22.

⁸² "It is shown, from the example of pain, that a perception may be clear without being distinct, but that it cannot be distinct unless it is clear." Descartes, R., *Principa* I, XLVI, AT VIII p. 22.

⁸³In the *Discourse*, Descartes concludes: "I concluded that I could take it to be a general rule that things we conceive of very clearly and distinctly are all true..." Descartes, R., AT VI, p. 33. In the same line, he held in the *Meditations*: "For in this first act of knowledge [cognitione] there is nothing other than a clear and distinct perception of what I affirm to be the case; and this certainly would be insufficient to make me certain of the truth of the matter, if it could ever come to pass that something I perceived so clearly and distinctly was false. And therefore, I seem already to be able to lay down, as a general rule, that everything I very clearly and distinctly perceive is true." Descartes, R., *Med* AT VII p.35 And in his reply to the second objections, we read: "Whatever we clearly understand to belong to the nature of some thing, can be truly affirmed of that thing." Descartes, D. AT VII, p. 150. According to some scholars, the evidence provided by intuition is a sufficient criterion of truth. Caimi, M., 2009, pp. XXVIII, XXXII. Frederick van de Pitte challenges this reading arguing that judgment is the only source of truth. He holds that "intuition is not the source of truth for Descartes, i.e., that while intuition is certainly a necessary condition for truth, it is *not* both the necessary and the *sufficient* condition for truth." van de Pitte, F., 1988.

of intuition, we cannot be deceived. In the *First Meditations*, the intuitive knowledge plays a fundamental role as what the mind grasps by the intuitive evidence marks the limits in the deconstruction of the building of knowledge. The evidence provided by intuition cannot be affected by natural doubt⁸⁴. The intuition provides fully certain and indubitable knowledge, and then this kind of evidence establishes the limits to methodical doubt⁸⁵.

To sum up, the knowledge provided by intuition is the ground of the building of knowledge. The method commands to reach these simplest ideas. The goal is to get as close as possible to those simple elements where no doubt is left. Intuition is the name that Descartes gives to the act of the understanding that reaches those first elements in the construction of the object of cognition. The mind is able to have access to what is real by means of intuition because intuition provides the simplest elements -thus, the most certain- of cognition. For this reason, a concept can have reference to an object just when it is grounded on those immediate and simple elements that the intuition provided. As it was for Scotus, the problem of intuition came along with the problem of the possibility to grasp what is fully determined, the object in its singularity. Moreover, the limits of intuition mark a limit for human understanding. For an infinite understanding could go even further in the distinction up to the point to reveal all the determinations that correspond to the thing. So even when intuition provides self-evident knowledge, it also represents the limitations of a finite intellect. Then, our mind can legitimately be related to what it represents - the Kantian question we posed in the very beginning - insofar it is grounded on what intuition provided. I can claim to be true whatever I perceive clearly and distinctly, i.e., intuitively. However, knowledge operates with concepts. Then, how can we guarantee that those constructions are truly grounded on those secure elements? If our mind, which operates with concepts, is only able to have a reference to what is real by means of intuition, how can we ensure that this truth we grasped is not "lost" in the process? As it has been noted, this can be only grounded on the metaphysical assumption of a non-deceiver God⁸⁶.

⁻

⁸⁴ Cf. Caimi, M., 2009, pp. XLVI.

⁸⁵ The first step of the Cartesian method is: "...never to accept anything as true that I did not *incontrovertibly* know to be so; that is to say, carefully to avoid both *prejudice* and premature conclusions; and to include nothing in my judgements other than that which presented itself to my mind so *clearly* and *distinctly*, that I would have no occasion to doubt it." Descartes, R., *Discourse* AT VI p. 18.

⁸⁶ This point has led to what has been called "the cartesian circle". The problem is whether the hyperbolic doubt affects the first intuitive principles. On one hand, Descartes claims that whatever I perceive clearly and distinctly is true. The first principles, such as the causality principle, are perceived by intuition, clearly and distinctly. Then, we must accept the principle as one of the first steps in the construction of knowledge.

As it was for Descartes, Leibniz also considered that intuition is the cognition of clear and distinct ideas. However, according to him, Descartes had not provided an accurate definition of the notions he was employing ⁸⁷. For Leibniz, the Cartesian account of the concepts of clarity and distinction—and the definition of intuition itself- was neither clear nor distinct. Leibniz provided a more accurate definition of the terms that Kant inherited ⁸⁸.

Leibniz holds that we have primitive ideas that can be decomposed into simple parts. Ideas can be analyzed into simple elements. These simple ideas are clear and distinct. Nature is made up of these simple elements which are "the true atoms of nature; in a word, the elements of things"89. Everything we can find in nature is a composition of these first elements: the monads. Perception is the temporal unity generated among multiplicity 90. It is a temporary state in which we do not necessarily attend to the unities that belong to this higher unity. This state is temporary because it is possible to attend to the unities that compound the higher unity. Intuition is the apprehension of the simple elements that compound the multiplicity which we temporarily perceive as a unity. Clarity and distinction are the marks of intuitive cognition. Cognition is clear when the idea is present to the mind, and the mind is capable of distinguishing it from any other idea. In a clear cognition, we can separate the representation from another representation. Leibniz specifies this definition of the clarity criterion establishing a relation with the principle of non-contradiction: clear representations can always be defined with a non-contradictory definition. It is always possible to give a non-contradictory definition of a clear representation. This possibility of demarcation defines the clarity criterion. However, in clear cognition, I cannot tell apart the determinations that make this idea different from the other. Even when I can claim that they are both different, I cannot determine the difference. While I can state that these ideas are different, I cannot establish how they differ. In this case, my cognition is clear but not distinct. A clear idea can be distinct or confused⁹¹. An idea is distinct when

-

The Cartesian proof of the existence of God relies on the endorsement of such principles. However, on the other hand, the hyperbolic doubt led us to deny the reliability of these first evident principles, which can only be accepted after the existence of God has been proved. Cf. Van Cave, J., 1998, p.101.

⁸⁷ Leibniz, G., G., IV, p. 422.

⁸⁸ Cf. Sánchez Rodríguez, M., 2013, p. 2.

⁸⁹ Leibniz, G., Mon. §3. (G., VI, p. 607)

⁹⁰ "The transitory state which incorporates and represents a multitude within a unity or within a simple substance is nothing but what we *call perception*." Leibniz, G., Mon. §14. (G., VI, p.608) ⁹¹ Leibniz, G., G., IV, p. 422.

the mind can identify the determinations that belong to the representation. In a distinct cognition, the mind is able to discern the elements that truly belong to the thing. Intuition is the apprehension of these first simple elements that correspond to the thing, and this kind of access guarantees that we have true knowledge of the thing and not a mere notion⁹².

The idea is distinct when I can get to these differential marks. In fact, we truly have an idea of the thing- and not a mere confused notion- when we have an intuition of the determinations that belong to the concept⁹³. A cognition is adequate when it is clear and distinct, and I can guarantee that the analysis of the idea has been carried out up to the end. Adequate cognitions are very rare for us, human beings. The limitation of knowledge consists, precisely, in this incapacity to represent distinctly every part of the universe. The level of determinations is a question of degree⁹⁴. Sense perception is just this degree of confusion where I cannot clearly identify the elements that compound my perception. Once I have analyzed the components of the substances and distinguished the parts that belong to them, I have intellectual cognition. The difference between intuitive cognition and intellectual cognition is a question of degree. The representations of the sensibility and understanding have the same root or, more precisely, concepts and intuitions do not come from different origins, but they have the same source. The difference between these two types of perception is the degree they achieve in the determination of the object. The intellect finds the distinctive marks that belong to the thing and turns this confused perception into a distinct one. Leibniz identifies sensibility with obscure and confused cognition and the understanding with distinct cognitions. These two faculties differ in the degree of clarity they can achieve. Actually, they are different degrees of the same function.

-

⁹² Every concept has content. As the concept is always composed of simple elements, it is never empty. An empty concept is not truly a concept but a mere notion, a chimere. There are not empty concepts but those that contain a contradiction. For the Leibnizian conception, all non-contradictory concepts have content and then, all knowledge can arise from them Cf. Caimi, M. 2005, p. 142.

⁹³ "When I can recognize one thing among others without being able to say what its differences or properties consist in, my knowledge is *confused* (...) But when I can explain the evidence I am using, the knowledge is *distinct* (...) But when everything which enters into a definition or an item of distinct knowledge is known distinctly, right down to the primary notions, I call the knowledge *adequate*. And when my mind simultaneously and distinctly understands all the primary ingredients of a notion, it has *intuitive* knowledge of it. This is very rare; most human knowledge is only confused, or *suppositive*." Leibniz, G., Disc., §24. (G., IV, p. 449 ss)

⁹⁴ "...this representation of the details of the whole universe is confused and can only be distinct with respect to a small part of things ..." Leibniz, G., Mon. §60. (G., VI, p. 616ss.) Also: Leibniz, G., Disc., §24. (G., IV, p. 450).

Now, it must be kept in mind that the truth of a proposition is grounded on its agreement with things as they are in themselves. A proposition is true if what is predicated of a subject actually belongs to it⁹⁵. Leibniz's account of intuition is grounded on his theory of substance. According to him, the substance is what is fully determined. The substance is the subject of every predicate that can be attributed to it while it cannot be an attribution of anything else. Nature can be considered a composition of these simple elements: the monads 96 . These simple things – which are the atoms of nature- are fully determined in every respect such that nothing can be added or subtracted from it. The universals are composed of these simple elements created by God. The substance is always perfectly determined. Then, every true proposition that we can hold that belongs to the subject is actually already included in it. As every predicate we can ascribe is already included in the subject, there is nothing that could be added to it. The distinction between the substance and its accidents is just a question of the level we reach in the determination. The accident of a substance is a concept that has not been completely determined⁹⁷. Every concept of an individual substance is an entirely determined concept. The task of thinking is to analyze the subject up to the simple elements. Once we get to those simple determinations that are the proper determinations of the things, the relation among them is resolved by mere calculus. The process of knowledge consists precisely in this process of determinations. The goal is to identify those properties that belong to the thing which, at the very beginning, are presented confusedly to the mind. The mind can go every time further in the analyzes so that those elements that were clear but not distinct can be determined and turned into distinct apprehensions. To enumerate all the determinations to be found in substances is what is demanded. Each new determination demands to be brought to clarity and distinction, and complete determination is the eternal task of thinking. Reasoning is precisely the act of the mind by which it discovers the intermediate ideas that make it possible to claim that a certain determination actually belongs to the thing 98.

Space is a determination required to individualize things. It is a determination that makes it possible to differentiate between a point and another. As, if these points were

.

⁹⁵ "Now it is obvious that all true predication has some foundation in the nature of things." Leibniz, G., Disc., §8. (G., IV, p. 432 ss.).

⁹⁶ Leibniz, G., Mon. §1. (G., VI, p. 607).

⁹⁷ Leibniz, G., Disc., §8. (G., IV, p. 432 ss.).

⁹⁸ The analysis is the "art of finding intermediate ideas" Leibniz, *New Essay*. (G., V, p. 348).

not differentiated in space, they would be the same point. But this contradicts the principle of sufficient reason and the principle of the best world possible. There would be no explanation of why God put this point in one place and not in any other. Space (and time too) is just a phenomenical determination of the substance. Leibniz maintains a relational conception of space according to which space is nothing more than relationships between substances that can be established and determined by rational analysis, without any intervention of sensibility. We perceive it as sensible just because we perceive it confusedly. Space and time can be reduced to relational intellectual properties of the things, as they are just relations among substances that can be perfectly analyzed by the intellect. Space is the order or relation of the coexistent parts of the universe. Space comes into existence at the same point that the parts of the universe are created. It cannot preexist them. Then, real things cannot be differentiated by their special location, as the spatial relations are nothing but the relationships among them. Now, the relation among things is established by the pre-established harmony as monads have no "windows". The relation among substances is established at the same time that the universe is created. The spatial relations among things are just confused perceptions of non-spatial internal properties of monads. Therefore, the spatial location of substances cannot be considered a distinctive mark of it that would make it possible to distinguish two equal substances. As a substance is what is completely determined in every aspect, there cannot be two substances perfectly alike. When we consider two things as equal, it is just because we have not fully analyzed the concept. If after being completely analyzed these two things share all their properties, they are actually the same thing as "nowhere is there perfect similarity"99. Then, if two things share all their determinations, they are, in fact, the same thing 100.

To sum up, on Leibniz's account, nature is a compound of these simple elements which we can reach when we have discovered all the determinations that truly belong to

.

⁹⁹ "nowhere (and this is one of my important new axioms) is there *perfect* similarity." Leibniz, G., Nature Itself §13. (G. IV, p.514)

beings that are perfectly alike, and between which it is not possible to discover some difference which is internal, or founded on an intrinsic denomination." Leibniz, G., Mon, §9. (G., VI, p. 608) In his *New Essay...*, Leibniz introduces the principle of the identity of indiscernibles as follows: "Every substantial being, be it soul or body, has its relation to every other substantial thing, which is peculiar to itself; and one must always differ from another by intrinsic denominations." (G., V, p. 100)). (...) "By virtue of insensible variations, two individual things can never be perfectly alike . . . and they must always differ more than numero. This at once puts out of court . . . a substance without action, the void in space, atoms and even particles not actually divided in matter, absolute rest, complete uniformity in one part of time, place, or matter." (G., V, p. 49).

the thing. Substances are individuated by their intrinsic properties. Therefore, an exhaustive analysis of its determinations would make it possible to know what they are. Intuition is the fulfilment of conceptual analysis. That is to say, there is no methodological difference between the cognition of universal principles and the cognition of particulars. On the contrary, there is a line of continuity between conceptual analysis by which we firstly determine universal properties of things, and the specification of those principles in an every time more definite determination of things that, ultimately, leads us to know things as they are in themselves. The intuitive cognition is just the accomplishment of the conceptual analysis, as the individuality rests on pure rational principles: the principle of contradiction (in regard to its logical determinations) and the principle of sufficient reason (in regard to its physical determinations). Returning to the central question of this thesis, i.e., how the mind can legitimately relate to what is real, we can conclude: first, Leibniz defined as eminently real what is perfectly determined. Then, conceptual analysis gives us the possibility of knowing things as they are in themselves. The principle of noncontradiction guarantees that we can have access to pure rational truths while contingent truths, such as those discovered in physics, are guided by the principle of sufficient reason. Now, contingent truths are just contingent for us. As every predicate necessarily belongs to the thing, all contingent truths are necessarily true from the point of view of things as they are in themselves insofar, they are grounded in the principle of non-contradiction and the principle of sufficient reason. There are not really accidental predicates of things, neither of them nor in the relations among them. The internal properties define the reality of each individual, and the relationships among these individuals are already established by the principle of sufficient reason. Now, there are many problems involved in Leibniz's account. In first place, his conception of the arrangement of nature is, as it was for Descartes, grounded on the assumption of a free willing God who created the world based on the principle of the best world possible. Secondly, his proposal depends on the not so well-argued conception of reminiscence. For Leibniz, our determinations of things are true knowledge and not mere chimeras because we have innate ideas which were introduced in us by a non-deceiver God. Thirdly, the principle of the indiscernible was rather problematic. This is one of the points of departure of the Kantian rupture with rationalism: the distinction between intuition and concept formation. One of the first attempts to establish a new distinction between intuitions and concepts can be found in On the First Ground of the Distinction of Directions in Space.

1.3. The Problem of Incongruent Counterparts

On the first foundation of the directions¹⁰¹ in space (1768), Kant introduces for the first time the paradox of the incongruent counterparts¹⁰². Kant drew different conclusions from this argument¹⁰³, that it is present throughout the entire Kantian work. The purpose of the introduction of the counterpart phenomenon in this paper of 1768 is to argue in favor of the Newtonian conception of absolute space. However, it is traditionally admitted that from 1770 on, Kant uses the phenomenon of counterparts to sustain that objects, singular things, cannot be determined by mere concepts. Conceptual determinations do not lead the mind to a complete determination of the object of cognition. The argument is used to prove the insufficiency of conceptual determination for a complete determination of objects. This argument will imply a rupture of the Kantian position with Leibnizian rationalism and will lead Kant to elaborate a new relationship of intuitive and conceptual representations. The notions of intuition and concept will be redefined. These new definitions will be the basis of the critical system that begins to be shaped in 1763 and acquires an increasingly elaborated expression during the "silent decade." ¹⁰⁴

1

¹⁰¹ David Walford, Paul Rusnock, and George Rolf hold that the term "Gegend" should be translated as "direction" rather than as "region." David Walford, who makes an exhaustive study of the difference between the concepts of *Gegend* and *Lage*, considers that in no way the term *Gegend* can be understood as a region. Walford considers that the confusion between these concepts would have led to a misunderstanding of the counterparts argument. Walford, D., 2001, pp. 409ss. Rolf, George y Rusnock, Paul, 1994. pp. 459ss.

¹⁰² James Van Cleve argues that Kant was the first philosopher to notice the importance of the problem of counterparts. He states: "Kant was evidently the first major thinker to notice the philosophical significance of such objects." Clave, J., van, 1999, p. 44. According to Paul Rusnock and George Rolf, Kant would have been familiar with this paradox since 1762/1763 and the first attempted solution in 1964. Rolf, George y Rusnock, Paul, 1994. p. 466. Rolf, George y Rusnock, Paul, 1995. p. 263.

¹⁰³ Cf. Vaihinger, 1892, p. 518 ss., esp. p. 523ss., Bennett, J., 1970, p. 175. Buroker. J. V., 1981, p. 3ss. Earman, J., 1991, p. 235. Kemp Smith, N., 1991, p. 45. Clave, J., van, 1999, p.44. Smith, K., 2003, p. 161ss. Hagar, A., 2008, p. 82. According to Kemp Smith, the argument of 1768 seeks to demonstrate that space is absolute, in 1770 that it is intuitive, and in 1783 that it is subjective. For Jill Vance Buroker, the 1770 presentation shows that space is a pure intuition. In 1783 and 1786, Kant would use the paradox to support the transcendental ideality of space. For Hans Vaihinger, in 1770 the paradox is resolved by accepting that space is a pure intuition and not a concept and in 1783 affirming that it is a form of intuition. For Jame van Clave, on the contrary, the presentation of 1770 and 1783 seeks to prove that the representation of space is intuitive. For John Earman, Kant uses the argument in 1768 against the Leibnizian conception of space; in 1770, to show that space is intuited and in 1783 and 1786 to sustain transcendental idealism. Smith, K. 1991, p. 45. Smith, K. 2003, p. 161ss., esp. pp. 164,165. Cleve, James van, 1991, p. 15. Burocker, J., 1981, p. 68. Vaihinger, 1892, p. 523. Earman, J., 1991, pp. 235,249. Clave, J., van, 1999, p. 44. Pippin considers that "The assumption of an absolute frame of reference (or space as a singular whole) seemed to him unavoidable," (...) "all we need note here is how crucial it was in turning him away from the Leibnizian view once and for all.") Pippin, R, 1982, p. 61.

¹⁰⁴ Jill Vance Burocker emphasizes that "the key to transcendental idealism is a series of arguments that appear in Kant's writings from 1768 to 1786, the author adds that the argument considered here "points out not only a radical change in Kant's thinking about space, but it is also the prelude to critical philosophy."

In his mature formulations, Kant explicitly uses the argument of the incongruent counterparts to show that the determination of the phenomenon cannot be carried out only by means of concepts¹⁰⁵. Kant shows that there are object determinations that cannot be elucidated by purely conceptual means. Specifically, the location of the phenomenon in space and time cannot be obtained by analyzing their intellectual marks. The spatiotemporal determinations cannot be obtained through an analysis of the conceptual marks of the phenomenon. Then, a complete determination can never be achieved through concepts. The argument will show that the reference of the mind to the object *in individuo* can only be guaranteed by a non-conceptual factor.

Kant begins the argument with a definition of equality, similarity, and congruence. Kant's point of departure is to be found in the definitions provided by Leibniz¹⁰⁶, followed by Wolff ¹⁰⁷ and Baumgarten¹⁰⁸.

_

Burocker, J., 1991, p. 316. Also: Burocker, J., 1981, p. 3. Robert Pippin shares this reading. He considers that by the argument of the counterparts, "Kant became convinced that a wholly relational view of space could not be defended, and, while for a time appearing to resort to some more Newtonian view, began his own search for a satisfactory solution short of the postulation of a metaphysical *Unding* like absolute space. The results of that search first appeared in their new critical form in his 1770 *Dissertation*, and a great deal of the case made there is preserved in the *Critique*. The assumption of an absolute frame of reference (or space as a singular whole) seemed to him unavoidable" Pippin, R, 1982, p. 61. Following this line, Brigitte Falkeburg states: "Kant's theory of intuition emerged from an intriguing puzzle concerning the mathematical foundations of his pre-Critical cosmology, the puzzle of incongruent counterparts. [...] Thus genetically, Kant's theory of intuition cannot be separated from his 1768 paper on incongruent counterparts." Falkenburg, B., 2006, p.157-158. Ezequiel Zerbudis challenges this interpretation. Zerbudis holds: "in contrast to what many scholars have supposed, there seems to be nothing in Kant's original presentation of the phenomenon of incongruous counterparts that should be taken as an indication of the need to postulate a separate intuitive faculty, which would be necessary for someone to be able to know the difference between a figure and its counterpart". Zerbudis, E., 2012, p. 327.

¹⁰⁵ The purpose of the argument in its mature presentation "is to show that there are characteristics of the phenomena that can only be known with sensibility; since they are inaccessible to the purely conceptual approach" Caimi, M., 1999, p. 111. Mario Caimi emphasizes that this is the purpose of the argument in the *Prolegomena*, taking into account that Kant's aim in the presentations of the argument in 1769 and 1789 is not so clear. Mario Caimi and Kemp Smith consider that the clearest presentation of this point is only reached in the *Prolegomena*. Smith, K. 2003, p. 163. Caimi, M., 1999, p. 111. On the contrary, Amit Hagar considers that already in the *Dissertatio* "Kant uses the idea of incongruent counterparts to illustrate (and not to prove) the intuitive character of spatial knowledge". Hagar, A., 2008, p.82.

¹⁰⁶ Paul Rusnock and George Rolf explain that Kant took the notions of "congruence", "equality" and "similarity" from the system of Leibniz. The technical sense of these terms should be understood in the light of the Leibnizian system: "Figures which have the same inner characteristics are called similar. Figures are congruent when capable of being moved to coincide, or when they differ at most by being in a different place (solo *numero*). Equality is still simply equality of magnitude. Leibniz believed congruence to be definable as the conjunction of similarity and equality." Rolf, George y Rusnock, Paul, 1995. p. 261.

¹⁰⁷ "When I can put one thing B instead of another A without prejudice to the magnitude, then it is that A and B are equal. I say without prejudice to the magnitude, that is, if substituting A for B is, in terms of magnitude, the same as if I had kept A." Wolff, C., 1719, §22. For Wolff, congruence is equality of what is similar. Wolff, C., Ont, §465.

¹⁰⁸ "Things that are the same according to quality are SIMILAR (\sim); according to quantity, EQUAL (=); according to both, CONGRUENT (@). Things that are different according to quality are DISSIMILAR (L); according to quantity, UNEQUAL (\neq); according to both, INCONGRUENT." Baumgarten, G., *Met*, §70.

According to the Kantian presentation, two objects are congruent when they share all their determinations. Even in the *Prolegomena*, Kant keeps the conceptions of congruence introduced by the Leibnizian tradition. Two things are congruent when they share all the determinations in relation to quality and quantity. When they share all the quantitative determinations, they are equal, and when they share all the qualitative determinations, they are defined as similar. If two figures are congruent, they should be able to completely cover each other. One of the figures must be able to fully replace the other without this substitution altering in the least any of the properties of the thing. Kant observes in the *Prolegomena*:

If two things are fully the same (in all determinations belonging to magnitude and quality) in all the parts of each that can always be cognized by itself alone, it should indeed then follow that one, in all cases and respects, can be put in the place of the other, without this exchange causing the least recognizable difference¹⁰⁹.

.

If two things are equal and similar, they are congruent, and then one of them should be able to replace the other. Kant holds that this is an *a priori* synthetic proposition grounded on the pure intuition of space¹¹⁰. However, the phenomenon of incongruent counterparts reveals that two figures can share all their internal properties, and yet they are not interchangeable. These figures are incongruent counterparts. The incongruent counterparts are defined as follows:

When a body is perfectly equal and similar to another, and yet cannot be included within the same boundaries, I entitle it the incongruent counterpart of that other¹¹¹.

.

^{109 &}quot;Wenn zwei Dinge in allen Stücken, die an jedem für sich nur immer können erkannt werden, (in alien zur Grösse und Qualität gehörigen Bestimmungen) völlig einerlei sind, so muss doch folgen, dass eins in alien Fällen und Beziehungen an die Stelle des andern könne gesetzt werden, ohne dass diese Vertauschung den mindesten kenntlichen Unterschied verursachen würde." *Proleg*, AA 4: 285.

^{110 &}quot;Um etwas zur Erläuterung und Bestätigung beizufügen, darf man nur das gewöhnliche und unumgänglich nothwendige Verfahren der Geometern ansehen. Alle Beweise von durchgängiger Gleichheit zweier gegebenen Figuren (da eine in allen Stücken an die Stelle der andern gesetzt werden kann) laufen zuletzt darauf hinaus, dass sie einander decken, welches offenbar nichts anders ais ein auf der unmittelbaren Anschauung beruhender synthetischer Satz ist…" *Proleg*, AA 4: 284.

¹¹¹ "Ich nenne einen Körper, der einem andern völlig gleich und ähnlich ist, ob er gleich nicht in eben denselben Grenzen kann beschlossen werden, sein incongruentes Gegenstück." AA 2: 382.

Congruence is defined as similarity of what is equal. Congruent things are capable of being enclosed in the same limits. We can move them and make them coincide. The paradox will be that two figures can be equal and similar without being congruent. The problem is that there are objects that share all their determinations and, however, one of them cannot be put in the place of the other. These objects have certain determinations that make them different which are not conceptual marks.

An example of incongruent counterparts is that of the spherical triangles of two opposite hemispheres¹¹². The triangles of each hemisphere can be congruent with respect to their sides and angles and, however, cannot be enclosed within the same boundaries¹¹³. Kant argues that the determinations and the relationships among them are equal. However, the triangle on one side of the hemisphere is not interchangeable for the one on the other side. One triangle cannot occupy the same place in the space occupied by the other. A complete description of their determinations is insufficient to specify this difference. In this case, it is shown that even when there is a difference between the figures, it is impossible for the understanding to apprehend it. The understanding cannot give an account of this difference as there are no internal differences that explain the fact that one figure cannot take the place of the other¹¹⁴. The construction of two triangles on a spherical surface shows that the figures corresponding to each of the hemispheres can be equal with respect to the marks that define them and not be congruent with each other¹¹⁵. Thus, the paradox is that being these figures completely equal and similar, they are not congruent. This demonstrates that mere concepts cannot define geometric knowledge, i.e., "there is geometric knowledge that cannot even be described by concepts"116.

Kant takes in the *Prolegomena* an example that he had already presented in *On* the first foundation ... and in the *Inaugural Dissertation* of 1770. The example is based on the possibility of distinguishing the right hand from the left hand. Both are equal in

-

¹¹² It is interesting to note that the example of the triangles is precisely the example that Wolff introduces to define congruence. Two triangles are congruent, if they are similar and equal and if they are congruent one should be capable of being moved to the place of the other. Wolff, C., Ont, 465. Mario Caimi and Rogelio Severo explain that the argument does not work for equilateral triangles. Cf. Caimi, M., 1999, p. 335. Severo, R., 2007, p. 519.

¹¹³ In one of his early works, Bertrand Russell uses this Kantian argument to argue that the concept of magnitude cannot be applied to space. He states: "The same irreducibility of space to mere magnitude is proved by Kant's hands and spherical triangles, in which a difference persists in spite of complete quantitative equality". Russell, Bertrand; 1956, p. 86n.

¹¹⁴ Proleg, AA 4: 285.

¹¹⁵ AA 2: 403.

¹¹⁶ Torretti, R., 1974, p.28.

regard to their parts. One could give a complete description of each of them while it still would be missing one feature that makes one hand different from the other. The understanding cannot tell any difference where there is one: one hand is left-orientated, the other is right- orientated. One hand cannot take the place of the other hand as the space that encloses the boundaries of one cannot enclose the opposite. The right and left hand is one of these cases in which, even when the objects are equal in their extension, they are not congruent with each other¹¹⁷. We will get the same result if we consider the right hand or the left hand as they are reflected in the mirror. Even if we can make a complete description of each of them, of our own hand and of the hand that is reflected, it would not be possible to establish through this characterization of their properties a distinctive mark that allows us to differentiate the original hand from the hand in the mirror. There are certain features of the object – as its spatial orientation- which are not revealed by analyzes of its marks. These solids or these figures, even though they are perfectly equal, cannot be substituted. Another example introduced by Kant is that of spirals that have opposite directions. In this case, as in the other cases, the conceptual determinations are insufficient to specify the difference present between the counterparts. The difference between spirals in the opposite direction, "we cannot make it intelligible by any concept whatsoever." ¹¹⁸ The exhaustive analysis of the marks of the spiral figures is insufficient to indicate their directionality. The orientation is not contained in the concept.

In the *Metaphysical Foundations of Natural Science* (1786), Kant presents the problem of incongruent counterparts in relation to motion and direction. Kant analyzes the case of a body moving in a circle. It changes direction as its movement continues; so that at one moment it goes to one side and then to the other¹¹⁹. The movement always follows the same direction but the side of the plane towards which it is moving changes. Then, the question is how to determine the side towards which the movement is directed. It should be possible to establish the difference between the movement towards one side and the other. This difference is not intelligible by mere concepts. There are no general marks that allow us to establish the direction of the movement. The discursiveness of

_

¹¹⁷ Proleg, AA 4: 286.

¹¹⁸ Proleg, AA 4: 286.

^{119 &}quot;In jeder Bewegung sind Richtung und Geschwindigkeit die beiden Momente der Erwägung derselben, wenn man von allen anderen Eigenschaften des Beweglichen abstrahirt. Ich setze hier die gewöhnliche Definition beider voraus; allein die der Richtung bedarf noch verschiedener Einschränkungen. Ein im Kreise bewegter Körper verändert seine Richtung continuirlich, so daß er bis zu seiner Rückkehr zum Punkte, von dem er ausging, alle in einer Fläche nur mögliche Richtungen eingeschlagen ist, und doch sagt man: er bewege sich immer in derselben Richtung, z.B. der Planet von Abend gegen Morgen (...) Allein was ist hier die Seite, nach der die Bewegung gerichtet ist?" *Proleg*, AA 4: 483.

understanding is insufficient to trace this difference. The problem is that for the understanding the two movements correspond in all aspects and then, they are identical from this perspective. However, there is "a genuinely mathematical internal difference"; the movements differ in their direction¹²⁰. The problem is how to explain this difference that the understanding cannot trace. The direction of motion is only intelligible intuitively. There is no problem here of greater or lesser darkness of the representation. It is not possible to explain the direction by means of the marks that make up the moving object. Space, Kant concludes, is not a property or relation of things but the pure form of intuition¹²¹. Space must be considered as a subjective form of our sensible intuition. Congruence is defined as the perfect equality and similarity in the determination of the object, which is only achieved through intuition¹²². Thus, the problem of counterparts, in its critical formulation, is introduced in direct connection with the requirement of a determination of the object that cannot be obtained conceptually. As we will further develop in chapter four, this conclusion depends on the peculiar definition that Kant gave of the notion of concept.

The determination of the object in a univocal way implies the possibility of distinguishing it from any other. However, Kant showed that an object can share with another all its intellectual determinations and still not be the congruent to it. But neither the determination of its parts nor the relation among them can explain why one object is different from the other. This is the case of the incongruent counterparts. Kant's incongruent counterparts argument shows that complete determination requires the individualization of space and time¹²³ but the determination of a unique place in space

1

¹²⁰ "keinen erdenklichen Unterschied in den innern Folgen geben kann und demnach ein wahrhafter mathematischer und zwar innerer Unterschied ist, womit der von dem Unterschiede zweier sonst in allen Stücken gleichen, der Richtung nach aber verschiedenen Kreisbewegungen, obgleich nicht völlig einerlei, dennoch aber zusammenhängend ist." *Proleg*, AA 4: 484.

¹²¹, Ich habe anderwärts gezeigt, daß, da sich dieser Unterschied zwar in der Anschauung geben, aber gar nicht auf deutliche Begriffe bringen, mithin nicht verständlich erklären (dari, non intelligi) läßt, er einen guten bestätigenden Beweisgrund zu dem Satze abgebe: daß der Raum überhaupt nicht zu den Eigenschaften oder Verhältnissen der Dinge an sich selbst, die sich nothwendig | auf objective Begriffe müßten bringen lassen, sondern blos zu der subjectiven Form unserer sinnlichen Anschauung von Dingen oder Verhältnissen, die uns nach dem, was sie an sich sein mögen, völlig unbekannt bleiben, gehöre." *Proleg*, AA 4: 484.

¹²² "Die völlige Ähnlichkeit und Gleichheit, so fern sie nur in der Anschauung erkannt werden kann, ist die Congruenz." *Proleg*, AA 4: 493.

¹²³ Henry Allison argues that Kant does not have an analogue of the problem of the counterparts for time. Allison, H.,1992, p. 168. According to Hans Reichenbach the problem of counterparts does not arise at all in the case of time. To refute this consideration, John Earman argues in this way: "The temporal analogue of a spatially extended figure would be a temporarily extended figure, for example, a temporal type vector." James Van Cleve also argues that the problem remains in the case of time. He argues that: "If you saw a movie or a micro-event from back to front, you would not be able to distinguish that something was not the same." James van Clave, R. Frederick, 1991, pp. 17 and 143. Sean Walsh holds that the problem of

and time is never reached by means of concepts. As it was shown, the conceptual marks are insufficient to identify spatial and temporal determinations. The spatio-temporal determinations required for the complete determination are not conceptual but intuitive. According to Kant, the possibility of determining the object in a unique way requires a factor that is not conceptual. The complete determination is never achieved through concepts. The possibility of satisfying the requirement of a complete determination of the object requires a non-conceptual factor, namely, intuition. The establishment of space and time as intuitions is introduced to make possible a unique determination of the object. Objects are individuated by means of intuitive representations: space and time.

This argument introduced in 1768 is one of his first attempts to explain how our imperfect thinking reaches what is fully determined. As we have noted, the need to introduce the intuitive factor in the process of cognition came along with the problem of the possibility of determining the object in its concreteness. Then, even when it is clear that Kant presented the argument with different formulations and reached different kinds of conclusions, the problem that he is trying to solve is the same: how can thinking have access to the object in its singularity? The postulation of absolute space or the distinction of faculties are just different attempted solutions to the same problem. The assumption of the two-faculty account of cognition was the definite answer that Kant found to this problem introduced in 1768. As we have exhibited, it was the problem that Leibniz introduced when he presented the principles of the identity of the indiscernible, and that was also present in Scotus formulation of the distinction between intuitions and concepts. As we have seen, in 1768, Kant had already in mind that objects can be individualized when they are specially located and, the determination of their special location cannot be done just analyzing its internal properties 124.

However, unfortunately, in 1768, Kant still lacked an accurate definition of the notions of concept and intuition. He uses these notions as he inherited them from medieval and modern philosophy. Concepts are abstractions of common marks of objects while intuition is the determination of singularity. The problem is that when Kant introduced the distinction in his *Critique of Pure Reason*, he did not give a proper account of the

_

temporary counterparts is found in *The First Foundations of Nature Science*, where Kant introduces the problem of movement. Walsh, Sean, 2007, esp., p. 421.

As Pippin explains: "Indeed, contrary to Leibniz's principle of identity, such bodies *were* individual bodies at all only by their already being spatially located." Pippin, R, 1982, p. 62.

definition of these two notions. As we showed, there seems to be a general agreement that Kant just introduced these terms without a proper clarification of what he properly meant.

Intuitions and concepts are different ways of giving unity to the diverse. Intuition is the unity of the manifold that sensibility can provide; the concept is the unity that the understanding provides. The unity of the concept is a product of the understanding, while the unity of the intuition is a product of sensibility. Intuitions are singular representations while concepts are universal representations. All our knowledge, as representations that refer to an object with consciousness, are intuitive or conceptual representations. Thus, all our cognitions are either intuitions or concepts. Intuition is a singular representation. The concept is a universal representation. The concept is a universal representation because it is a representation that is generated from what is common to all objects that fall under it. The concept is generated by abstracting what is common in many objects ¹²⁵. Then, "if a representation is not a common representation: then it is not a concept at all."126 On the contrary, intuition is a representation of singular things. Intuitive representations give us the singular object and then allow us to obtain completely determined knowledge. The complete determination can only be given by the individual object, because "only singular things or individuals are completely determined." ¹²⁷ Therefore, the possibility of completely determined knowledge is sensibility only possible as an intuitive representation; that is, "there can only be totally determined knowledge as intuitions (not as concepts)." 128 Thus, in regard to intuitions, the logical determination can be complete, but "in regard to the concepts, the logical determination can never be considered as achieved" 129.

Thus, it is clear that independently of the way in which Kant characterizes the peculiarity of intuitive representations, the faculty of intuitions is introduced so as to explain the way in which thinking relates to singular real things. As we will see, the peculiarity of the Kantian distinction will be grounded in a brand-new way of conceiving

[&]quot;The genus is representation in general (*repraesentatio*). Under it stands the representation with consciousness (*perceptio*). A perception that refers to the subject as a modification of its state is a sensation (*sensatio*); an objective perception is a cognition (*cognitio*). The latter is either an intuition or a concept (*intuitus vel conceptus*). The former is immediately related to the object and is singular; the latter is mediate, by means of a mark, which can be common to several things." Kant, I., *KrV*, A320 /B376-7.

^{126 &}quot;wenn eine Vorstellung nicht repraesentatio communis ist: so ist sie gar kein Begriff." Kant, Ak XXIV, p. 908.

¹27 AA 9: 99.

¹²⁸ AA 9: 99.

¹²⁹ AA 9: 99.

the limitations of knowledge. This new approach to the issue will clarify the particular problem that the relation of concepts to intuitions implied.

1.4. The Reference to the Given as a Consequence of our Finitude

We hold that the passivity of intuition is one of the expressions of the finitude of human thought. The reference of intuition to affection is a consequence of the imperfection of our knowledge. To argue in this direction, we will study the fourth observation to Transcendental Aesthetics and Kant's correspondence with Marcus Herz.

1.4.1. Original Intuition and Derivative Intuition in the Fourth Observation to Transcendental Aesthetics.

In the fourth observation on the Transcendental Aesthetic, Kant claims:

IV. In natural theology, where one conceives of an object that is not only not an object of intuition for us but cannot even be an object of sensible intuition for itself, one is careful to remove the conditions of time and space from all of its intuition (for all of its cognition must be intuition and not thinking, which is always proof of limitations). But with what right can one do this If one has antecedently made both of these into forms of things in themselves, and indeed ones that, as *a priori* conditions of the existence of things, would remain even if one removed the things themselves? - for as conditions of all existence in general they would also have to be conditions of the existence of God. (B72)

Kant begins his remark on the Transcendental Aesthetic introducing the theological problem that would cause a realistic conception of space and time. He poses a dichotomous position: either space and time are objective forms of all things, or they are subjective forms of our sensible intuition. If space and time are conditions of things in themselves, they would also be conditions of the existence of God. Then, in order not to condition the divine existence, space and time must be considered subjective forms of our intuition. Thus, the first part of this observation begins with the warning that if space and time are made forms of things in themselves, then God himself would fall into the form of space and time. The beginning of this fourth remark led some scholars to consider that the central issue of this section is a theological problem. For Vaihinger, for example, the crucial point of observation is to confirm the doctrine of the ideality of space and time. In order to prove this point, Kant would have introduced a problem of the philosophy of

religion. The question that must be answered is: "How does God behave in regard to his existence and his way of knowing in relation to space and time?" ¹³⁰. The general goal of the observation is, according to Vaihinger, to attack a realistic conception of space using a problem of the philosophy of religion. According to Kemp Smith, in the fourth observation, Kant continues the arguments against Newtonian realism. Kant introduces the theological problem that "If space and time condition all existence, they will condition even divine existence, and so must render God's omniscience, which as such must be intuitive, not discursive, difficult of conception¹³¹." However, we consider that the main point of this section is to be found in the second part of its formulation. According to this interpretation we propose, the question that Kant presents is not only related to a theological problem, but also to an epistemic one. The philosopher explains the relation between intuition, sensibility, and affection that had been introduced in the first paragraph of Transcendental Aesthetics. Kant follows the exposition in this fourth remark explaining why our intuition has a relation to affection. The finite intuition:

...is called sensible because it is not original, i.e., one through which the existence of the object of intuition is itself given (and that, so far as we can have insight, can only pertain to the original being); rather it is dependent on the existence of the object/ thus it is possible only insofar as the representational capacity of the subject is affected through that. (B 72)

Kant explains in this passage why human intuition is related to affection. Our intuition is related to affection because it is a finite intuition. Our intuition is sensible since it is not original. The intuition of human beings is a derivate intuition. For this type of intuition, the existence of the object cannot be produced by the mind. The existence of the object of this intuition is not posited by thinking. On the contrary, this intuition depends on the

_

¹³⁰ Vaihinger, H., 1892, p. 505.

¹³¹ Kemp Smith, 1918, p. 159 ss. Furthermore, it is noteworthy that commentators who focus on the study of Transcendental Aesthetics have not given relevance to this section. They just analyze these passages superficially. Lorne Falkenstein, in his famous study of intuition in Transcendental Aesthetics, does not dedicate any line to comment on this passage of Aesthetics, even though his book is devoted to a study of the concept of intuition in this section. Likewise, the article by Charles Parson only makes reference to this section in a footnote without further development. Georg Mohr analyzes the concept of intuition in this fourth observation, but according to him, there is no introduction of conceptual novelties here, but responses to the detractors of the first edition. Falkenstein, L., 1995; Mohr G., 1998, pp. 122, 127. Parson, C., 1998.

object and, it is precisely for this reason that the cognitive power endowed with a finite intuition requires that the object affects it. Dependent beings are not able to produce the objects of experience. The original intuition, by contrast, is characterized in this passage as one that does not depend on the object to have a representation of it. For an original thinking, the existence of the object does not require anything but itself. This type of intuition is characteristic of the original Being while for finite thinking beings, intuition is always derivative, i.e., dependent. Finite cognition depends on the object to conform its experience. If the mind were capable of producing the object, it wouldn't require a relation to affection. The recognition of the role of affection is the acknowledgment of our finite condition. In this way, Kant explains the relation between intuition, sensibility, and affection. As Kant noted in the first paragraph of the *Critique*, not every intuition has a necessary relation with sensibility and thus with affection. Kant claimed at the very beginning of the Aesthetic that an object must be given "to us humans." Here, he clarifies his point: intuition is sensible insofar it is not original but derivative. Being a derived intuition is for Kant to be dependent on intuition. This intuition "[is not] such that the existence of the object of intuition is given by it." Derivative intuition requires that an object affects it; that is, "it is possible only because the representative capacity of the subject is affected by it" (B74). Thus, each type of intuition corresponds to one of the two different types of intellect. The intellect of the original Being has an original intuition. The intellect of a dependent being has a derivative intuition. Kant clarifies that the dependent beings can moreover be differentiated in regard to the form of their sensibility. Space and time are forms of human intuition but there could be sensible intuitions with other forms of sensibility. However, this does not affect this feature of the dependent intuitions. All entities that are not independent, such as God, have a sensible intuition. For dependent entities, there is no possibility of intellectual intuition. Therefore, Kant affirms that "all finite thinking beings must necessarily agree with human beings in this regard (though we cannot decide this)..." The original intuition only corresponds to the original Being as long as it is independent. Human intuition is sensible "it is derivative (intuitus derivativus) and not original (intuitus originarius)." In this way, Kant states that the object must be given to us by means of affection because our intuition is proper of dependent beings. As the finite intellect is unable to produce the existence of objects, the intellect requires a receptive faculty to produce its representations. In this way, Kant determines that affection is a necessary element for the formation of representations by a peculiar way of conceiving the limitations of finite thinking. Here the comparison with the divine intellect is used to specify the peculiarity of human intuition. This observation presented in Aesthetics had already been developed by Kant. To show this, we will analyze this contraposition between the finite and the infinite intellect as it is posed in the letter of Kant to Marcus Herz of 1772.

1.4.2. The Contrast of *Intellectus Archetypus* and *Intellectus Ectypus* in Kant's Letter to Marcus Herz of February 21, 1772

The exchange of correspondence between Kant and Marcus Herz is one of the richest within the Kantian epistolary¹³². Particularly, the letter sent by Kant to his disciple on February 21, 1772, has been studied by numerous interpreters of his work, as it is considered that this letter exhibits the Kantian critical turn. However, it is discussed what position should be attributed to the letter within the Kantian system. For Cassirer, this document marks "the true hour of birth of the Critique of Pure Reason." 133 Kirk Dallas Wilson considers that the typical critical distinction between intuition and concepts "emerges from the important letter to Marcus Herz of February 1772 in which Kant first raised the critical question." Wolfgang Carl also understands that the letter anticipates the developments of the *Critique*. Carl argues that the task of deduction of the categories carried out in the KrV is defined here 135. This document shows a rupture with the Dissertation of 1770, especially by the exclusion of the possibility of an intellectual intuition. According to de Vleeschauwer, on the contrary, the epistle has been traditionally misunderstood. What is reflected here is a balance of the past and not a program. The text "begins with a retrospective view from the Dissertatio" ¹³⁶. Lewis Beck also considers that there is no reason to see in the letter the outline of a project that anticipates the future developments of the Kantian program. Beck argues against Wolfgang Carl. He concludes that, contrary to what Carl thought, the rupture between the Kantian Dissertation of 1770 and the Critique of Pure Reason is after 1772, and not earlier¹³⁷. Andrés Lema Hincapié holds that the letter includes both: "critical anticipations and mere dogmatic repetition" ¹³⁸. We will not go into the numerous controversies raised by this correspondence. We will focus on the function that the distinction between an ectype and an archetype intellect plays in this letter, as it reveals that the way in which

-

¹³² Cf. Zweig, Arnulf, 1999, p. 3.

^{133 &}quot;Nicht mit Unrecht hat man von diesem Briefe gesagt, daß er die eigentliche Geburtsstund der »Kritik der reinen Vernunft« bezeichnet." Cassirer, Ernst, 1921, p. 135.

¹³⁴ Wilson, K.D., 1975, p. 249.

¹³⁵ Carl, Wolfgang, 1989, pp. 5 ss. Other scholars arguing in this direction are Norman Kemp Smith, Jennifer Mensch, Beatrice Longuenesse, and Fernando Moledo. Kemp Smith, Norman, 1918, p. 219ss. Mensch, J., 2007, esp. p. 110. Longuenesse, B., 1998, p. 17; Moledo, F., 2014, pp. 66ss.

¹³⁶ Vleeschauwer, H.-J., 1962, p. 63.

him, the problem formulated in this letter cannot be understood as the "positive formulation of the critical problem". Philonenko, A., 1969, p. 94.

¹³⁸ Lema-Hincapié, A., 2004.

Kant conceives the limitation of human understanding exhibits a rupture with rationalism which leads to reshaping the notion of intuition. Specifically, Kant starts relating the limitation of the human mind with the necessary reference of intuition to affection. Thus, arises the question of how to relate the concepts that emerge purely from the understanding with an element that the mind cannot create by itself.

Kant claims that he had been considering the extent and limits of human knowledge. In this context, he poses the problem of how to ground the relation between a representation and its object. Kant asks how a representation can legitimately relate to the object it represents. He asks: "... on what foundation rests the relationship of what we call representation in us with the object? 139 The difficulty is not only to explain the relationship between the representation and the object. Moreover, what must be elucidated is how the representation can *legitimately* relate to what is represented. Two possibilities are outlined that could give an answer to this question. The first possibility is that the intellect was completely ectypic. In this case, the understanding would obtain the material for its logical elaborations from the data provided by the senses. The objects would be the real cause of the representations. The convergence of the representation with the object that it represents would be explained as a cause-effect relation. According to this analogy, the object would be the cause and the representation the effect. The representation would be formed from the material provided by the sensation. The content of the representation would be what the object provides as its cause. Therefore, the validity of the representation would not present difficulties in this case as:

If a representation comprises only the manner in which the subject is affected by the object, then it is easy to see how it is in conformity with this object, namely, as an effect accords with its cause, and it is easy to see how this modification of our mind can

-

¹³⁹ According to Kemp Smith, this problem, as it is posited here, is the one that is present in the *Critique of Pure Reason (KrV)* that Kant introduced in A 84-92 / B 116-24. This scholar uses this letter to Herz to shed light on these passages of the *Critique*. Lema Hincapié follows Kemp Smith. He considers that "the letter does formulate the essential critical problem of the objectivity of representations" José Gómez Caffarena also understands that "the letter is the first explicit expression of what we can call the fundamental critical problem of intellectual knowledge." Arnulf Zweig also claims that Kant had here reached "a formulation of what was to become one central problem of the *Critique*: how are synthetic a priori judgments possible." Many other critics share this interpretation. Fernando Moledo argues that by 1772, Kant not only had posed the critical question, but he already had in mind the clue to give answer to it. For this reason, the Kantian revolution of thinking is to be found around 1772. However, against this reading, Alexis Philonenko argues that in this letter, the problem of *Critique* is still not formulated in critical terms. Kemp Smith, N. 1918, p. 219ss. Lema Hincapié, A., 2004, p. 60. Caffarena, J., 1996, p. XXVIII. Moledo, F., 2017. Philonenko, A., 1969, p. 97.

represent something, that is, have an object. Thus the passive or sensuous representations have an understandable relationship to objects...¹⁴⁰

The second possibility is that the intellect was fully archetypal. The representation would be in this case absolutely active with respect to its object. According to Kant, a fully active mind is capable of producing its object in the act of representing it. The mind creates the represented object. In this way, the material content of the object would be caused by the operation of the mind itself. Therefore, here the validity of this representation is not a problem either because:

...if that in us which we call "representation" were active with regard to the object, that is, if the object itself were created by the representation (as when divine cognitions are conceived as the archetypes of things), the conformity of these representations to their objects could also be understood¹⁴¹.

This is the way how divine knowledge relates to its objects. The intellect of God is an archetypal intellect, the ground of the existence of objects. According to Kant, human thinking is as archetypal as the mind of God when it operates with quantities. Mathematical thinking proceeds in the same way as archetypal understanding. In mathematics, the mind has pure quantities as data. Therefore, the production of the representation can be explained making reference to spontaneity and its principles ¹⁴². The problem of the validity of representation is presented to our intellect because the matter for the construction of knowledge is not just a quantity. The objects of human cognition are also determined by sensible qualities. Therefore, it is here that the relation between representation and its object becomes problematic. The problem is the construction of sensible experience. The difficulty of explaining the legitimacy of the relation between the representation and what is represented becomes particularly complex when the concepts of understanding, which we have a priori, aim to have reference to "things". In

¹⁴⁰ Kant, I. AA 10:130. We follow the translation of Arnulf Zweig. Kant, I. 1999, pp.133 ss.

¹⁴¹ Kant, I. AA 10:130.

¹⁴² Kant claims: "In mathematics this is possible, because the objects before us are quantities and can be represented as quantities only because it is possible for us to produce their mathematical representations (by taking numerical units a given number of times). Hence the concepts of the quantities can be spontaneous and their principles can be determined a priori." Kant, I. AA 10:131.

this way, the philosopher restricts the initial conflict of the validity of representations in the following way:

But in the case of relationships involving qualities - as to how my understanding may, completely a priori, form for itself concepts of things with which concepts the facts¹⁴³ should necessarily agree, and as to how my understanding may formulate real principles concerning the possibility of such concepts, with which principles experience must be in exact agreement and which nevertheless are independent of experience – this question, of how the faculty of the understanding achieves this conformity with the things themselves" is still left in a state of obscurity¹⁴⁴.

Explaining the validity of representation is not a problem either for the divine intellect or for the human mind when it operates with pure quantities. The concordance of the representation with the object is a difficulty inherent to the human intellect in shaping its sensible experience. The concepts of the understanding lie a priori in the mind, but our intellect cannot fully construct its experience because the latter does not only contain mere quantities but also qualities. Thus, it raises the question of how concepts that spring out of the mind can correspond to those represented objects that the mind cannot produce by itself. There seems to be an insurmountable darkness in relation to our intellectual faculty: where the conformity with things come from.

As an attempt to clarify the problem, Kant introduces in this letter two types of possible intellect: the ectype and the archetype. The first is characterized as a reproductive understanding, while the second is a productive one. The archetypal understanding can ground things. It can bring objects into existence. The ectype understanding, on the other hand, requires things to provide the data so it can operate with them. It cannot create the data by itself. Thus, Kant notes, the correspondence of the representation of the subject with the object could be explained because the representation is an effect of the object that is the cause of the representation- or because the representation is the cause of the object. If the mind were archetypal, the object would be created by the act of representing; since the archetypal intellect is one whose intuition is the very grounding of things. It

¹⁴³ For an analysis of the distinction between *Dingen* and *Sachen* in this letter, see Beck, L., 1989, pp. 24 ss. Carl, W., 1989, pp. 6ss.

¹⁴⁴ Kant, I., A 10:130.

constructs them. On the contrary, the ectype intellect must take the data from the sensible intuition of things. The difference between both types of intellect is structural. It is not a mere question of degree as it was for Leibniz. Our understanding cannot be the causal principle of objects. The intellectual concepts of the ectype understanding do not bring the objects of experience into existence¹⁴⁵. As Kant explained, the problem of the correspondence of representation with the object concerns only the intellect as it is neither merely reproductive nor purely productive. The intellectual representation of our finite intellect requires the object to provide the data to form the experience. But pure concepts of understanding are not mere abstractions of sensible material. Then, the problem is to explain the correspondence between thinking and things for an intellect that cannot fully create them. The reference of intuition to affection is an expression of this limitation.

In this correspondence, Kant introduces the notion of God's intuition as a model that contributes to defining certain features of a finite intellect; namely, its necessary relation to affection. The conclusion we reached is that the application of concepts is a problem only for an understanding that cannot create the objects of experience. Finite beings require the object to be given. On the contrary, for God, objects are created in the very act of thinking. As we saw, this contrast between the ectypus intellect and the archetypal has an analogous formulation in the fourth observation to the Aesthetics. There, it was pointed out that the intellect of God can produce the object *materialiter*. Therefore, the original Being does not require sensible affection. We, men, as finite dependent beings, need an affecting object for the constitution of our experience.

As we saw, the receptive nature of intuition and its dependence on an affecting object is the first mark attributed to intuition in *KrV*. In the Introduction to the Aesthetics, Kant stressed that "at least for us, humans" the object must be given to us and that for this to be possible the object must affect the mind in some way. Kant introduced the pronoun "for us" to stress this point. The study that Jakob Sigismund Beck made of this section in his *Erläuternder Auszug* ... offers an indication that in the Introduction to Aesthetics Kant had the same type of argument in mind as those he developed in the fourth

_

¹⁴⁵ "Thus the possibility of both an *intellectus archetypus* (an intellect whose intuition is itself the ground of things) and an *intellectus ectypus*, an intellect which would derive the data for its logical procedure from the sensuous intuition of things, is at least comprehensible. However, our understanding, through its representations, is neither the cause of the object (save in the case of moral ends), nor is the object the cause of our intellectual representations in the real sense (*in sensu reali*). Therefore, the pure concepts of the understanding must not be abstracted from sense perceptions, nor must they express the reception of representations through the senses; but though they must have their origin in the nature of the soul, they are neither caused by the object nor do they bring the object itself into being." Kant, I., A 10:130.

observation to the Aesthetics and in the letter to Herz. Reading the first lines of Aesthetic, Beck points out: "the content of the representation is given and not produced. Intuitions are, for example, the representations of external objects that we obtain as long as we are affected, and their content is given." To human intuition, Beck opposes divine intuition. God produces the content of his representations¹⁴⁶. In this way, the Kantian turn "at least for us humans" - introduced in the second edition - is retaken by Beck as "... on the contrary, for God."¹⁴⁷.

From these developments, we can conclude: the distinctive feature of human intuition according to the first paragraph of Aesthetics is its relation to receptivity. Kant determines that for a finite rationality, it is necessary the reference to affection to know something at all. This restriction is explained in the fourth observation of Aesthetic, and it is also developed in the epistle to Herz. Men require sensible intuitions to be given since their intellect is not purely archetypal. Men, as finite beings, cannot create the object *materialiter*. Therefore, a finite intellect, like the human, requires the object to be given.

¹⁴⁶ Beck, S., 1793, p. 8.

¹⁴⁷ Therefore, as Vaihinger stated, this Kantian allusion to other thinking beings should not be taken as a mere stylistic turn. Vaihinger, H., 1892, p. 345. As Dieter Heidemann has recently shown, Kant opposes the concept of human intuition to that of "intuition in general". "Intuition in general" includes other kinds of intuition as the intuition of God or any other being. Kant uses this notion to highlight the peculiarities of finite intuition. Heidemann shows that even when the concept of other intuitions can only be defined negatively, there is a positive use of the notions. Heidemann, D., 2019.

Conclusion

In this first part, we have shown that there is a general agreement among scholars on two main issues. First, the distinction between intuition and concepts plays a central role in the building of the critical system. This dichotomy is the ground upon which the Kantian theory is constructed. However, there is a second general agreement within Kantian studies. The Critique opens with a series of definitions that are not justified in the first passages of the *Critique*, where they are introduced. They are merely assumed. Therefore, the first problem that we are dealing with is that the core of the Kantian theory of knowledge rests upon a distinction merely introduced by Kant. The core of the Critique seems to be resting on a series of unjustified assumptions. The second problem is to understand the main features of intuitions and concepts. We have studied the general characteristics of intuitive and conceptual representations. For Kant, all representations are divided into intuitions and concepts. The concept is a mediated representation. It cannot refer to the object in individuo. Intuition was defined as an immediate and singular representation of the object while the concept is a universal and mediated representation. However, as we saw, there is no general agreement on this issue. The third problem that we found is the relation of intuition to affection. Kant affirms that sensible intuition has a necessary reference to an element external to the mind. There must be an affection for the construction of the experience. While the forms of knowledge lie a priori, the matter of knowledge is given a posteriori. We studied the problems generated from this relationship of intuition to affection. We observed that even when this problem could be solved, it still remains to explain the relation between these two sources of knowledge, which are heterogeneous with each other. Concepts are a product of understanding, while sensibility provides intuitions. As we have seen, the problem of explaining the relationship between these heterogeneous faculties was highlighted by Kantian contemporaries, and it also represents a theoretical endeavour for contemporary researchers of Kantian philosophy.

In the second part of this section, we studied how the differentiation between intuitions and concepts came to be the technical difference that Kant employed. We showed that the main marks on the Kantian notion of intuition were already advanced by Scotus: immediacy, singularity, and the relation with sensibility (and, therefore, with the concept of existence). We saw that Duns Scotus was the first to give a precise distinction

of these notions. As we exhibited, from the very beginning, the distinction between intuitions and concepts came along with the traditional medieval debate of the relation between universals and particulars. The intuitive way of cognition is the way that the intellect has to achieve the particular, and the particular is what is completely determined. We studied how this problem is also present in Descartes and Leibniz. According to both of them, the intellect was potentially capable of knowing what is real: what is perfectly determined, and intuition was the type of representation that named this kind of cognition. Focusing on Leibniz's account, we showed how he conceived intuitive representations. Paradigmatically real is what is perfectly determined: substances. A complete analysis of the determination of a concept would make it possible to fully know things as they are. Intuition is the accomplishment of this analysis. The mind has a legitimate relation to the objects of experience when we have completed the analysis, and we have shown that there is not any contradiction in the concept. Only then we can claim that our concept is a real concept and not a mere empty notion. That would be the Leibnizian answer to the question we posed at the very beginning: "What is the ground of the relation of that in us which we call 'representation' to the object?" Our representations have a legitimate relation to objects when we can assure that there is not an internal contradiction in the concept, i.e., when we know intuitively.

In the third part, we focused on the Kantian proposal. Particularly, we studied how Kant conceived his new conception of the distinction between intuitions and concepts in dialogue with Descartes and Leibniz. The study of the incongruent counterparts argument showed that satisfying the requirement of a complete determination of the object demands an extra conceptual element: intuition. The analysis of the conceptual marks of the object is insufficient for its univocal determination. Its location at a point in space and in an instant of time cannot be determined by mere concepts. The requirement of a complete determination of the object is never achieved by means of concepts but it is only satisfied by intuition. The reference of thought to the object *in individuo* can only be guaranteed by intuition. The reference to intuition is established so as to guarantee the possibility of a complete determination of the object.

Then, we showed that the necessary reference to what is given is one of the expressions of the finitude of the human mind. Derived intuition requires an object to be given. The original intuition can produce its object in the act of thinking it. The original intuition is proper of the original Being. Ours is a derived intuition. This opposition was present before the developments of the *Critique*. We studied the letter of Kant to Marcus

Herz of 1772. In this letter, Kant explained that the intellect cannot have access to the particular by mere analyses. For Descartes and Leibniz, the limitation of our mind was mainly expressed for its incapability to have access to all the determination of things, i.e., to fully analyse concepts. There is a difference of degree between concepts and intuitions because there is a difference of degree between the model of a perfect mind (infinite) and ours (finite). On the contrary, for Kant, intuitions and concepts are heterogeneous representations because it is not the case that we know less than a perfect model of cognition, but we know different. The impossibility to know things in individuo by pure concepts is an expression of this peculiar way of conceiving the imperfection of human knowledge. For Kant, the relation between the representation and what is real is no longer grounded on the possibility of a complete analysis but on the possibility of giving content to concepts, which merely by themselves are empty. The introduction of the possibility of empty concepts comes along with the requirement of an external element to give content to the conceptual representations. Now, we can only know something under the condition that those concepts that belong to the understanding can be applied to particulars without losing their universality. However, the universality of the concept is external to the concreteness of the object in its individuality, in the sense that space and time – those conditions that enable to individualize the object- belong to the forms of intuition while concepts are products of the understanding. At the same time, the matter of experience can only be given a posteriori. We saw the numerous problems that Kant faced at splitting the two sources of knowledge. In his overcoming of rationalism, Kant shows the division of two heterogeneous faculties could solve the problem within the rationalistic conception. It seems that we have two possibilities: either we claim that individuals can be reached by a process of the understanding, or we claim that they are heterogeneous elements of thinking. The first option led to the problems that Kant outlined, such as the problem of incongruent counterparts. The division of faculties was meant precisely to overcome this problem. However, the second option ended up in the problems that we summarized. We will argue that the proposal of Paul Natorp is able to deal with both problems. As we shall see, Natorp redefines the distinction between intuition and concepts in the discussion of the problem of method. The transcendental method, which we will argue is a synthesis method, will guarantee to give an answer to the Kantian question: on what foundation rests the relationship of our representation in us with the objects. In Chapter 2, we will study how the problem was introduced by other contemporary approaches: psychologism and logicism. They misunderstood the nature of the problem. We will exhibit that both currents have a wrong conception of the relationship between intuitions and concepts due to methodological errors. Natorp will show that both positions are unsuccessful to explain the issues raised by Kant.

Chapter 2. Contemporary Approaches to the Problem of the Distinction between Intuition and Concepts

Introduction

As we exhibited, the problem of the relationship between intuitions and concepts has a long tradition that results in the Kantian formulation of the problem. The question of the relation between intuitive and conceptual representations is the core of Kantian philosophy. In Chapter 1, we exhibited that one of the central problems of knowledge – on what grounds rests the relation of our representations with the objects - can be formulated in terms of the relation between intuition and concepts. We studied the problems involved in the distinction and how they were revisited by Kantian contemporaries. They considered that Kant could not give a satisfactory solution to the problem that he posed. As we saw in the previous section, the relation between intuition and concepts was one of the most discussed aspects of the Kantian proposal. contemporary philosophy, the problem of the relationship between intuitions and concepts emerges as one of the central issues. Philosophers argue about what elements thinking introduces and which ones are given to it. The Kantian question remains unsolved. As we analyzed in the first chapter, the question of the relationship between intuition and concept was also presented by Kant in the following way: How can thought legitimately represent the object? How can the universality of the concept relate to the singularity of the object? Natorp's proposal is grounded on the Kantian paradigm. However, his position is presented in a context where different schools had already tried to give a solution to these problems. The aim of this chapter is to study how Natorp approaches the problem of the relationship between intuitions and concepts in dialogue with his own contemporaries. We will analyze how Natorp presents his proposal in dialogue with the philosophical tendencies of the time: psychologism and logicism. We will exhibit that both currents have a wrong conception of the relationship between intuitions and concepts based on methodological errors.

In the first place, we will study the proposal of psychologism and then that of logicism. Natorp will show that both positions are two types of dogmatic proposals, unsuccessful to explain the issues raised by Kant. In chapters 3 to 5, we will focus on Natorp's own position. However, it is necessary first to explain why the solutions of the time were infertile for him.

2. 1. Against Psychologism

Introduction

This section aims to analyze Natorp's criticism of psychologism. Natorp will argue that the main confusion of psychologism is due to a methodological error. More precisely, Natorp considers that an accurate conception of the relation between intuitions and concepts demands abandoning the standpoint of subjectivity. The subjective point of view will lead to considering the object as a fact given to intuition. The object will be determined in advance. According to this point of view, the task of concept formation consists in a process of abstraction. The concepts are abstractions of the marks that belong to the object that is given to intuition. The position will receive the name of psychologism. As we shall see, psychologism will have a misconception of the relation between intuitions and concepts due to a methodological error. Natorp will exhibit that the problems of this perspective are grounded on the assumption of the subjectivity standpoint. Psychologism takes subjectivity as the starting point of the investigation and considers the object as what is opposed to it. Natorp will show that psychologism starts from an incorrect understanding of the philosophical method. Particularly, the mistake of psychologism consists in grounding logic on psychology. Psychologism confuses the study of the laws of knowledge with the study of the legality of psychical life. The problem of the genesis is confused with the problem of validity. From this methodological error, psychologism considers the data given to intuition as a first element in the formation of knowledge. Starting from the problem of the formation of knowledge, psychologism conceives the immediate data as what is truly real, as the first for the act of knowing and the foundation of all objectivity. For psychologism, what is given to intuition is the starting point for the production of concepts. According to this perspective, starting from a given initial data would guarantee the possibility of objective knowledge. However, Natorp will show that from this perspective, concepts are merely abstractive. From the finite human standpoint, the intuitive representation is what is given to senses, and concepts are the abstractive marks from what is given. Natorp will argue that thought does not require anything external to itself in the production of its object. The laws of thinking do not originate from a process of abstraction from given intuitive contents. On the contrary, thinking creates the objectivity through its laws. Objectivity consists in this dependency on thinking. Natorp will prove that this dependence is precisely a guarantee of objectivity. More specifically, it will be shown that the only possible way to conceive a relation between the laws of thought and its objects is to base what is objective purely and exclusively on the legality of thinking.

First, we will examine the emergence of the debate on psychologism in the nineteenth century. Our goal is to show the relevance of Natorp's position in the philosophical debate of the time. Second, we will study some of the most representative positions. We will focus on Beneke's thesis, one of the precursors of psychologism. Then, we will study Helmholtz's ideas, as a representative of physiological Neo-Kantianism. Finally, we will focus on Natorp's objections against the subjective method to show how this methodological error of psychologism leads to an incorrect understanding of the relationship between intuitions and concepts.

2.1.2 Introduction to the *Psychologismus-Streit*

Natorp's criticism of psychologism is framed by what was known as *Psychologismus-Streit*. The debate on psychologism was one of the most important disputes in German philosophy at the end of the 19th century, and it is concomitant with the emergence of psychology as a scientific discipline independent of philosophy¹⁴⁸.

By the end of the nineteenth century, philosophy is experiencing a crisis. This is recognized both by numerous philosophers of the time¹⁴⁹ and by contemporary scholars¹⁵⁰. Philosophy had an "identity crisis."¹⁵¹ On the one hand, philosophy experiences a strong rejection of post-Hegelian speculative idealism, which is in decline after Hegel's death. There is a generalized rejection of all forms of purely abstract speculation. For the philosophers of nature, the Hegelian philosophy represented a 'complete nonsense'¹⁵². On the other hand, the evolution of particular sciences led to a reconsideration of the task of philosophy. For many thinkers, the return to Kant was motivated by the loss of credibility suffered by philosophy which started with this fall of speculative idealism¹⁵³. Natorp shares this vision of the state of philosophy. In *The Logical Foundations of the Modern Mathematics*, he considers that the philosophy abandoned the sobriety that for many centuries it had shared with the exact science,

⁻

¹⁴⁸ Windelband considers the separation of psychology from philosophy as one of the paradigmatic scientific facts of the 19th century. Cf. Windelband, W., 1903, p. 519. Külpe, on the contrary, considers that by that time there still had not taken place a total separation between psychology and philosophy. Külpe, O., 1921, pp.76 ss.

¹⁴⁹ Külpe, O., 1907, p.11; Cassirer, E., 1950, p. 3ss. Windelband, W., 1903, p. 511, 513, 519. Heidegger, M., GA1, p. 5; Helmholtz, H., 1950, p.147 ss. Windelband states that the nineteenth century "is far from being a philosophical century". Windelband, W., 1903, p. 511.

¹⁵⁰ Dufour, E., 2003; Kusch, M., 2005, p.2; p.8; Gonzalez Porta, M., A., 2005, pp. 36ss. Beiser, F. 2014, p. 15ss.

¹⁵¹ This term was first used by Herbart Schnädelbach. Cf. Beiser, F., 2014, p.15.

¹⁵² "Hegels Naturphilosophie erschien den Naturforschern wenigstens absolut sinnlos. Von den vielen ausgezeichneten Naturforschern jener Zeit fand sich nicht ein einziger, der sich mit den Hegel'schen Ideen hatte befreunden können." Helmholtz, H., 1950, p.147.

¹⁵³ Oswald Külpe argues: "Als dann mit dem Niedergang der Hegelschen Philosophie das Vertrauen zu dieser Wissenschaft überhaupt erlosch und eine gründliche Emanzipation der Einzelwissenschaften von ihrer Führung und Bevormundung einsetzte, da schien den Philosophen keine bessere Hilfe möglich zu sein, als die Rückkehr zu Kant." Külpe, O., 1907 p.11. Following the line of Külpe, Martin Heidegger holds in one of his first published works: "When, with the decline of Hegel's philosophy, the particular sciences energetically freed themselves from the tutelage of philosophy and threatened to repress it completely (with positivism the precarious situation and the philosophy-dependent task was noticed), the only salvation was seen in the "return to Kant". Heidegger, M., GA1, p.5.

ending up falling into the empty speculation opposed to the rigorous thinking of the mathematics.¹⁵⁴

The return to Kant is a reaction to the challenge presented by, on the one hand, the fall of Hegel's speculative idealism, and, on the other, the total emancipation of the sciences with respect to philosophy.

Hence arises the question of the relationship that philosophy has with the emerging scientific disciplines that are now emancipated from it. Philosophy faced two dangers. The first danger is to fall into a speculative metaphysics that cannot give a proper explanation of any fact. As Ernst Cassirer explains, some thinkers argued that philosophy does not contribute to the development of science. Moreover, philosophy could be an obstacle to its progress¹⁵⁵. The second problem that philosophy has is to be reduced to a particular area of positive science. Philosophy is not only faced with the problem of justifying its method, but it must also give an account of what its proper object of investigation is¹⁵⁶. Thus, while philosophers must dispute their objects of study to positive science, some scientists of nature consider that philosophy is not only useless but harmful for the progress of knowledge¹⁵⁷. In this context, empirical psychology emerges as a science, and with it the philosophers who seek in this discipline a kind of refuge from the end of speculative idealism¹⁵⁸.

Some of these thinkers take psychology as a new fundamental branch of philosophy¹⁵⁹. Friedrich Beneke is one of the main representatives of this current. Beneke

¹⁵⁴ Natorp states: "Die alte, nach beiden Seiten fruchtbringende Verbindung zwischen Philosophie und Mathematik schien eine Zeitlang sehr gelockert. Was die Mathematik vielleicht einmal der Philosophie zu danken hatte, die Strenge des Beweisverfahrens, ja den ganzen Begriff des formalen Aufbaus einer Wissenschaft, das hat sie längst aus eigener Kraft und eigenem Trieb so in sich aufgenommen, dass sie mit gutem Grunde glauben darf, darin von der Philosophie nicht viel mehr lernen zu können. Diese dagegen schien die nüchternen Bahnen, in denen sie manches Jahrhundert mit den strengen Wissenschaften Hand in Hand gegangen war, zeitweilig ganz zu verlassen und sich in spekulativen Abenteuern zu gefallen, die das streng geschulte Denken des Mathematikers zu allererst zurückstossen mussten." Natorp, P. ZLGNM, p. 177.

¹⁵⁵ Cf. Cassirer, E., 1950, p. 4.

¹⁵⁶ Cf. Beiser, F., 2014, p. 18.

¹⁵⁷ Helmholtz holds: "Die Naturforscher wurden von den Philosophen der Borniertheit geziehen; diese von jenen der Sinnlosigkeit. Die Naturforscher fingen nun an, ein gewisses Gewicht darauf zu legen, dass ihre Arbeiten ganz frei von allen philosophischen Einflüssen gehalten seien, und es kam bald dahin, dass viele von ihnen, darunter Manner von hervorragender Bedeutung, alle Philosophie als unnütz, ja sogar als schädliche Träumerei verdammten." Helmholtz, H., 1950, p.147.

¹⁵⁸ As Beiser claims: "The sciences now seemed to cover the entire *globus intellectualis*, so that there seemed no special subject for philosophy." Beiser, F. 2014, pp. 16ss. Windelband explains that these philosophical schools arise in the broader context of the rise of materialism. Windelband, W., 1903, p. 513 y 519.

¹⁵⁹ Cf. Beiser, F. 2014, p. 16 ss. Anderson, Lanier, 2010, p. 288.

believes that philosophy is the first science, the science on which the rest of the sciences depend. This 'science of science' is ultimately called psychology. Psychology is the starting point of all philosophy¹⁶⁰. Psychology will be the grounding science of philosophy. Logic, ethics, and aesthetics are applications of psychology as a fundamental science. However, the incipient institutionalization of psychology as a science must be distinguished from the accusation of psychologism¹⁶¹. The term 'psychologism' was first used by Eduard Erdmann in 1866 as an accusation towards Frederick Beneke¹⁶². His criticism points to the attempt of some thinkers to make psychology the grounding science of philosophy and science in general. This term denotes rather a "philosophical accusation." ¹⁶³

Beneke has been considered the pioneer of the 'back to Kant' ¹⁶⁴. Paradoxically, he was the first philosopher accused of psychologism. For Beneke, the starting point of

_

¹⁶⁰ Beneke, F. 1933, p.2. For Beneke, psychology is: "...the center of all philosophy as a whole: the sun from which all other philosophical sciences receive their light. Only in this way is it possible to achieve true unity and true order, true universal validity for philosophy. All philosophical concepts are certainly a product of the human soul; and only by knowing the way and the way in which they have been generated is how can they receive their supreme clarity. In the same line, in his brief of 1833 (*Philosophy in its relations with experience, speculation and life*), Beneke holds: "Nicht nur als Anfangs oder Mittelpunkt; nicht nur als Grundlage für alle philosophie Erkenntnis haben wir die Selbst Erkenntnis oder psychologische zu betrachten, sondern alle übrige philosophische Erkenntnis. Können wir nur und in dieser gewinnen. In den Begriffen aller übrigen philosophischen Wissenschaften denken wir nichts Anderes als psychischen produkten, welche demnach auch nur als solche in voller Wahrheit und Tiesse gewürdigt werden können." Beneke, F., 1833, p. 14.

Martin Kusch compiled various definitions of the concept of psychologism in order to show that although the objection of psychologism was widespread at the time, what these authors understood by psychologism varied greatly. Thus, for example, Oswald Külpe defines psychologism as "the unjustified application of psychological consideration in the field of theory of knowledge." "Man pflegt die unberechtigte Anwendung der psychologischen Betrachtungsweise in der Erkenntnislehre als »Psychologismus « zu bezeichnen. Die Psychologie hat es nämlich nur mit der tatsächlichen Beschaffenheit und Entwicklung der seelischen (und also auch der Erkenntnis-) Vorgänge zu tun, die Erkenntnislehre dagegen untersucht den Erkenntniswert der letzteren, d. h. ihre Bedeutung für die Erfassung von Gegenständen (ihre »objektive Gültigkeit«)." Külpe, O., 1921, p. 39. However, his differentiation between psychology and theory of knowledge could well be considered as a psychologist from other perspective. The difficulty consists in determining who exactly the 'enemy' is.

¹⁶² Cf. Kusch, M., 2005, pp. 98 ss.

¹⁶³ Jacquette, D., 2003, p. 43. This controversy on the theoretical level has deep consequences in the institutional sphere. As numerous studies noticed, the problem was not only theoretical but also the university positions in the faculties were at stake. In 1913, a group of 107 philosophers in Germany, Austria and Switzerland wrote a petition claiming that no more positions were given to experimental philosophers. They claimed that all chairs of philosophy were becoming chairs of psychology. Natorp signs this petition and speaks publicly against that university chairs were given to experimental psychologists. According to this request, it is questioned that thinkers who study metal life occupy positions corresponding to philosophy. Natorp - along with other thinkers such as Husserl, Rickert and Riehl - theoretically and institutionally resists the dissolution of philosophy in empirical psychology. This institutional separation can be considered as the translation of theoretical separation. Cf. Kusch, M., 2005, p.186 ss. Beiser, F., 2014, p.18.

¹⁶⁴ Brandt Burke holds: "The historical importance of Beneke as the real pioneer of "the movement back to Kant," has never been sufficiently recognized, or more than that, it has not been recognized at all." Brandt, B., 1895, p. 29.

philosophical research is the reflective moment of self-awareness. Man is conscious in the reflection of the mental acts that he carries out to obtain knowledge 165. This awareness of mental acts is the foundation of the possibility of psychology. The psychology that describes the processes found in self-perception is empirical psychology. Empirical psychology is the basis of philosophy¹⁶⁶. The possibility of obtaining knowledge should be sought in the mental mechanisms that allow the formation of representations. Beneke believes that philosophy must identify the origin of the formation of representations. Being is being represented¹⁶⁷. The truth is based on mental representations. Then, philosophy must study how representations are generated in the soul of man¹⁶⁸. Beneke believes that logic is certainly the core of philosophy. However, logic depends on psychology ¹⁶⁹. Psychology will be responsible for explaining the principles that govern the formation of knowledge in mental representations. Mental representation requires two conditions. In the first place, a soul that has the senses as instruments. Second, it requires an affecting object. The sensations are the first elements in the elaboration of the representation and, therefore, the starting point of the investigation 170. The intuitive moment is required for the explanation of the process of knowing because it is the first required moment in the genesis of the representation. Psychology reveals the conditions that lie in the mind for the formation of these representations that constitute knowledge. Thus, Beneke proposes a foundation of philosophy in psychology. Through the psychological foundation, philosophy is prevented from the two dangers outlined above. On the one hand, philosophy avoids empty speculation. On the other hand, it follows the method of natural science. This path initiated by Beneke, as a continuator of the currents of modern empirical psychology, is deepened in subsequent years ¹⁷¹. With the growth of the institutionalization of psychology as a science, the theoretical interference that psychology has on the philosophical level also increases. Beneke thought that

_

¹⁶⁵ Brandt holds: "And again, only on the basis of inner experience can philosophy, and in particular scientific knowledge of the human soul, be established with certainty and steadfastness." All this is only to give special prominence to inner consciousness as a fundamental datum of individual experience. And with the recognition of this fact, we reach the fundamental starting point of Beneke's psychology and philosophy". Cf. Brandt, B., 1895, p.51 s.

¹⁶⁶ Messer, A., 1920, p. 92.

¹⁶⁷ Beneke, F., 1840, p. 67.

¹⁶⁸ It is interesting to note that the central concept of Beneke's investigation is not the mind but the soul. The soul is the determining element of human life and one of the first conditions in the formation of representations. The senses are defined as instruments of the soul. Cf., Beneke, F., 1871, §2.

¹⁶⁹ Beneke, F., 1842, p.21.

¹⁷⁰ Beneke, F., 1871, §2.

¹⁷¹ Oswald Külpe sees in Beneke a developer of the studies initiated by Tetens in the eighteenth century. Külpe, O., 1921 p. 82.

psychology, as a grounding science of philosophy, should follow the method of natural sciences. Later, many authors will deepen this conception. Thus, arises physiological psychology. Not only were the foundations of logic sought in the life of consciousness but, more particularly, in the physiological processes that are carried out in the formation of mental representations. Wilhelm Wundt and Hermann von Helmholtz are pioneers in this direction.

2.1.3 Psychologism in Neo-Kantianism.

Hermann von Helmholtz, who worked with Wundt¹⁷², is one of the representatives of physiological psychologism. Helmholtz is one of the first Neo-Kantians and one of the first thinkers of the nineteenth century who seeks to ground the theory of knowledge in the physiology of the senses¹⁷³.

Helmholtz defines himself as a representative of natural philosophy¹⁷⁴. This place that Helmholtz occupies is of particular relevance considering that Marburg's Neo-Kantianism emerges as one of the first reactions against the psychologization of logic. Helmholtz not only receives the influence of the psychology of Wundt, but he is also influenced by Kantian and Fichtean idealism. His proposal emerges as a peculiar form of association between these two influences. On the one hand, his work as a scientist and his work with Wundt, on the other, his studies in Kantian and post-Kantian idealism. Helmholtz tries to reconcile philosophy with the sciences of nature. From Helmholtz "a new and peculiar relation between empirical sciences and philosophy is created." ¹⁷⁵

According to Helmholtz, the philosophy and science of nature share the same question. They investigate the relationship between representations and reality. For Helmholtz, the core of the problem of knowledge is expressed in the Kantian question: "In what sense do our representations correspond to the reality?", the question that, as we

¹⁷³According to Köhnke, Helmholtz belongs to the "programmatic" Neo-Kantian phase. For Alois Riehl, Helmholtz "was the first to declare that Kantian ideas were still alive." He states: "Er war der Erste, der es aussprach, dass Kants Ideen noch leben." Riehl, A., 1904, p. 1. Beiser considers Riehl is wrong when he states that Helmholtz is the first Neo-Kantian. Beiser, F., 2014, p. 196.

¹⁷² Cf. Kusch, M., 2005, p. 197.

¹⁷⁴ "Ich habe umso mehr Veranlassung, die Frage nach dem Zusammenhang der verschiedenen Wissenschaften hier zu erörtern, als ich selbst dem Kreise der Naturwissenschaften angehöre…" Helmholtz, H., 1950, p. 145.

¹⁷⁵ Cassirer, E., 1998, p.12. For Riehl, the merit of Helmholtz's philosophy lies in recovering the relation between philosophy and science, and not in his physiological reading of Kant. Riehl, A., 1904, p. 2.

showed, was reduced to the problem of the relation of intuitions and concepts. Helmholtz holds:

Das Grundproblem, welches jene Zeit an den Anfang aller Wissenschaft stellte, war das der Erkenntnistheorie: "Was ist Wahrheit in unserem Anschauen und Denken? in welchem Sinne entsprechen unsere Vorstellungen der Wirklichkeit? Auf dieses Problem stossen Philosophie und Naturwissenschaft von zwei entgegengesetzten Seiten; es ist eine gemeinsame Aufgabe beider. Die erstere, welche die geistige betrachtet, sucht aus unserem Wissen und Vorstellen auszuscheiden, was aus den Einwirkungen der Körperwelt herrührt, um rein hinzustellen, was der eigenen Thätigkeit des Geistes angehört. Die Naturwissenschaft im Gegentheil sucht abzuscheiden, was Definition, Bezeichnung, Vorstellungsform, Hypothese ist, um rein übrig zu behalten, was der Welt der Wirklichkeit angehört, deren Gesetze sie sucht. Beide suchen dieselbe Scheidung zu vollziehen, wenn auch jede für einen andern Theil des Geschiedenen interessiert ist. ¹⁷⁶

According to Helmholtz, this question of the relation between concepts and intuition is the common point between philosophy and the science of nature. Both science and philosophy try to understand the relation between thinking and reality. They want to explain the relationship between our representations and what is real. Philosophy and natural science are included in the problem of the *Erkenntnistheorie*. The philosophy and science of nature deal with this problem of knowledge in general but each of them from a different perspective. Philosophy studies the problem of the generation of representations in the mind. Its task is to distinguish in the representation the element that corresponds to reality from the element of the cognitive faculty. That is to say, one must separate in the representation that which belongs to the mind from what corresponds to the world. The field of investigation of philosophy is the mental process. The science of nature, on the other hand, deals with the objective side. His field of research is that which corresponds to the world, the reality. However, Helmholtz believes that both philosophy and natural science seek to answer the question of how our representations relate to reality. For both, it must be explained how concepts relate to intuitions. The problem of knowledge arises in terms of the correspondence of the human mind with reality, and reality is that which must be achieved by thinking. Philosophy focuses its study on the

^{176 &}quot;in welchem Sinne entsprechen unsere Vorstellungen der Wirklichkeit?" Helmholtz, H., 1879, p. 6.

subjective pole of knowledge, in the psychic life, establishing the limits, justification, and extension of empirical knowledge. Science focuses on the objective pole, the content of experience. For Helmholtz, philosophy and natural science share the same fundamental basis. They are just two sides of the same problem. According to Helmholtz, Kant expressed this fact clearly. Kantian philosophy has the same basis as natural science 177.

Helmholtz identifies the structure of our organs that determine the representations with the Kantian forms of knowledge, physiologizing the *a priori* forms of Kant. The subject has certain structures that determine the way in which we experience the world. These structures can be established by a physiology of the senses because it is the structure of the senses that determines the particular form that the human experience takes. Helmholtz believes that his theory of perception will confirm the thesis presented by Kant¹⁷⁸. Indeed, men have a priori forms that determine experience. However, these forms must be sought in the structure of the organs. For this reason, the main task of theoretical philosophy is the physiology of the senses.¹⁷⁹

In his research on the concept of space, Helmholtz shows that the peculiar form that spatiality acquires is determined by empirical factors. The form of space is built from the relationship between the affection and the peculiar constitution of the senses. Philosophy must explain the generation of the representation of space from its genesis in empirical consciousness. Helmholtz studies the psychological representation of space. This representation of space depends on the possibility of the subject to voluntarily perform movements that vary the perception of objects. Those sensations that are altered by these movements are the spatial sensations. The representation of space results from our subjective construction as we perform certain types of movements are the spatial sensations. The representation of space results from our subjective construction as we perform certain types of movements that a priori representation of space must be distinguished from space properties. Space is an a priori representation but its properties are determined a posteriori. That is, the priority of space does not indicate an a priori proof of the character

-

¹⁷⁷ Helmholtz, H., 1950, p. 146.

¹⁷⁸ Helmholtz, H., 1879, p.8.

logy and science simply on the level of physiology, however. He takes it a step further by also considering the psychology of perception, that is, the psychic acts that are necessary for perception. Helmholtz is far from thinking that we perceive the world just by having sensations; he goes on to consider some of the many psychic acts of inference and judgement—most of them automatic and subconscious—necessary to convert sensations into perception". Cf. Beiser, F., 2014, pp. 198, 200.

¹⁸⁰ He holds: "Und eine gegebene, vor aller Erfahrung mitgebrachte Form der Anschauung würde der Raum sein, insofern seine Wahrnehmung an die Möglichkeit motorischer Willensimpulse geknüpft wäre, für die uns die geistige und körperliche Fähigkeit durch unsere Organisation gegeben sein muss, ehe wir Raumanschauung haben können." Helmholtz, H., 1879, p. 16.

of its properties. For this reason, we are allowed to affirm the transcendental character of space as a form of intuition, but we are not allowed to affirm the a priori character of laws of geometry. The structure of the eye determines, by its own internal constitution, the general features of the visual and, in this sense, it is a form of intuition. However, the particular colours that the eye sees, "the relations among them and the order in which they appear are effects of external causes that are not determined by any law of our organization" ¹⁸¹. The same occurs with the representation of space. Therefore, even if it is accepted that the representation of the space is a priori, the specific axioms of space must be verified a posteriori¹⁸². For Helmholtz, concepts such as space and number should be elucidated by reference to their empirical genesis in the mind¹⁸³. The problem of knowledge must be addressed in its formation in the subjective pole. Thus, philosophy has contact with the science of nature through the theory of perception. The main philosophical concepts are explained in the doctrine of sensible perception. In this way, the results of the empirical sciences validate the results of the Kantian system. Helmholtz shows how the Kantian theory finds a translation in the physiology of the senses. The reference to what is given is required by this physiology of senses. Helmholtz tries to explain the genesis of the representation in the mind. From this point of view, a theory of sensation is required and, concomitantly, the reference to something that is given to the mind. As it was for Beneke, the intuitive moment is represented by what is given to the mind as the first element of the formation of the representation. On the other hand, concepts of experience are abstractions from what is given. For this reason, the answer to the question of the relation between thinking a reality -between concepts and intuitionsis grounded in a theory of correspondence. It can only be satisfied by an a posteriori proof. From this perspective, the construction of the object of experience depends on what is given. The object experience cannot be fully constructed by thinking. On the contrary, thinking depends on the object. The standpoint of psychology and physiology led to this result.

We can identify the following common features of psychologism. First, empirical psychology is the basis of all philosophy. Then, logic must be based on a theory of mental

¹⁸¹, Unser Auge sieht alles, was ist sieht, als ein Aggregat farbiger Flächen im Gesichtsfelde; das ist seine Anschauungsform. Welche besonderen Farben bei dieser und jener Gelegenheit erscheinen, in welcher Zusammenstellung und in welcher Folge, ist Ergebniss der äusseren ein Einwirkungen und durch kein Gesetz der Organisation bestimmt. Ebenso wenig folgt daraus, dass der Raum eine Form des Anschauens sei, irgend etwas über die Tatsachen, die in der Axiomen ausgesprochen sind." Helmholtz, H., 1879, p.23. ¹⁸² Helmholtz, H., 1879, p.28.

¹⁸³ Cf. Cassirer, E., 1950, pp. 57 ss.

acts. Logic depends on psychology. Then, the sciences that depend on logic will also be submitted to psychological laws. Second, logic must focus on the study of the origin of representations. Representations originate from the senses. The sensations are the first elements in the elaboration of the representation and, therefore, the starting point of the investigation. Thought requires an intuitive element in the construction of knowledge. Third, psychologism argues in favour of a subjective foundation of knowledge. The problem of knowledge must be addressed in its formation in the subjective pole. The subject's mind and its processes are what should be investigated. Fourth, the problem of validity is based on the problem of genesis. The legitimation of the act of knowledge must be sought at the origin of the representations. Logic is grounded on the discipline that explains how representations originate in the mind. Fifth, psychologism considers that the foundation of the real requires sensation as a starting point. The intuitive element, considered as a posteriori data is essential for the construction of the object of knowledge. The mind cannot produce the object of experience by itself. There is an element that must be given for the construction of the experience. Knowledge requires sensibility as a passive faculty to receive representations. The passivity of sensibility indicates a reference to an element that the subject cannot construct.

2.1.4. Natorp's Critique of Psychologism

The work of Natorp *On the objective and subjective basis of knowledge* is the first *manifesto* of the Neo-Kantian school against psychologism¹⁸⁴. Even though Cohen had already raised some objections against the subjective orientation of knowledge, it is Natorp who systematically develops for the first time the problem of the subjective path of the foundation of knowledge. Natorp will incorporate these arguments into his *Introduction to Psychology*, a work published the following year of this research. As noted, this work is influential in the dispute against psychology. In his *Logical Investigations*, Husserl highlights the influence that Natorp's arguments had on his own productions. Husserl expressly refers to *Social Pedagogy*, the *Introduction to Psychology*, and the article published in the *Philosophische Monatshefte*, *On the objective and subjective basis of knowledge*. Husserl emphasizes that it was these last two works that had the greatest impact on his thinking¹⁸⁵.

Natorp will show that psychologism confuses a particular science, psychology, with a fundamental science: logic. The problem of knowledge should not be studied according to its genesis in the consciousness of the individual. On the contrary, one must seek a fundamental science that proceeds in such a way that it can guarantee the legitimation of knowledge in general. The subjectivist perspective takes as a starting point of the investigation what is immediately given to intuition. The intuitive aspect of the process would involve this relation to something that is immediately given in natural experience as an external element to thinking. This requirement emerges as a consequence of the subjective point of view. Husserl confuses the problem of the genesis of the representation with the problem of the validity. This methodological error, as we shall see, will lead to the loss of any notion of objectivity. Natorp will show that the objectivity can only be guaranteed if it is exhibited how the thinking process can produce its objects. The mind constructs the cases in the creation of laws 186. An idealistic conception of the

¹⁸⁴ Cf. Edgard, S., 2008, p. 54.

¹⁸⁵ "Auch in einigen anderen, nicht minder wesentlichen Punkten berühren sich meine Prolegomena mit diesem Werke des scharfsinnigen Forschers, welches mir für die Bildung und Darstellung meiner Gedanken leider nicht mehr hilfreich sein konnte. Dagegen konnten auf mich zwei ältere Schriften Natorps , der oben zitierte Aufsatz aus den Phil. Monatsh. XXIII und die Einleitung in die Psychologie anregend wirken — wie sehr sie mich auch in anderen Punkten zu Widerspruch reizten." Husserl, LU, *Prolegomena*, p. 156. § 41

¹⁸⁶ This point will be developed in Part III.

law will be defended as opposed to the naturalistic notion of psychology. Finally, Natorp will show that the problem is not resolved by appealing to a transcendental subjectivity.

Logic and psychology

The problem concerns the foundation of the method of logic. The question is whether the foundation of logic should follow a subjective path or an objective path. The problem is to determine if the starting point of the investigation should be oriented to the subjective pole, to the agent of knowledge, or to the objective pole, to knowledge as a result. As we observed, the defenders of psychology, even with their multiple differences, agree that the foundation of knowledge must be found in the subject. Psychologists agree that the problems posited by logic can be solved by attending the subjective processes that give rise to the act of knowing. The central problem here is whether in the foundation of knowledge the determining factor is the subjective or the objective. The subjective side represents the subject of knowledge, it is the activity or experience of the subject. A subjective study of knowledge will investigate the factual experience of the cognitive agent. The objective side represents what is known, that is, the content of knowledge. The product is the objective side while the agent of the process is the subjective side side.

Natorp begins his argument by accepting that knowledge has two dimensions. On the one hand, knowledge is an objective determination. It means what must be known. In this sense, knowing means relating to the mechanisms of conceptual determinations of the object. On the other hand, knowledge is also an activity, an experience of the subject that carries out the knowledge process. Knowledge includes these dimensions that are correlated. These two dimensions cannot be separated. However, the problem is to establish the path for the foundation of knowledge. Knowledge means both: the process of knowing and the result¹⁸⁸.

¹⁸⁷ The rational finitude that realizes the activity of knowing is an "abbreviation" of the logical space (absolute subjectivity) in which it is inserted. Natorp claims: "Unser wirkliches Denken begnügt sich sozusagen mit Abbreviaturen des Denkens…" Natorp, P., QQ, p. 16.

^{188 &}quot;Erkenntniss aber stellt sich von vornherein zweiseitig dar: als "Inhalt" (Erkanntes oder zu Erkennendes) und als "Thätigkeit" oder Erlebniss des Subjects (als Erkennen). Zwar sind in jeder Erkenntniss beide Beziehungen miteinander gegeben und eng verbunden; es gibt so wenig ein Erkanntes ohne Erkennenden, wie einen Erkennenden ohne Erkanntes. Aber doch muss in abstracto beides unterschieden werden, und offenbar wird eine, die Erkenntniss in ihrem eigenen Gesetze begründende Theorie nur eine von beiden Beziehungen unmittelbar betreffen können. Es fragt sich somit, welche von beiden in der Begründung der Erkenntniss als die erste, zu Grunde liegende, bestimmende anzusehen sei." Natorp, P., UOSB, p. 260.

Natorp explains the reasons why the subjectivist conception is untenable. First, the foundation of subjective knowledge constitutes a metabasis eis allo genos, a change to another genus¹⁸⁹. There is a confusion of the levels of knowledge. The subjectivist position confuses the grounds and what is grounded. The ground is the objective and the subjective is what is grounded. Logic is a fundamental science, psychology is derivate. Psychology is a special science. Logic is the science of science. These two levels cannot be mixed. There is a gap between logic and psychology. It can be conceded that knowledge is made up of a subjective and an objective side. However, logic deals with the objective laws of knowing. It does not study the individual subjectivity. The problem of the ideality of the law cannot depend on the effectiveness of the psychic process 190. Second, after all, the subjective foundation leads to the abandonment of any idea of objectivity¹⁹¹. Grounding logic in psychology also implies abandoning the very idea of objectivity of knowledge. Objectivity would not be properly objectivity if it were grounded on the process of each factual subject. The choice of a subjective path of knowledge foundation makes all objective validity a mere subjective validity. Universal and objectively valid knowledge depends on a process that is valid only from the point of view of the subject. Then, the very concept of objective validity is abandoned if the science that should give the conditions of universal validity can only provide the subjective mechanisms of the formation of representations. Thirdly, subjective foundation falls into a vicious circle. Logic must explain the problem of the objective validity of knowledge. If logic depends on psychology, this science of consciousness lacks the necessary parameters to establish whether its arguments are valid or not. The task of finding the ultimate foundation of logic implies the grounding of objective knowledge that psychology itself cannot offer. To ground the logic in psychology, psychological legality should have a foundation that guarantees the objectivity of its propositions, even the thesis that the parameter of truth depends on the psychic

⁻

¹⁸⁹ Originally, the expression comes from Aristotle (in Posterior Analytics I.7., 75a 38). John K. O'Connor makes an investigation of the sources of the use of this expression. He analyzes the use of the expression in Aristotle and argues that to gain a better understanding of the problem in Husserl one should go back to Brentano. Cf. O'Connor, J., 2008.

¹⁹⁰ "Zwar räumten wir bereitwillig ein, dass kein Erkanntes sei ohne Erkennenden; dass Erkenntnis allein gegeben sei im Erlebnisse des Subjects, im Bewusstsein des Erkennenden; aber, so wie diese Beziehung zum Subject hier nicht den Fragepunkt bildet, so finden wir uns auch bei der Beantwortung der Frage nicht genöthigt, auf sie zu recurriren. Jeder Recurs auf das Subject des Erkennens, auf die Art der Betheiligung des Bewusstseins dabei, muss uns vielmehr von vornherein als *metabasis eis allo genos* erscheinen." Natorp. P., UOSB, p. 262.

¹⁹¹ As Scott clearly explains: "As Natorp sees it, accepting a psychologistic or subjective method for logic entails giving up the very idea of objective knowledge". Scott, E., 2008, p. 57.

processes¹⁹². Psychology aims to state true propositions. The claim that the truth is based on psychic processes must be true as well. However, psychology depends on a certain conception of the truth that validates this statement. Natorp acknowledges that this argument is insufficient since logic must also prove the truth of its propositions 'logically'. 193 Similarly, the dependence of logic on psychology implies the abandonment of the possibility of logic in general¹⁹⁴. Logic must be grounded on itself. It must have an immanent foundation. All other sciences must be based on it because logic is the science of sciences. If logic is grounded on psychology, it is not logic anymore 195. The objective truth of the principles of knowledge cannot be based on the subjective experience of the cognitive subject; since if we make logic depend on psychology, the very claim to find the legal foundation of knowledge is suppressed. Therefore, "logic has nothing to say about thinking as a fact, or as a psychological process"196 because what must be found are the fundamental concepts and principles that give this first science an autonomous validity. In this sense, logic is the opposite of psychology. The latter deals with the empirical aspect of subjective process while the former seeks the principles of the general validity of objective knowledge. Only then, "the autonomous and purely objective foundation of truth" can be guaranteed. The subjective path would lose the very meaning of the concept of truth since objective validity cannot depend on empirical subjectivity. Then, the possibility of determining the objective validity of knowledge depends ultimately on the possibility of establishing an objectivity at some point independent of subjectivity. Natorp argues that this demand led to the mistaken belief that there are data that the subject apprehends that are independent of objectivity.

The dogmatism of the given

¹⁹² "Handelt Logik von dem Kriterium der Wahrheit, von dem, was allgemein, weil auf gesetzmässige Art, die Wahrheit einer Erkenntniss bestimmt, so darf doch die Gültigkeit dieses Kriteriums nicht abhängen von einer Erkenntniss, die nur nach diesem Kriterium als wahr zu behaupten wäre." Natorp, P., UOSB, p. 264.

¹⁹³ Natorp, P., ZLM, p. 270.

¹⁹⁴ "Entweder also, es gibt keine Logik, oder sie muss mit dem Anspruch auftreten, ganz auf eigenem Grunde zu bauen, nicht von irgendeiner andern Wissenschaft ihre Fundamente borgen zu sollen." Natorp, P., UOSB, p. 264.

¹⁹⁵ "Allem voraus, die subjectivistische Ansicht unannehmbar macht, ist die Erwägung, dass der ganze Sinn der Logik, als einer allgemeinen, die Wahrheit der Erkenntniss begründenden Theorie, aufgehoben wird, wenn man, wie die Consequenz jener Ansicht es fordert, Logik von einer besonderen Wissenschaft, Psychologie, ihrem Princip nach abhängen lässt." Natorp, P., UOSB, p. 264.

¹⁹⁶ Natorp, P., LGEW, p. 99.

¹⁹⁷ Natorp, P., UOSB, p. 148.

Natorp identifies the type of independence required. There are two possible senses of such independence. One is that objects are completely exterior to the mind. This response would invalidate the very concept of objectivity since being an object – as will explain in Chapter 3- is to be a term of an act of thinking. Thinking is a discursive process. It implies establishing relationships. The terms required by the concept of relationship are nothing outside it. The terms do not precede the relationship, but they are established by it as requirements ¹⁹⁸. The object, as a term of the relationship that represents knowledge, is nothing outside of this relationship. The object is placed in front of knowledge and, nevertheless, is grounded by it. Certainly, one could ask how the object can be independent of the act of knowledge and, at the same time, be grounded by it. Natorp answers that this independence is generated by virtue of the process of establishing laws.

The establishment of laws involves a process of abstraction. However, the abstraction does not depart from a given sensa data. From his perspective, the process of abstraction consists in disregarding certain marks of the objects that are given to senses and taking into consideration only certain determinations in order to form a concept. The abstraction depends on the object that is given to the senses. This was the perspective of psychologism, which defines the process of abstraction explaining the genesis of the representation. According to Natorp, on the contrary, abstraction must be defined entirely positively as the choice of a point of view that guarantees the unity of determinations. This is the only legitimate way to interpret the concept of abstraction. The process of the concept formation involves neither disregarding marks nor the removal of marks of a given object to intuition. The negative definition of abstraction is misleading 199. The positive definition of the notion of concept must show the parameter that allows articulating the multiple determinations of the object, that is, the unity of the determination. This unity of determination allows us to establish in advance what elements will be considered in the object of knowledge. The marks that are not considered as part of the object will be a corollary of the chosen point of unity. The choice of the point of view provides which marks are part of the object. The choice of this point of

¹⁹⁸ "Dem Inhalte nach aber ist Denken: Setzen von Beziehung, nichst anderes. Beziehung fordert Termini; aber auch nicht diese gehen der Beziehung voran, sondern die Beziehung setzt auch erst die Termini." Natorp, P., LGEW, p. 99.

¹⁹⁹ "Ich glaube, man erklärt die Abstraction, wo nicht überhaupt untriftig, so doch unzulänglich und nicht von dem richtigen Anfang her, wenn man sie bloss negativ versteht: logisch, als den Abzug eines Merkmals; psychologisch, als Ausserachtlassen, Absehen, Abwenden oder Abziehen des geistigen Blicks von einem bestimmten, thatsächlich doch in der Vorstellung enthaltenen Moment." Natorp, P., UOSB, p. 270.

view establishes the selection of determinations and how to establish the relationship among them. The object is nothing but this complex of relationships that are determined by the choice of the point of view. Abstraction is not a process in which a mark belonging to the object is eliminated but the choice of a determining unity that defines which marks constitute the object under consideration and their forms of relationship. This articulating unity of multiplicity is the law²⁰⁰. The multiplicity of the marks that define the object is only the correlation of the unity that articulates it. The required abstraction is found in the concept of law. In this way, the law can guarantee the independence of the object of knowledge. Likewise, the law can be related to the singular case without losing its universality. The law produces its instances, and the object of knowledge is produced by the law. For idealism, the meaning of the case for knowledge is only to be an instance of legality. The case is not only the subjective appearance but precisely, the particular with respect to the universal that is the law²⁰¹. The correlate in the subjective pole is the appearance. The appearance of the phenomenon is always changing, it changes according to the changes in the state of the subject. On the contrary, the law forms its case in such a way that the object constructed by it is a unity completely independent of any subjective state. This abstraction of the law guarantees its validity regardless of any modification in the state of the subject. The objectively valid is, precisely, what was articulated by the unity of the law 202 .

According to Natorp, this mistake of psychologism consists in a misunderstanding of the meaning of the law. The psychologist's account cannot trace a distinction between law and states of facts. A law can be considered a fact only if by facts it is understood 'being the case', something that could be verified. In this general sense, the law can be considered a fact. However, the problem is to identify the law with a temporarily determined event. The expression: $2 \times 2 = 4$ is a fact in the sense that it is the case. However, in no way does this imply that the operation entails a temporary character. The

2

²⁰⁰ "Der Gegenstand bedeutet positiv das Gesetz; er bedeutet die beharrende Einheit, worin die wechselnde Mannigfaltigkeit der Erscheinung gedanklich geeint und festgestellt wird." Natorp, P., UOSB, p. 271.

²⁰¹ "Für den Idealisten hat im Gegentheil das Einzelne in der Erkenntniss nur Bedeutung kraft des Allgemeinen, dessen Einzelnes es ist; es entlehnt somit alle Geltung, die es in der Erkenntniss beanspruchen kann, vom Allgemeinen, darf dagegen ursprünglich und von sich selbst nichts gelten wollen." Natorp, P., UOSB, p. 278.

²⁰² "Sowie aber die gesetzmässige Auffassung den Gegenstand, das objective Gültige vertritt, so ist die Erscheinung, vor der Reduction aufs Gesetz und damit auf den Gegenstand, der concreteste Ausdruck der Subjectivität. Erscheinung ist die noch nicht im Gesetz objectivirte, mithin noch subjective Vorstellung, sowie die durch die Erhebung zum Gesetz, zum Standpunkte des Allgemeingültigen, zur Einheit gebrachte Vorstellung die gegenständliche ist." Natorp, P., UOSB, p. 273.

law is not a general expression for facts if we define facts as temporarily determined phenomena. Natorp holds:

Gewiß, jedes Gesetz sagt aus, was allgemein stattfindet; sofern man also jedes Stattfinden ohne Unterschied Tatsache nennt, ist jedes Gesetz eine allgemeine Aussage über Tatsachen. Es ist in diesem Sinne Tatsache, daß 2 x 2=4, und Tatsache, daß Widersprechendes nicht gleichermaßen wahr ist u. s. f.; aber zu dem Schluß: also sind alle Gesetze Ursachgesetze, gelangt man nicht durch diesen allgemeinsten Sinn der Tatsache, sondern durch das stillschweigend mitgedachte spezifische Merkmal zeitlicher Bestimmtheit. Ursachgesetze sind Zeitgesetze des Geschehens, und nur sofern man unter Tatsache, im auch zulässigen engeren Sinn des Worts, Geschehen versteht, deckt sich "Gesetz von Tatsachen" und "ursächliches Gesetz". Aber daß 2 X 2 = 4, ist kein Geschehen in der Zeit, weder ein einzelnes noch ein allgemeines, sondern ein Stattfinden, das an gar keine Zeitbedingung gebunden ist oder sie irgendwie einschließt. Dasselbe gilt von den logischen Gesetzen; sie sind nicht Zeitgesetze, folglich nicht ursächliche Gesetze, weder physische noch psychische, oder in solchen begründet, sondern von einer fundamentaleren Ordnung; denn das ursächliche Gesetz ist vielmehr dem logischen, ebenso wie dem mathematischen, unterworfen, nicht das logische, das mathematische dem ursächlichen. ²⁰³

The laws of nature depend on the laws of logic, but logic does not depend on any other science. The laws of logic are constructed without being events in time, that is, events determined by the law of causality. This does not mean that the laws of logic do not apply to temporary events but that temporary events presuppose the laws of logic. The determination of events in time presupposes the laws that regulate all determination in general. For example, any temporary determination implies the possibility of determining the event as identical to itself, that is, A = A. This logical law, the law of identity, grounds the event that takes place. However, no fact of nature can ground this fundamental logical law. This does not imply denying the temporal nature of the thinking process. Indeed, the process of thinking can be considered a phenomenon in nature. Thinking can also be studied as long as it is conditioned by causal laws. It is not denied that there is a process that takes place. It is affirmed that the validity of the laws of logic does not depend on the legality of the generation of representations. The establishment of the laws that regulate how effective thinking is consummated is a problem of a different field. Logical laws

²⁰³ Natorp. P., SP, p. 18.

have universal validity while the legality of the succession of representations is limited. The legality of the thinking process is conditioned while the laws of logic have an unconditional validity²⁰⁴.

The methodological error of psychology leads to conceiving the data of immediate experience as the first in the order of knowledge. It considers that what is given to perception is the most objective since it is what subsists regardless of the act of knowledge. As we showed, the data is the ultimate goal, and its independence is only guaranteed by its reduction to the law. Objectivities are nothing but the products of laws-construction. It is only the unity of the determination of the law that determines the reality of phenomena²⁰⁵.

Against transcendental psychology

Transcendental psychology does not escape the aforementioned problems. Natorp argues against one of the rival schools of the time: phenomenology. Husserl tried to solve the problems of psychologism by appealing to a transcendental consciousness. He claimed that the transcendental approach to subjectivity would avoid the problems involved in psychologism. However, according to Natorp, this resolution of the problem does not escape the critique made against the subjective foundation of knowledge. In the first place, transcendental psychology also confuses the problem of the gestation of knowledge with the question of the problem of its sources of validity. The contents of knowledge are not subjective representations, but ideal elements that can be verified independently of any subject. This distinguishes a scientific phenomenon from a mere fact. The ultimate goal of the process of thinking is to find the legality of the appearance. It seeks to turn the mere appearance into an object. For this reason, the point of view of thinking must be the objectivity of the law. The knowledge cannot in any way obtain legitimation in the subjective processes of the mind. Thinking as a subjective process can only have appearances as its objects. The subjective path that is rejected is not only the one that

²⁰⁴ "Der Inhalt eines logischen Satzes ist nicht, dass unter solchen und solchen Bedingungen Gedanken sich so, unter andern anders verbinden, sondern dass, ohne jede einschränkende Bedingung, gewisse Gedankenverbindungen wahr , davon abweichende falsch sind. Diese Unbedingtheit der logischen Gesetze würde fraglich werden, wenn die überaus bedingte zeitliche Gesetzlichkeit des Vorstellungslaufs für die logischen Gesetze einstehen sollte." Natorp. P., SP, p.19.

seeks the foundation of knowledge in individual consciousness²⁰⁶. Natorp not only rejects the foundation in the individual consciousness but also every attempt to ground knowledge in its genesis. The subjective path affirms that knowledge can be grounded by explaining the processes that are its origin. The rejection of the subjective path of foundation denies both. On one hand, Natorp argues that knowledge cannot be grounded in processes of an individual mind. On the other hand, it is generally denied that knowledge is legitimized according to the activity of consciousness in general, whether it is individual or supra-individual. In the second place, transcendental psychology incurs in the same mistake of psychologism by taking immediate experience as the first factum of knowledge. The point of view of subjectivity leads to assume as a first factum what is given to sensible intuition. The subjectivity standpoint led to this result. Natorp recognizes the peculiarities that phenomenology gives to ordinary experience. Natural knowledge is immediately related to objects. Immediacy is one of the fundamental conditions that phenomenology uses to take the natural experience as an original factum²⁰⁷. Natorp, even recognizing the immediacy of that kind of cognitive relationship, will show why this is not a sufficient condition to grant the privilege that phenomenology gave it. The problem with this type of knowledge is its union with the present perceptual experience. The spontaneous knowledge, which starts from perception, "always takes place in the presence of objects", or, phenomenologically expressed, "has the object in flesh and blood". However, this peculiarity of natural knowledge prevents characterizing this immediate experience as that first factum. Natural experience is anchored in immediate perception, while the prosecution of the explanatory system requires turning on these perceptions to constitute them as representations. For Natorp, there is no possibility of immediate access to the experiences. Even the path for a psychological analysis, which aims to move away from the realm of the merely empirical experience,

²⁰⁶ In this sense, we disagree with the reading of Edgar Scott. According to Scott's interpretation, Natorp only rejects that knowledge can be grounded in relation to the factual awareness of a particular cognitive subject. The problem would be to ground knowledge in the mechanism of a particular consciousness. However, according to Scott, Natorp would not reject the foundation of knowledge in a general consciousness. On the contrary, intersubjective consciousness would be the external parameter that guarantees the independence of knowledge with respect to the consciousness of the individual. The author holds: "At the very least, this suggests that subjectivity consists in being particular to the representations of individuals. That is, in order for objective knowledge to be independent of 'the subjectivity of knowledge', it need not be independent of all consciousness. Rather, being independent of 'the subjectivity of knowledge' means being independent of only 'this or that' consciousness. It means being independent of any particular, individual consciousness. [...]. The intersubjective or shared body of knowledge provides an independent standard against which any individual knower must measure her representations" Cf. Edgard, S. 2008, p.58.

²⁰⁷ Husserl, *Ideas I*, §24.

can only return from the objectification of the human spirit. Thus, the methodology that starts from ordinary experience turns into an "absolute idealism", since it stops the flow of objectification of living in the rigid conceptual elements that cannot run parallel to the progress of the spirit. On the contrary, the critical method does not face the danger of absolutism. Faced with this absolutism of the regressive path, it is pointed out that the transcendental regressive method hides a progressive element while its reduction occurs from the fact of the development of the spirit²⁰⁸.

We can summarize Natorp's criticisms of psychologism as follows:

First, Natorp's criticism is based on the distinction between the logical and psychological aspects of cognition. To be a grounding science of knowledge, logic must be an objective science. The logic does not deal with the process of knowing. Its task is to find the laws that govern knowledge as a result. Psychologism confuses the laws of knowledge with the rules that regulate the psychic life of empirical subjects. Consequently, it aims to legitimize a fundamental science (logic) in a special science (psychology). This conception is circular. Psychology cannot provide by itself a definition of truth, but it requires a concept of truth that legitimizes its own propositions. In its rejection of the definition of truth offered by logic, psychology becomes circular. This error leads to confusing the problem of genesis with the problem of validity. The foundation of knowledge does not require an explanation of the way in which knowledge is generated but should explain the way of legitimization of cognitions.

Second, we showed that this methodological mistake leads psychology to take the object of knowledge as something given. This mistake is the product of "naive thinking" that considers objectivity as something that is given to the mind. From this perspective, the completely determined object is given to sensible intuition, and the mind generates its concept by a process of abstraction. The object is what is given to intuition and the concepts are constructed abstracting certain marks from the objects. Natorp shows that thinking does not require anything external to itself in the construction of its object. On the contrary, the objects of cognition are generated, they are produced and not given. As we explained, this process of production of objects is the generation of laws. The law is not an abstraction of concrete cases but produces its instances. Knowledge seeks to

²⁰⁸ "Die transzendentale Methode, als immanente, ist in solcher Gefahr nicht; sie ist selbst fortschreitend, entwickelnd, auch unendlicher Entwicklung gewachsen." Natorp, P., KMS, p. 199.

establish the case as an instance of the law. There is no element given to thinking. Having started from the subjective point of view, psychologism considers the immediate data as the paradigmatically real, as the first for the act of knowledge. According to Natorp, the conception of psychology leads to the loss of the concept of objectivity as it makes logic dependent on psychology. For Natorp, the validity of knowledge is precisely independent of the mental processes of factual subjects. Thus, on the one hand, the object is in some sense the most dependent, since it is nothing more than a construction of thinking expressed in the law. However, as opposed to mere appearances, the fact is also independent. The law guarantees its independence from all subjective consciousness. In fact, the only guarantee of independence, required by the object of knowledge, is its foundation in the law. The laws of logic are not facts conditioned by time. On the contrary, temporary events involve the laws of logic. The facts, the temporarily determined events, suppose the laws of thinking.

Psychologism starts from the methodological error of taking the subject as a starting point and the object as what is contrasted as part of psychic life. The object given to intuition is the first element. Concepts are generated through a process of abstraction. As we saw, this approach cancels every possibility of the foundation of knowledge and, more specifically, every possibility of the foundation of objectivity of facts. Therefore, an accurate approach to the relation between conceptual and intuitive representations necessarily demands the abandonment of this point of view. Natorp explains how considering the construction of concepts as laws solves the problems raised above. We will further develop this relationship between concepts and law in chapters 3 and 4. In what follows, we will study why the point of view of logicism is also insufficient.

2.2. Against Logicism.

Natorp argued that the perspective of psychologism confused the problem of the genesis with the problem of the validity of knowledge. Logic, Natorp showed, should not start from any initial data given to thinking, as psychologism considered. The intuitive given content cannot be a point of departure. Rather, logic must show how thinking constructs its objects by its own means. To establish his position, Natorp will argue against the representatives of logicism. As Frege explains, the logicism program aims to show that "arithmetic is a branch of logic and need not borrow any ground of proof whatever from either experience or intuition"²⁰⁹. Thus defined, it could seem as if the logicist program has only to do with the problem of the foundation of mathematics. However, it must be borne in mind that by the nineteenth century, 'logic' was almost a synonym of a theory of knowledge²¹⁰. The problem was not only to ground mathematics but also to clarify the role that the mathematical determinations play in the constitution of objectivity²¹¹. Then, to show how mathematics is grounded in pure thinking is also the first step to exhibit how objectivity, in general, can be constructed by conceptual determinations. Natorp also aims to show that logic is built on the basis of pure thinking, without any reference to anything given to intuition, in the Kantian sense of the word. He argues that thinking can build the foundations of experience. This perspective was shared by that time with the representatives of logicism. This philosophical current also held that the process of cognition could not depart from something given to intuitions. They wanted to exhibit that logic could be grounded by thinking. Thus, logicism challenged the core of the Kantian program by denying any reference to any intuition in the construction of objectivity. There is no place for a distinction between sensibility and understanding, nor between intuition and concepts because thinking alone can produce the form and the content of objectivity²¹².

Natorp agrees on the need of a logical foundation and of a revision of the distinction between intuitive and conceptual representations. However, Natorp did not

²⁰⁹ "Logicism is the thesis that mathematics is reducible to logic, hence nothing but a part of logic". Carnap, R. 1931, p. 91. According to Frege, arithmetic is a branch of logic. For this reason, it does not depend on intuition. Frege claims to have proved this point in his *Grundlagen der Arithmetik*. Cf. Frege, G., 1893, p.1. ²¹⁰ Cf. Dufour, E., 2010, p. 19.

²¹¹ Dewalque, A., 2009, p.45.

²¹² Frege holds: "...das von jedem durch die Sinne oder selbst durch eine Anschauung apriori gegebenen Inhalte absehende reine Denken allein ans dem Inhalte, welcher seiner eigenen Beschaffenheit entspringt, Urlheile hervorzubringen vermag, die auf den ersten Blick nur auf Grund irgendeiner Anschauung möglieh zu sein scheinen." Frege, G., 1879, p. 55. Later, we will highlight the role of intuition for geometry.

share with the logicists all the points of their program. For Natorp, logicists also misunderstood the relationship between intuitive and conceptual representations on the grounds of a methodological mistake. If the mistake of psychologism was to depart from the perspective of what is given, logicism will absolutize the perspective of the concept. Natorp showed that investigation cannot depart from the perspective of the genesis of the representation in the mind. Psychology cannot be the ground of knowledge. Natorp will argue that formal logic is insufficient for a foundation of knowledge. The foundation of knowledge in formal logic assumes a separation between the principles of knowledge and the object that is known. He will argue that the foundation of knowledge requires a transcendental logic. The aim of this section is to analyse Natorp's arguments in relation to the inadequacy of the position of logicism. Formal logic is insufficient for a logical foundation of the sciences. This methodological error will lead to logicism to a misunderstanding of the relationship between intuition and concepts. As we shall see, the position of logicism in relation to the formation of logic will lead to a separation between the universal and the particular. Arguing against this direction, Natorp introduces the conception that thinking can produce the object of knowledge by itself. However, the process of the formation of concepts will be considered in a very different way from that proposed by the logicist program. Natorp will maintain a synthetic grounding of knowledge. In this conception, the creation of the instances will be explained by reference to the formation of law. The conception of logicism will lead to an analytical conception of the foundation of knowledge. First, we will briefly focus on the debate with logicism to put Natorp's arguments in context. Second, we will study Frege's positions, considering that Natorp argues primarily against this conception²¹³. Third, we will analyze Natorp's common points with this position and the main objections. We will show that the methodological error of logicism leads to an incorrect formulation of the relationship between intuitions and concepts.

2.2.1. Introduction to the Debate.

As we mentioned, at the beginning of the nineteenth century, philosophy was experiencing a crisis. One of the reactions to this crisis was psychologism. However, another line of philosophers argued that philosophy must follow the mathematical

²¹³ According to Charles Parsons, Natorp seems to be only familiar with *The Foundation of Arithmetic*. Parsons, C., 2014, p.13.

method. The debate can be divided into two closely related problems. The first problem concerns the relationship between philosophy and mathematics. According to Natorp, the relationship between these two disciplines underwent a profound modification with the decline of Kantian philosophy. On the side of mathematics, the progress achieved by this discipline at the beginning of the 19th century led to the loss of the close relation it had with philosophy. Mathematicians believed they can dispense with the instruments provided by philosophical disciplines and intended to ground their procedures with purely mathematical methods. The philosophical analysis of the exact sciences was still largely based on the assumptions inherited from Aristotelian logic, which is increasingly insufficient to ground the course of the new mathematics²¹⁴. Besides, the development of non-Euclidean geometries contributes to the discredit suffered by philosophy. Philosophical presuppositions, this time more related to the Kantian paradigm, cannot account for the new developments in geometry. Then, new systems emerge that try to overcome the logical foundation of the exact sciences based on the Aristotelian-Kantian assumptions, and to generate a more fruitful and consistent logical system with the new mathematical model. Philosophy moves away from the rigor that formerly the mathematical method had provided. According to Natorp:

Die alte, nach beiden Seiten fruchtbringende Verbindung zwischen Philosophie und Mathematik schien eine Zeitlang sehr gelockert. Was die Mathematik vielleicht einmal der Philosophie zu danken hatte, die Strenge des Beweisverfahrens, ja den ganzen Begriff des formalen Aufbaus einer Wissenschaft, das hat sie längst aus eigener Kraft und eigenem Trieb so in sich aufgenommen, dass sie mit gutem Grunde glauben darf, darin von der Philosophie nicht viel mehr lernen zu können. Diese dagegen schien die nüchternen Bahnen, in denen sie manches Jahrhundert mit den strengen Wissenschaften Hand in Hand gegangen war, zeitweilig ganz zu verlassen und sich in spekulativen Abenteuern zu gefallen, die das streng geschulte Denken des Mathematikers zu allererst zurückstossen mussten.²¹⁵

The metaphysical tendencies of post-Kantianism increasingly led philosophy to depart from the rigor to which it originally aspired. The mathematical method was a way to find

⁻

²¹⁴ Cf. Jacquette, D., 2006, p.11 ss; Detlefsen, M, 2004, p.55.

²¹⁵ Natorp, P., LGNM, p.177.

that rigor of philosophical thinking that seemed to be lost after Kant. Mathematics was its refuge to ground conceptually the procedure of logic. The rigor of mathematics and its formal character provided the philosophy with a safe method of analysis. This rigor that philosophy had lost could be recovered based on the firm ground of the mathematical method. Just as Spinoza and Leibniz had done in modernity, the crisis of philosophy could be faced by adopting the model that science, especially mathematics, applied successfully. Rigorous reasoning could be guaranteed if they could adopt the mathematical method. The possibility of defining symbols and creating a system of rules that defined how these signs relate to each other would avoid the vagueness of natural language. Philosophy could have the rigor of the mathematical method by following procedure in combinatorial art and calculation. Philosophical reasoning could guarantee its validity by following these calculation rules. The logicists to which Natorp refers are continuators of this current of reasoning initiated by Leibniz²¹⁶.

Secondly, at the end of the 19th century and the beginning of the 20th century, a revolution of the logical method arises and, with it, a debate regarding the relationship between logic and mathematics. The question is whether mathematical relationships can be deduced from logic or if they constitute a peculiar domain of relationships. According to some thinkers of the time, mathematics can be based on a series of fundamental logical concepts. Mathematical laws could be derived from a logic of thinking. Mathematics would be a branch of formal logic. The axioms of mathematics could be derived from the laws of logic. If arithmetic is an extension of logic, a study of the fundamental logical principles would suffice to provide this discipline with a solid foundation. Therefore, in general, the project of arithmetic logic is a characteristic problem of the time. It seeks to find the legality that determines the mathematical object. As Natorp explains there are two separate closely related problems. On the one hand, logic receives the influence of mathematics. Logic itself starts receiving a mathematical treatment. On the other hand, mathematics aspires for a logical foundation. Mathematicians want to show that the objects of mathematics can be built without any reference to intuition but purely

²¹⁶ Cf. Placencia, L., Espinosa, R., 2017, p. XI.

conceptually²¹⁷. This problem occupies Natorp's thinking from his early writings²¹⁸. The question is, on the one hand, to establish the role that those mathematical determinations play in the construction of the object in general and, on the other, to investigate whether the determinations of the mathematical object could be established from logical laws. This problem about the status of the mathematical object has a peculiar relevance to Natorp's thinking. The problem is not only on the status of mathematics but the more general problem of the nature of the object of cognition. The problem is whether the object of knowledge, which is grounded primarily on mathematical determinations can be reduced to pure logical relationships. Lastly, the question is whether the object can be purely conceptually determined. Natorp will agree with the logicist program on the need to ground knowledge in logic, which is one of the main points of the logicist program. Indeed, the object of experience is firstly defined by mathematical determination, and mathematics requires a logical foundation. While their ideas differ internally, representatives of nineteenth-century logic converge on some fundamental ideas. The question can be put into two separate problems. On the one hand, logic aspires to proceed mathematically. It aims to proceed purely conceptually without any reference to intuition. On the other hand, mathematicians were searching for such a method to avoid any reference to intuition, to an intuitive given content. Within this problem, the current of logicism emerges. In what follows, we will study the main thesis of this project, focusing on the problem of the relation between intuition and concepts.

2.2.3. Main Thesis of Logicism.

Rejection of mathematical psychology

One of the common points of this current is its adversary. Logicists reject the foundation of logic in psychology and, more specifically, mathematical psychologism²¹⁹.

_

²¹⁷ Natorp explains: "Ich hätte nicht den Mut, mich, als Nichtfachmann, an Mathematiker zu wenden, wenn ich nicht sachliche Gründe dafür zu erkennen glaubte, dass die Logik, die Erkenntniskritikenge Fühlung mit der Mathematik zu suchen hat; nicht um sie zu belehren, mehr, von ihr zu lernen, genauer, ihre Mitarbeit an einigen ihrer schwersten Aufgaben zu erbitten, die ohne die Mithilfe der Mathematik nicht zu bewältigen sind. Ich denke dabei nicht so sehr an einen besonderen Zweig unserer Wissenschaft, dem, nachdem er lange in aristotelischer Tradition erstarrt war, durch die mathematische Behandlung neues Leben zugeführt worden ist: die Syllogistik, sondern ich denke an die ganz allgemeine Tendenz der neueren Mathematik, sich zu einer rein logischen Gestaltung durchzu arbeiten, so dass die Berufung auf "Anschauung" mehr und mehr überflüssig wird." Natorp. P., EGM, p. 2. See: Porta, González, M. A., 2011, p. 205 ss.

²¹⁸ Cf.: Natorp. P., QQ, ZLGNM, EGM, NTE, LGEW.

²¹⁹ Cf. Natorp, P., LGEW, p. 3.

For the logicists, the study of the formation of mental representations has no relation to the formal sciences. In fact, the intrusion of psychology into logic has hindered the task of a philosophical foundation of mathematics. For logicism in general, logical laws cannot be considered abstractions of the thinking process. The process of formation of the subjective representation must be distinguished from the conceptual definition of legality. The logical law is not obtained from a determination of the thinking process. The logic consists neither in the study of the processes of forming representations nor, consequently, in the investigation of the possibility of the convergence of mental representation with things. Subjective representations cannot have the force of law and, therefore, cannot be the foundation of mathematical statements. Psychologism confuses the logical law that governs mathematical statements with the natural laws that rule mental processes. The knowledge of the subjective formation of a representation does not allow us to know any property of the legality of thinking. The law that governs objectivity cannot be obtained from an intuitive fact given to sensation. The objectivity of a factum presupposes its being independent from the point of view of the subject. If the fact were dependent on the subjectivity, it would be a subjective fact. The objectivity presupposes the independence of the fact from our sensations. While psychology analyzes the problem of subjectivity, logic studies objective thinking, and "there is nothing more objective than arithmetic."220 The domain of the objective is heterogeneous with respect to the scope of the merely subjective.

The logicists agree on the need to radically separate the logic from empirical psychology. They oppose the possibility of founding logical concepts such as validity or truth in psychic mechanisms. Psychology cannot determine the conditions of truth, since being true and taking something for true are completely different phenomena. The laws of the genesis of representation and its correspondence with a represented object have no inherent connection with the truth. The same applies to the study of historical genesis. The study of the historical conditions in which a mathematical discovery takes place has no relation to the conditions of validity of mathematical truth, a problem that constitutes the object of logic. Psychology confuses the question of the fact with the question of validity. The problem of logical justification must be separated from the problem of how

²²⁰ Frege holds: "Nein, mit Gefühlen hat die Arithmetik gar nichts zu schaffen. Ebensowenig mit innern Bildern, die aus Spuren früherer Sinneseindrüncke zusammengeflossen sind. Das Schwankende und Unbestimmte, welches alle diese Gestaltungen haben, steht im starken Gegensatze zu der Bestimmtheit und Festigkeit der mathematischen Begriffe und Gegenstände." Frege, G., 1884, p.v.

the process of thinking takes place. The problem of logic is restricted to the domain of pure thinking. There should be no intrusion of intuitive data taken from the experience. The problem of the logical law is restricted to the purely conceptual. For the logicists, the study concerning the problem of the legality of thinking cannot have any reference to intuition, be it pure or empirical. The logical law concerns the realm of pure thinking, the realm of pure concepts. To ground the legality of thinking, one cannot appeal to pure intuition either. While they agree that the laws of logic are necessary and sufficient foundations of arithmetic, logicists also reject the idea that pure intuition is necessary for the foundation of the science of numbers. Against Kant, logicism believes that the idea of number is independent of both empirical intuition and pure intuition. The logical law on which mathematics is based is a product of pure thinking²²¹. Thinking is particularly free in arithmetic because it has freed itself from its restriction to intuition, in this respect the representatives of logicism agree. For Frege, for example, this point differentiates arithmetic from geometry. Geometric laws "govern the domain of what can be intuited spatially" while arithmetic laws govern the domain of everything thinkable²²². Arithmetic is in this sense an analysis of the laws of thinking itself. Frege is one of the authors who develops extensive arguments against psychologism and devotes much of his work to prove that arithmetic is based on logic and does not depend on any kind of intuition.²²³

The definition of logic

The representatives of logicism agree on the need to base mathematics on logic. Logic must provide the basis of thinking, and with it the fundamental legality of everything that falls under its scope. Logic is the science of thinking²²⁴. However, as we saw, the concept of thought must be separated from that of the act of subjective thinking²²⁵.

The goal of logic is to investigate the foundations of true thinking and has no relation to subjective psychic acts. The problem of knowledge is independent of the act of thinking of a factual subject. Logic does not study the genesis of representations - how

²²¹ Natorp, P., LGEW. p.4.

²²² Cf. Frege, G., 1884, p. 20 ss.

²²³ "In meinen *Grundlagen der Arithmetik* habe ich wahrscheinlich zu machen gesucht, dass die Arithmetik ein Zweig der Logik sei und weder der Erfahrung noch der Anschauung irgendeinen Beweisgrund zu entnehmen brauche." Frege, G. 1893, p.1.

²²⁴ Frege, G., 1979, p.5.

²²⁵ For Frege, this is the definition of the science of logic. However, this same definition is what has led to the error of psychology. Frege, G., 1979, p. 4.

they are empirically obtained. The problem of logic is the problem of the justification of truth. We legitimize our statements through the laws of inference. For this reason, the laws of valid inferences are the object of the study of logic. We must distinguish the way in which inferences are effectively made from their legitimation. Logic is the science of inferences in this last sense²²⁶. Logic includes the entire domain of thinking, providing the laws that determine every being. This science is not restricted to any particular field²²⁷. In this sense, logic is the ground of every object of cognition, because it bases the legality of every being. Everything that falls into this area is governed by this legality. According to this conception, logic provides the most general truths of thinking, and then the logical laws that govern all constructions of thinking. 228 Concepts are expressions of thinking functions. They are expressions of how thinking proceeds in the construction of its objects. Logic can be defined as a science of concepts. Mathematics is based on the laws of thought and arithmetic purely and exclusively in this area. Logic is the science of the purely conceptual field and mathematics is based on this logical space. The fundamental idea on which logicism is based affirms that mathematics rests on a series of fundamental logical concepts that are a pure product of thinking. The logical realm is that of pure thinking. Arithmetic, as it is based on logic, is also a pure a priori science. Therefore, a demonstration of the logical derivation of arithmetic is also proof of its analytical and a priori character. According to this conception, arithmetic is "a more widely developed logic, and each arithmetic statement would be a logical law, although a derived law."229 Therefore, the fundamental objective of logicism is to reduce the fundamental concepts and principles of mathematics to purely logical concepts and principles. The logic thus conceived is self-sufficient, it does not require a subsequent logic that legitimizes it. For this conception, logic requires neither a metaphysical foundation nor a theory of knowledge. On the contrary, logic is the founding science that provides the foundations of mathematics and does not require further legitimation.

²²⁶ Frege gives the following definition of logic: Logic is concerned only with those grounds of judgments which are truths. To make a judgment because we are cognizant of other truths as providing justification for it is known as inferring. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic. Frege, G., 1979, p. 3.

²²⁷ For Frege, the task of logic: "... is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter. We must assume that the rules for our thinking and for our holding something to be true are prescribed by the laws of truth. The former is given along with the latter. Consequently, we can also say: logic is the science of the most general laws of truth. Frege, G., 1979, p. 128.

²²⁸ "[t]o say that the laws of logic are the most general laws of truth is to say that they are the most general truths" [Ricketts, 1986, p. 80]. Quoted in: Suillivan, P, 2004, p. 727.

²²⁹ Frege, G., 1884, p. 99. Frege does not include geometry because the latter requires an intuitive element.

Validity and truth

The verification of the validity of mathematical reasoning is a logical problem, a matter of calculation.

The validity of logical propositions does not depend at all on the mental mechanisms that generate the representation of the law. The genesis in empirical consciousness is the problem of psychology. Logic, on the contrary, focuses on the study of valid inferences. Validity does not refer to how we actually think but to how we should think. Therefore, arithmetic propositions cannot be justified through the explanation of their empirical genesis in the mind. One of the consequences of mathematical psychology is that the truths of mathematics would be limited to the contingency of the peculiar constitution of the nature of the human mind. The validity of arithmetic judgments would be restricted to the field of human knowledge. On the contrary, mathematical truths are valid for the whole scope of the rational and not only for this or that particular rationality. Logical truths are universally valid, independent of all time and space. Logicism considers that arithmetic judgments have universal validity. Frege argues that the conception of psychology culminates in an elimination of the very concept of truth. The conception of psychologism necessarily leads to relativism. If logic were founded on the mental act, mathematics - which relies on logic - would lose all possibility of holding any objective validity. For psychologism, thinking proceeds according to natural laws or laws whose essence does not differ from natural laws. Then, the concept of truth loses all its meaning. Psychologism confuses the laws of thought with the natural law. The psychological analysis is precisely opposite to a rational procedure. The validity of logical reasoning cannot be based on the contingency of the subjective act of thinking. Logic governs every true being (Wahrseins) and not the holding it as true (Fürwahrhaltens) of a particular subject²³⁰.

The pursuit of truth is the characteristic feature of all science. All science aspires to the truth²³¹. However, they do not have the truth as an object of study²³². On the contrary, truth is for logic its most proper object. Just as ethics is the science that studies

⁻

²³⁰ Frege, G., 1893, p. XVII.

²³¹ In general, "the objective of scientific work is the truth". Frege, G., 1979, p.2.

²³² Frege, G., 1979, p. 3. In relation to this point, Peter Suillivam comments: "Frege's commitment to the a priority of logic is intelligible only if he recognized a distinction of kind, and not merely of degree, between the most general laws of truth and laws of special sciences". Sullivan, P., 2004, p. 727.

the concept of good, and aesthetics the concept of beauty, logic is the science of truth. The laws of logic are the laws of true thinking²³³. This science exhibits the laws of thinking and the laws that thinking should follow in the search for truth in general. As the laws of logic are the most general truths, this science can be defined as the "display of the content of the concept of truth."²³⁴ The way to display the content of this concept is by displaying the laws of inference²³⁵. The laws of truth are the laws of inference. The truth of each logical law can only be justified using another logical law²³⁶. For logicism, the validity of mathematical reasoning is based on its subjection to the laws of logic. Meanwhile, mathematics relies on logic, the possibility of mathematics to arrive at true propositions is based on the subjection to the laws prescribed by this fundamental science. The verification of the validity of mathematical reasoning is a logical problem. The legitimacy of a mathematical judgment is based on the logical law. Valid judgments are those that are based on the laws that logic prescribes. Logicists consider that it is necessary to generate a symbolic system and define calculation rules that allow them to operate with these symbols. Logic would be the discipline that determines the correct formation of symbols and defines the legitimate modes of relationships. The need to generate a formal vocabulary - free from natural language ambiguities - and syntax rules is recognized; that is, definitions of terms and laws that regulate the relations among the defined elements. Definitions generated through logical language should not reproduce the structure of natural language but the language of pure thinking. The symbolism must be generated so that it can be an expression of this legality of pure thinking. The logical definition must be based on the structure of thinking and not on natural language. Furthermore, the definitions that can be extracted from natural language are taken from the experience. On the contrary, the logical vocabulary is creative. Logic creates concepts and gives the rules, also purely conceptual ones, that allow operating with concepts as terms. These concepts are creations of thinking and not results of an abstraction from intuitive content, as concepts of natural language. The symbols of logic are not abstracted from any element outside thinking but are created by thinking itself. The syntax of logical language is not an abstraction of the actual use of natural language but the expression of

²³³ "The word 'true' can be used to indicate such a goal for logic, just as can 'good' for ethics and 'beautiful' for aesthetics". Frege, G., 1979, p. 128. Also, p.4.

²³⁴ "It would not perhaps be beside the mark to say that the laws of logic are nothing other than an unfolding of the content of the word true." Frege, G., 1979, p.3.

²³⁵ For this reason, Frege defines logic both as the science of valid inferences or as the science of truth. Frege, G., 1979, p.88

²³⁶ CF. Frege, G., 1893, p. XVII.

the way in which thinking operates in the formation of the concepts. The conceptual system is complete, no further grounding of the definitions or of the rules of operation is required. The logical system has intrinsic legitimacy. Logical language must be separated from natural language²³⁷. However, considering the differences mentioned, an analogy can be drawn, logic is to thinking what grammar is to language. The generation of this logical language is one of the greatest contributions of logicism and, particularly of Frege²³⁸. As long as the justification of arithmetic judgments rests on the possibility of their derivation from logical laws, the calculation can be reduced to derivation functions. The mathematical calculation is legitimized in the logical deduction. For Frege, for example, the fertility of a definition is determined by the possible use that can be made of it in the deductions. A legitimate definition, being free of contradiction, can always be an element of a demonstration. However, the absence of contradiction is not a firm probative foundation. Therefore, it is necessary to examine the general logical principles that govern the chains of reasoning. Frege identifies the calculation operation with the logical derivation. "to calculate would be to deduce." ²³⁹

Pre-eminence of the analysis over synthesis

According to logicism, all the statements of arithmetic are analytical since they can be derived from logical laws. The distinction between analytical and synthetic judgments must be understood in relation to the legitimacy of the judgment and not the problem of its genesis. The problem of its formation is not relevant to mathematics. Empirical psychology studies the genesis of judgment. For the problem of arithmetic statements, it must be considered the way in which they are legitimized and not how they are formed. The problem is the validity of the judgment and not its genesis in the empirical consciousness. The judgments of arithmetic will be considered analytical if its foundation can be obtained purely from logical laws. Arithmetic judgment will be considered synthetic if to ground its legitimacy, it is necessary something outside the primitive laws of thinking. Frege explains in his *Foundations* that arithmetic truths are

²³⁷ Frege, G., 1979, p. 6.

²³⁸ Imbert, C.,1972, p. 139

²³⁹ Frege, G., 1884, p. 99.

²⁴⁰ Cf. Frege, G., 1972, p. 26 ss. §3.

the result of a logical derivation and that, according to the definition he has provided, arithmetic judgments should be considered analytical and non-synthetic judgments.

The Kantian definition of analytical and synthetic must be reformulated, both for concepts and judgments. The Kantian confusion is based on an overly narrow definition of synthetic judgments. Thus, in the first place, as we mentioned, Kant would have confused the problem of genesis with the problem of justification. The Kantian distinction between analytical judgments and synthetic judgments would fall into the same error as a psychologism, in a confusion of the problem of genesis with the problem of validity.

Kant's second mistake is to define the concept as a sum of properties. This error originates in a prejudice inherited from Aristotelian logic. The definition of the concept as a sum of properties and the attributive conception of judgment derives from this fundamental error: taking elements of natural language to express formal language. Kant's mistake when considering arithmetic judgments as synthetic judgments is based on this incorrect definition of the notion of concept. The understanding of the concept as abstraction of common marks led Kant to consider that arithmetic judgments are nonanalytical judgments. The definition of concepts as the sum of properties and the definition of judgments as the attribution of a predicate to a subject must be rejected. These definitions are fruitful for an understanding of natural language, but they do not express the way in which pure thinking operates. The concept must be understood as a function. The function defines the extent of its content, and the content is limited to what is expressed in the function²⁴¹. The rejection of the synthetic character of the judgments of arithmetic is based, first, on the rejection that knowledge requires at any point to resort to intuition. For Frege, the judgments of arithmetic are built on the basis of pure thought. Second, Kant inherits the prejudices of Aristotelian logic in his definition of the notion of concept. This conception of concepts explains the process of formation of concepts following the model of Aristotle metaphysics, based on the relation of substance and accident. Frege considers that it is necessary to reformulate the notion of concept. Third, Kant distinguishes the analytic from the synthetic by confusing the problem of genesis with the problem of validity. It is necessary a redefinition of what is understood by the distinction between analytical and synthetic. On this basis, Frege states that the analysis can be amplifying²⁴². Frege considers that analytical judgments allow us to increase our

²⁴¹ For Imbert, "the core of Frege's work is the identification of the concept and function." Imbert, C., 1972, p.208n.

²⁴² Frege and Couturat argue in the same direction. Cf. Natorp, P., LGEW, p.19.

knowledge in some way. Through these judgments, certain aspects are revealed that, although contained in other concepts, had not been put in evidence. According to the Kantian conception, judgments of this kind should be considered synthetic. For Frege, the legitimation of the judgments of arithmetic can be obtained purely from logical laws. While they can be derived purely from logical laws, these judgments should be considered analytical. However, new elements are extracted in the conclusions of the arithmetic reasoning that were not contained in the previous laws. In this sense, the judgments of arithmetic are analytical and amplifying. That is, analytical judgments allow us to increase our knowledge. The demonstration of an arithmetic judgment may require a variety of definitions. The grounded judgment was contained in the definitions but required a peculiar act so that its legitimacy could be revealed²⁴³. The application of the laws of logic allows us in this sense to expand our knowledge without implying that its propositions are synthetic²⁴⁴. The laws of logic have an intrinsic foundation. A logical law, as we have already observed, can only be grounded if it can be legitimized by another logical law²⁴⁵. The application of a law is valid if this law can be reduced to another subsequent logical law. The propositions of logic are analytical. To prove that the laws of arithmetic are all analytical, it must be shown how they are deduced from the laws of logic. This task would require demonstrating that every arithmetic statement can be deduced from a logical law²⁴⁶.

In summary, Frege's rejection of the synthetic character of arithmetic is based, first, on a redefinition of the concepts of analysis and synthesis. Second, for Frege, arithmetic judgments can be amplifying without any reference to intuition. Third, Frege reformulates the notion of concept. The concept should not be understood as a sum of

²⁴³ Several mathematicians of the time share the conviction that it is necessary to separate the geometry of arithmetic at this point. Cf. Detlefsen, M., 2004, p.54.

²⁴⁴ For Michael Detlefsen, Frege fails to prove that mathematical inferences can be analytical and "epistemically productive." Detlefsen observes: "Frege's conception of mathematical inference was thus faced with two apparently competing demands: on the one hand, the need to endow analytic judgments with tacit content so as to enable analytic inference to be epistemically productive; and, on the other, the need to restrict the mechanisms producing tacit content in such a way as to guarantee that synthetic content can never be tacitly contained in what passes for analytic content. In the end, I believe, he failed to meet these two demands adequately. He did not succeed in providing a set of basic laws and a criterion of tacit content the pair of which were guaranteed to permit only the production of analytic truths as tacit contents of the basic laws. Nor did he manage to ensure that the epistemic productivity sustainable by means of his mechanisms of tacit content production are capable of matching those which may be observed to hold in arithmetic." Detlefsen, M., 2004, p. 64.

²⁴⁵ "Die Frage nun, warum und mit welchem Rechte wir ein logisches Gesetz als wahr anerkennen, kann die Logik nur dadurch beantworten, dass sie es auf andere logische Gesetze zurückführt." Frege, G., 1893, p. XVII

p. XVII. ²⁴⁶ Cf. Frege, G., 1884, pp.101 ss.

properties of things. The Fregean conception is based on a redefinition of what is understood by the distinction between analytical and synthetic. Positively, the judgments of arithmetic are analytical because their propositions can be justified from the laws of logic.

Application of mathematics

As we mentioned, the laws of arithmetic are based on pure thinking. They have no reference to any intuition, neither pure nor empirical. Therefore, the arithmetic applied to intuition loses its distinctive feature and incorporates a foreign element to it. The laws of number apply to objects of thought. Arithmetic laws determine the relations of pure thought as opposed to the natural laws that regulate the order of empirical phenomena. Natural law is the term that mediates between arithmetic and its application to phenomena. The laws of arithmetic may govern the domain of natural law. Therefore, it can be said that the laws of numbers are laws of laws.²⁴⁷ The arithmetic law, subject to the logical law, governs the domain of concepts. Arithmetic can rule in the order of intuition only because it regulates the judgments of physics, which establish the laws of the natural world. Arithmetic is based on pure concepts. The laws of logic are necessary and sufficient conditions for the legitimation of the judgments of arithmetic.

For Frege, the laws of arithmetic differ from the laws of geometry. The distinction between the mode of legitimization of geometry and arithmetic is one of the features of Frege's logicism²⁴⁸. Frege's position on geometry is much closer to Kant's conception. According to Natorp, this is the breaking point of logicism in two directions. Dedekind and Cantor follow Frege as they appeal to intuition. They consider that intuition is necessary to ground geometry. Russell and Couturat oppose this idea and propose to ground mathematics in general on pure thought. As we will see later, Russell and Couturat are closer to Natorp than to Frege at this point²⁴⁹. In relation to its origin, the laws of

²⁴⁷ Cf. Frege, G., 1884, p. 99.

²⁴⁸ Cf. Detlefsen, M., 2004, p. 64.

²⁴⁹ Natorp holds: "Unter den Mathematikern etwa seit Kants Zeit findet man denselben Zwiespalt der Ansichten: eine ältere, deutlich von Kant beeinflußte Richtung, die aber nur noch wenig Anhänger zu zählen scheint, hält an einem Sonderanteil der Anschauung neben dem reinen Denken bei der Begründung der Mathematik, wenn nicht der ganzen, dann doch der Geometrie, noch immer fest; gerade die vorwärts strebenden aber, an der Spitze Frege, Dedekind, Cantor und schon früher Graßmann, im Ausland, um nur die jüngsten und eifrigsten zu nennen, Russell und Couturat, verwerfen diesen Dualismus ganz und arbeiten mit Anstrengung daran, den Bau der Mathematik rein auf logischem Fundament zu errichten." Natorp. P., LGEW, p. 3.

number are a pure product of thought. Arithmetic originates only in thinking and does not require any reference to intuition. On the contrary, geometry is based on thinking but needs intuition for the construction of its objects. The geometric law is not just an extension of the logical law, as in the case of arithmetic. Therefore, the judgments of geometry are synthetic, because to legitimize their statements the laws of geometry cannot be based exclusively on the logical law as in the case of arithmetic. Arithmetic, as we mentioned, can always be justified by resorting to the logical law. Therefore, all the arithmetical judgments are analytical. Arithmetic, as it is a pure construction of thought, is identified with logic. The laws of number apply to the entire field of thought. Arithmetic, unlike geometry, regulates everything conceivable and not only the objects of possible experience. The universality of the arithmetic law is based on the universality of the logical law. The scope of the laws of arithmetic is universal. Arithmetic, as an extension of logic, valid for every object of thinking. Conversely, the whole field of thinking is governed by the laws of number. The whole field of thought is countable, and it is subject to the laws of arithmetic. Geometry is applied to what can be intuited spatially, which does not cover everything conceivable. The laws of geometry do not have the universality of the laws of arithmetic.

In sum, we could identify the following main features of logicism. In this characterization, we have attended to the central points on which Natorp focuses. From the above, we can highlight Natorp shares with logicism the following thesis: Logicism rejects any attempt to ground logic on intuition, be it pure or empirical. The laws of logic owe nothing to any intuition. The domain of logic is purely conceptual. The laws of thought are not founded on intuitions, neither empirical nor pure. Logic is the science of thought. Concepts are the pure functions of thinking. Arithmetic is grounded on logic. Then, arithmetic has a foundation in pure concepts of thought. Arithmetic, as long as it has a foundation in logic, is legitimized in the laws of thinking. Some logicists, such as Frege, consider that there is a necessary reference to intuition in geometry. The logicists agree that the conceptual foundation is a necessary and sufficient condition for arithmetic. In the case of geometry, some consider it a necessary but not sufficient condition. Logic is the science of valid inferences. Thus defined, the logic must generate a symbolic system and define calculation rules that allow operating with these symbols. Logic would be the discipline that determines the correct formation of symbols and defines the legitimate modes of relationships. The verification of the validity of mathematical reasoning is a logical problem, a matter of calculation. The validity of mathematical reasoning is based

on its subjection to the laws of logic. A judgment is admitted if it is derived from a valid inference. For this reason, the rules of calculation can be reduced to rules of inference. The mathematical calculation is legitimized in the logical deduction. The Kantian distinction between analytical judgments and synthetic judgments must be reformulated. Analytical judgments are those that do not require anything external to thinking for its legitimization. The problem is in relation to validity and not in relation to the genesis of knowledge. The laws of logic are analytical as long as they have immanent legitimacy. The laws of arithmetic, while they can be derived from logical laws, are analytical as well. The judgments of arithmetic are analytical because their foundation can be obtained purely from logical laws. Kant confused the distinction because he inherited the prejudices of Aristotelian realism. Kant confused the problem of genesis with the problem of validity. The Kantian error is based on a realistic definition of the notion of concept. The concepts of logic are creative and not the results of an abstraction process, as in natural language. There is no necessary reference to intuition so that the judgment can be amplifying. Thought is amplified by virtue of its own creations without reference to intuition. Therefore, the judgments of arithmetic are analytical and amplifying. There is an identification between logic and arithmetic. Arithmetic is "a more widely developed logic, and each arithmetic statement would be a logical law, although a derived law," "calculate would be to deduce." The laws of arithmetic can only be applied to the phenomena of experience only mediately. Arithmetic governs the natural world by regulating the laws of the science of nature. The universality of the law is applied only indirectly to objects of knowledge. The laws of logic apply to objects indirectly. The objects of nature are, in relation to the laws of logic - and, consequently, in relation to the laws of arithmetic-given. For some representatives of logicism, the laws of geometry, unlike those of arithmetic, have an extrinsic foundation. The construction of the geometric object requires appealing to the intuition of space. Therefore, the judgments of geometry are synthetic.

2.2.3. Points in Common with Logicism.

⁻

²⁵⁰ Frege, G., 1984, p. 99.

Natorp's first common point with logicism is the rejection of mathematical psychology²⁵¹. Natorp shares with the logicists the conviction that it is necessary to separate the problem of genesis from the problem of validity. The laws of thinking, the object of study of logic, must be separated from the laws that govern subjective consciousness²⁵². Logic must be independent of any other science, including psychology. Logic and psychology are sciences with different objects of study. Logic can be defined as the science of thinking. However, this conception can lead to a confusion between the legality of pure thinking and the laws that govern the psychic life of the individual. Logic is not an empirical science. In his arguments against psychologism, as we exhibited, Natorp showed the need to distinguish between the mental fact and the principles of cognition, not determined by empirical legality; that is, by natural causality²⁵³. The logical laws do not determine a temporary event. In the same way, the sciences that are based on logic, such as mathematics, are completely independent of the mental processes that the subjects carry out. Therefore, the legitimation of the statements of mathematics cannot be obtained from a study of the genesis of representation. The recognition of the validity of a statement of mathematics does not require the possibility of representing this truth as a mental content. The problem of the validity of mathematical judgments cannot be solved through an analysis of the formation of representation in the subject. Frege and Natorp argue in the same direction. ²⁵⁴ The logical law, on which the legality of mathematics is based, is not a fact. The law is precisely what opposes the subjective point of view. The law grounds the possibility of objectivity²⁵⁵. Natural law regulates events. The logical law is the expression of the relations of thinking. Logic, as a universal science, cannot depend on a particular science, i.e., psychology. The method of logic cannot follow the same

²⁵¹ Cf. González Porta, M., 2006, p.166. As Gonzales Porta rightly observes, there are two currents of antipsychologism, that of Neo-Kantianism and the position of Frege and Husserl, which he calls "logical realism". Gonzáles Porta explica: "existen dos variantes diferenciables en la crítica al psicologismo, una, la neokantiana, y otra, la representada por Frege y Husserl, y a la cual en el actual contexto me referiré como "realismo lógico"". Gonzáles Porta, M. A., 2021, p. 166.

²⁵² Natorp, NTE, p. 343.

²⁵³ Natorp, P., L, p.10.

²⁵⁴ Cf. Frege, G., 1979, p.2. Natorp, P., SP, p. 18.

²⁵⁵ "ein Gesetz besagt überhaupt ein allgemeines Stattfinden; oder allgemeinen Bestand einer Relation. Darin muss aber nichts von Zeitbedingungen enthalten sein, d. h. es gibt Gesetze, die nicht Zeitgesetze eines Geschehens sind. Und zwar sind diese was von Zeitbedingungen unabhängigen Gesetze, nämlich die der Logik und Mathematik, fundamentaler als alle Gesetze, welche Zeitbedingungen einschließen, denn Zeitbestimmung setzt selbst erstens die allgemeinen Gesetze der Bestimmung, d. h. die logischen Grundgesetze, und zweitens Grössenbestimmung (Zählung und Messung), mithin die Gesetze der Mathematik voraus." Natorp, P., L, p.10.

delineations as the psychological method because the problem of logic is the relations of the contents of thinking and not the genesis of the mental life²⁵⁶.

The conditions for the possibility of a judgment are not conditions for the formation of the representation. Frege and Natorp differ with respect to the solution on how to establish the validity of the judgment. However, both agree on the need to make this distinction between the conditions that allow the formation of a representation and the conditions for the possibility of judgment. Both also claim that the introduction of something given to intuition comes along with the standpoint of psychologism. The genesis of the representations requires an element given to an intuitive faculty. The account of the formation of the representation demands that something is given. Both agree that this problem of the genesis has nothing to do with the logical problem, the problem of the theory of knowledge. The latter do not deal with the problem of empirical origin. Frege and Natorp also agree on the need to ground mathematics in logic. Then, the validity of the mathematical judgments depends on the way in which this foundation is carried out. This foundation cannot be established by the analysis of a study of the mind.

The necessity of a logical foundation

The second point in common with the logicist consists in the admission of a logical foundation of mathematics. Both for Natorp and for logicism, logic is the fundamental science of thinking²⁵⁷. Logic as a universal science of knowledge must provide the foundations to the remaining sciences²⁵⁸. Natorp recognizes the merit of logicism at this point. Logicism has correctly undertaken the task of a purely logical foundation of the exact sciences²⁵⁹.

Logic will be the fundamental science of thinking, and it will exhibit the laws that regulate it. Each particular science rules over a certain field of objects, the logic is universal. The whole field of thinking is regulated by the laws of logic. Neo-Kantianism

²⁵⁶ "Die Methode der Logik is also weder kausal (psycologisch oder biologisch) noch teleologisch, sondern im gleichen Sinne rein objektiv wie die der Mathematik." Natorp, P., L, p. 11.

²⁵⁷ For Éric Dufour, the conception of logic as the center of philosophy is a common element of German philosophy of the 19th century. The discrepancy is generated in relation to what each current means by logic. Dufour, E., 2010, p.20.

Pulkinm Jarmo: "The neo-Kantians, too, supported the idea that mathematics should be based on a logical foundation. However, their conception of the logical foundation differs greatly from that of Russell and Frege". Pulkinm J., 1986. p.20

²⁵⁹ Natorp, P., KMS, p.196.

in general converges with logicism at this point. The sciences in general - and, in particular the exact sciences - must be legitimized from their foundation in the laws of logic. Therefore, the logical foundation allows the scope of thought to remain as a unified whole. The foundation of science in logic prevents the separation of thought in heterogeneous domains irreconcilable to each other. Each particular area may have special laws that regulate it. However, all of them will be subject to the legality of thought.²⁶⁰ Natorp considers that the fundamental law of consciousness demands the unity of all manifold in thinking²⁶¹. Logic is the unifying science of thinking. Logic, as an expression of the legality of thinking, exhibits the action of thinking in its unity. The logic is then, the fundamental science, which provides the general laws for all particular cognitions.

Mathematics, as a special science, also requires a foundation. This foundation will be carried out by that science that studies the laws of thinking in general: logic. The science of nature derives its foundation from mathematics and mathematics is grounded on logic. The definition of the concept of thinking as a whole allows this unification and, consequently, logic as a science of the expression of the laws of this unifying unity. Therefore, both the laws of mathematics and those of natural science are regulated by the laws of logic; that is to say, they are grounded on the logical law.

Natorp agrees with logicism and with Frege in particular, on the necessity of a logical foundation of the exact sciences. Both authors argue in the same direction. Frege and Natorp propose a logical foundation of mathematics²⁶². Logic expresses the laws of thinking and with it, the laws that regulate thinking in general. For both, logic is the science of thought. The discrepancy consists in the way in which this foundation is carried out and what is understood by logic in each case.

Need to reformulate the concepts of analysis and synthesis

Both Natorp and Frege, and other representatives of logicism, share the conviction that it is necessary to reformulate Kantian concepts of analysis and synthesis. In particular, the

^{260 &}quot;Das Denken ist im Wesentlichen uberall dassellbes: es kommen nicht je nach dem Gegenstände verschiedene Arten von Denkgesetzen in Betracht. Die Unterschiede bestehen nur in der grösseren oder geringeren Reinheit und Unabhängigkeit von psychologischen Einflüssen." Frege, G., 1884, p. iii.

²⁶¹ "Durch das Grundgesetz des Bewußtseins ist Einheit alles Mannigfaltigen oder Gesetzlichkeit bedingungslos gefordert." Natorp, P., PS, p. 34.

²⁶² Natorp, P., LGEW. p.1.

analytical and synthetic nature of the judgments must be redefined. Frege and Natorp consider that Kant was not deep enough in this distinction. This is the third point of convergence of Natorp with logicism. As Frege did, Natorp also argued that the distinction between analytical and synthetic judgment must be reformulated.

The judgment coordinates two concepts, establishing a peculiar relation among them. In the affirmative judgment, "S is P", S is the subject and P is the predicate. The judgment is analytical if the predicate is contained in the subject. The judgment is synthetic if the predicate introduces an element that is not contained in the subject. Kant considered that the judgments of mathematics are synthetic because they require a reference to intuition. Thought by itself can construct neither the object of arithmetic (the number) nor the object of geometry, objects in space. Arithmetic judgments require a reference to intuition in determining the number in time. A priori synthetic judgments of arithmetic, such as "5 + 7 = 12", require the pure intuition of time. A priori synthetic judgments of geometry, such as "the line is the shortest distance between two points," require the pure intuition of space.

Natorp, as Frege, is critical of the way in which Kant understood the distinction between analytical and synthetic and considers that this distinction must be reformulated. The Kantian distinction is based on a conception inherited from Aristotelian logic that conceives judgment as a relationship between subject and predicate. Frege and Natorp share the idea that judgment as an expression of the structure of thinking does not take the form of natural language. The grammatical form of judgment is insufficient to establish the relational character of thinking. Therefore, the logical study of judgment does not converge with grammatical analysis. The way in which thought establishes relationships overcomes the restricted mode of the expressions of natural language²⁶³. Natorp shares this idea with Frege and considers that the most proper expression of thinking is the function. It would be more precise to relate the action of thinking to functions than to judgments. The judgments are ways of bringing the manifold to a unity but the way in which the manifold is reunited in various modalities does not reflect the structure of the Aristotelian form of judgment. Rather, this type of judgement is possible based on an original way of establishing relationships whose most precise expression is the function. More precisely, the judgments should be considered as a function. Only then, judgments could be considered as an expression of the action of thinking. In this

²⁶³ Frege, G., 1979, p.6.

sense, judgment is the expression of the functional character of thinking²⁶⁴. Frege would fully share this idea. Frege states:

Kant scheint den Begriff durch beigeordnete Merkmale bestimmt zu denken; das ist aber eine der am wenigsten fruchtbaren Begriffsbildungen. Wenn man die oben gegebenen Definitionen überblickt, so wird man kaum eine von der Art finden. Dasselbe gilt auch von den wirklich fruchtbaren Definitionen in der Mathematik z. B. der Stetigkeit einer Function. Wir haben da nicht eine Reihe beigeordneter Merkmale, sondern eine innigere, ich möchte sagen organischere Verbindung der Bestimmungen.²⁶⁵

Natorp, in this same direction, maintains:

Also aus keinen voraus gegebenen, gleichsam feststehenden Denkpunkten und mit diesen zugleich gegebenen, ebenso festen Lagen solcher Punkte gegeneinander, sondern aus dem Quell einer unerschöpflichen Denkbewegung, aus dem Quell der Methode allein kann das synthetische Urteil, das eigentliche Urteil überhaupt als synthetisches, sich erzeugen. Allerdings stumpft Kant selbst die Schärfe dieser radikal idealistischen Einsicht wieder ab, wenn er den Urakt der Synthesis beschreibt als die "Handlung, verschiedene Vorstellungen zueinander hinzuzutun" und "ihr Mannigfaltiges" zu einer Erkenntnis zu begreifen. Danach scheinen die letzten Elemente, in der fragwürdigen Gestalt von "Vorstellungen", doch wieder vor der Erkenntnis, selbst vor dem Urakt des Erkennens, dem Akte der Synthesis, voraus gegeben sein zu sollen. ²⁶⁶

²⁶⁴ "Zwar folgeweise läßt jedes von diesen sich auch in Form eines Urteils aussprechen, aber nur hinterher; primär ist von Faktoren, oder besser noch (mit Kant von Funktionen (Einzelleistungen) des Urteils zu reden." Natorp. P., LGEW, p. 28. We will study later Natorp's definition of concept and judgment.

²⁶⁵ Frege, G., 1884: §88.

²⁶⁶ And he follows: "Aber hier ist nun Kant sehr leicht aus seinen eigenen Voraussetzungen zu korrigieren. Man braucht nur zu fragen: sollen diese Elemente vor dem Grundakt der Synthesis voraus einen "gewissen Inhalt" schon haben oder nicht? Aber die Synthesis soll ja vielmehr das sein, was sie "zu einem gissen Inhalte erst vereinigt. Also waren sie vordem — Vorstellungen zwar, aber ohne gewissen Inhalt? Vorstellungen, in denen — nichts Bestimmtes vorgestellt war? In der Tat darin liegfs: nichts Bestimmtes. Die Bestimmtheit des "Was", das ist genau, was der Urakt der Erkenntnis als Akt des Bestimmens erst zu erbringen hat." Natorp. P., LGEW, p. 46.

As we shall see later in detail in chapter 4, Natorp agrees with Kant that the mathematical judgments are synthetic. However, he rejects the idea that the synthetic nature of the mathematical judgment is related to some need to appeal to intuition. For Natorp, the judgment of mathematics is synthetic, but it is, in turn, a pure product of thinking without any reference to intuition. Synthesis is the expression of the possibility of thinking progression. Natorp's definition, as we will see later in detail, diverges from Frege's account. However, both authors agree on the need to reformulate the distinction established by Kant.

In short, Natorp shares with Frege the need for a logical foundation of the exact sciences. Exact sciences require a logical function, and in this foundation, there should be no reference to intuition. The principles of mathematics are based on logic, and logic is a purely conceptual science that owes nothing to pure or empirical intuition. Mathematics is based on logic and then, on pure concepts. For Frege, arithmetic has a mediate application. Arithmetic governs the natural world by regulating the laws of the science of nature. Natorp also shares with Frege that it is necessary to develop a better understanding of the analytical and synthetic nature of the judgments. For both, it is necessary to reformulate the notion of concept, rooted in the ancient Aristotelian logic. The realistic conception of the notion of concept must be replaced by an idealistic notion. Likewise, they converge on the idea that an extension of knowledge can occur without any reference to intuition. There is no necessary reference to intuition so that the judgment contributes knowledge and can thus be amplifying.

2.2.4. Natorp's Criticism of Logicism.

Frege and Natorp share the idea that it is necessary to establish a new logical foundation of mathematics. Both thinkers also believe that knowledge requires a new logical legitimation, which is not purely speculative or psychological. However, they disagree on the way in which this task should be carried out. Natorp considers that formal logic is insufficient to ground the procedure of mathematics. Formal logic is insufficient to legitimize both the truths of mathematics and those of natural science. The purpose of the logical groundings of the exact sciences is, as the proposal of logicism, to ground mathematics as *a priori* science. However, this foundation will be carried out in a very

different sense from that defended by authors such as Russell and Frege²⁶⁷. Natorp thinks that the task of a genuine logical foundation of the exact sciences had not been developed yet²⁶⁸. Contemporary logicism does not overcome the ancient conception of logic. Modern logistics continues the tradition of Aristotle. The reformulation of the sense of 'the logical' is necessary because this concept was particularly misunderstood by the logicians of that time. Logicism did not go much further than Aristotle at this point. They have not understood the fruitfulness of the logical. The purpose of logic is to make understandable the construction of the object of knowledge. Logic cannot start from that object as a mere fact. The logic thus conceived is a productive science. One cannot start the logical investigation under the unjustified assumption of the separation of knowledge, as if we had the act of knowing on the one hand, and the object of knowledge, on the other. A foundation of the exact sciences demands a logic that shows how the objects of these sciences are constructed in and by thinking, thus exhibiting the inseparable relationship between thinking and object. A genuine logical foundation requires a transcendental logic, which studies the unfolding of the process of thinking in the production of its object of cognition. Formal logic is insufficient to provide this foundation. A transcendental logic is required to show the legitimacy of knowledge in the explanation of its conditions of possibility. These conditions will make possible the foundation of knowledge, which is expressed in the physical-mathematical sciences. Natorp rejects that formal logic can be the ground of science. The conception of the mathematical foundation in formal logic implies an identification between logic and mathematics. Mathematics is based on the logic for the legitimization of its procedure. However, the logic to be the foundation of mathematics operates according to the laws of calculation. Logic is turned into a branch of mathematics. There is no proper foundation. Mathematics follows the methods of logic, but logic has a mathematical formulation. In conclusion, there would be no real difference between logic and mathematics. Rather, there would be an identification between the two sciences²⁶⁹.

²⁶⁷ "Auch das gegenwärtige Buch unternimmt eine rein logische Begründung und behauptet damit den Apriori-Charakter der Mathematik, aber in einem anderen Sinne als die Vorgenannten". Natorp. P., LGEW, p.3.

²⁶⁸ "Einer solchen Vorbereitung bedarf es, weil schon die Aufgabe selbst, so wie sie hierverstanden wird, nicht allgemein als solche anerkannt ist." Natorp. P., LGEW, p.2.

²⁶⁹, Conturat (31, S. 230), der vielleicht am weitesten nach dieser Seite geht, äußert sich darüber immerhin zögernd. Auf der einen Seite sieht er in der Mathematik einen Teil der Logik: sie sei ganz logisch der Form nach, aber beschreibe in ihrem Inhalt nur einen Teil des Umfangs der Logik; auf der anderen Seite will er die Logik rein rechnerisch gestalten, macht sie also unleugbar zu einem Zweig der Mathematik. Wäre es dann nicht folgerichtig, die Verschiedenheit von Logik und Mathematik überhaupt zu verneinen? Denn

Vicious circularity

The procedure of logicism is circular. Logicists claim that the legitimacy of mathematical propositions can be obtained from their deduction of laws of logic. However, logic itself is a science that operates by deductions. In fact, the justification of a logical law is made, according to logicism, exhibiting its derivation from another logical law. Then, logic as a deductive science must provide the basics of deduction, but the legitimization of its laws can only be done by deductive means. There is a circularity in the foundation. The logicists, on the one hand, want logic to be a purely deductive and calculative science and, on the other, that it is capable of legitimizing that calculation procedure only on its own. Thus conceived, logic can neither ground the mathematical procedure nor ground itself. According to Natorp's characterization, as we observed in the last section, the procedure of logicism would be the following: definitions of the symbols that will be admitted into the system are formulated, the rules that express the way in which it will be legitimate to connect these symbols are defined and, hence, a mechanical process is performed²⁷⁰. The only restriction to the way of linking symbols is the principle of noncontradiction. A genuine understanding of this process is not only not necessary, but the introduction of elements outside logic can disrupt the procedure that is purely calculative. The clarification of the meaning of these symbols is not necessary at all to carry out the derivation. Formal logic does not provide the ultimate foundation that makes knowledge possible. The logic thus defined cannot provide a real understanding of the process of knowledge. For the conception of logicism, the logical analysis "is limited to a mere composition of arbitrary symbols carried out with equally arbitrary rules." ²⁷¹ For Natorp, unlike logicism, the question of the groundings is unavoidable, and formal logic is not enough to provide such a foundation. It is necessary to introduce a more fundamental science than formal logic that exhibits the principles of the process of knowledge. Indeed,

-

²⁷¹ Natorp, P., ZLGNM, p.182.

wenn zugleich A in B und B in A ganz enthalten ist, so sind nach einem bekannten Satze der rechnerischen Logik beide notwendig identisch." Natorp, P., LGEW, p.5.

²⁷⁰ For Philip Jourdain, Natorp pays little attention to the work of mathematical logicians he criticizes severely. In particular, he tries to show that Natorp misunderstood the procedure of Couturat. Jourdain points out: "But mathematical logicians do not think that they can justify the principles of logic deductively, and do not, of course, attempt the task of beginning with definitions. They begin with primitive ideas as such primitive proposition as are necessary to make deductions". Philip. J., 1911, p. 554. J. J. Maxwell, in his note to this review, tries to refute Jourdain's critique by showing that Natorp did not misunderstand the logical symbolists but, rather, Philip Jourdain misunderstood Natorp. Maxwell, J., 1912.

the analysis of the scope of the logical derivation is necessary, but the task of a genuine understanding of the procedure, which can only be found in the study of the logical foundations, cannot be skipped. Recognizing these grounding is the task of philosophy²⁷². Natorp rejects taking these principles as if they were simply given, and they required no further explanation. This type of circularity, unlike the circularity of the critical method, is vicious. It does not constitute an elucidation or an explanatory basis. The impossibility of finding these last principles means that logical knowledge has a restricted domain. The logical principles hold a purely relative validity as long as they are always valid "in relation to a certain system of definitions and a certain sequence of demonstrations, never absolutely."²⁷³ The validity of the laws of logic becomes relative. In this conception, the universality that logic must have as a grounding and objective science is lost.

The error of this conception has its origin in the uncritical acceptance of the prejudices inherited from Aristotelian logic. Logicists consider that the fundamental principles are found in the immediate evidence or that they are simply given to the understanding. Although modern logic is enriched, like classical (i.e., Aristotelian) logic, it simply declares its principles and concepts. The simple declaration of principles is a sufficient condition to accept them. Ultimately, the error of logicism is the same as that of psychologism, which is grounded on naive realism. They both rely on the acceptance of data as given to thinking. The only necessary task is analysis. Logicism is thus closely related to naive realism, also of Aristotelian roots. The error of the dogmatism of Aristotelian logic is to depart from certain assumed and unjustified definitions, as if they were simply declared, given to thinking. This is the fundamental error of naive realism that takes things as given to intuition and considers that the task of thought is reduced to operating on the given content. Therefore, for this conception, logical development can take place purely by means of analytical judgments, which are limited to expressing

²⁷² "Die Konsequenz dieses Bestrebens muss dahin führen, dass man nicht zufrieden ist, in der Mathematik überhaupt, wie in jeder Wissenschaft, logisch zu verfahren, d. h. Widerspruch zu meiden und, was man behauptet, zu beweisen, sondern dass man sich die weitergehende Aufgabe stellt, auch als Voraussetzung nichts zuzulassen, was irgend noch aus fundamentaleren Voraussetzungen herleitbar, also noch nicht schlechthin einfach ist. Die Frage aber nach den letzten Voraussetzungen einer so fundamentalen Wissenschaft, wie die Mathematik, führt unmittelbar in das Herz der Philosophie als Erkenntniskritik." (...) "Aber, neben der Aufgabe der Entwickelung der Konsequenzen aus gegebenen Voraussetzungen besteht jedenfalls noch die andere, des Zurückgehens auf die letzten erreichbaren Grundlagen." Natorp. P., EGM, p.2. Also. Natorp, P., ZLGNM, p.182.

by Couturat. Natorp affirms: "Ausdrücklich sollen (nach Couturat, S. 39) die Grundbegriffe und Grundsatze als solche (d. h. undefinierbar und unbeweisbar) allein gelten allemal in bezug auf ein bestimmtes System von Definitionen und eine bestimmte Beweisfolge; nie schlechthin." Natorp, P., LGEW, p. 8.

implicit contents in the data, be it intuitive data, as in the case of psychologism, or abstract data, as in the case of logicism. For both, naive realism and logicism, understanding only operates by analysis of the given contents. However, the analysis could not provide clarification of its fundamental principles. In sum, formal logic is insufficient both to ground itself and to give a foundation to mathematics. This kind of grounding is circular and does not constitute an explanatory basis. This conception ignores the essentially productive character of thinking, to which the object is not given as data but produced. Logicism confronts the universality of the concept to the concretion of the object instead of showing how the object is a construction of the conceptual procedure of thinking. In the next chapter, we will exhibit how this is accomplished by transcendental logic.

Preeminence of the synthesis over analysis

This analytical perspective of the task of knowledge must be revised. Thought, as we will see in detail later, is essentially productive. The preeminence given to the analysis starts from the assumption that there are certain data given to thinking as a piece of information that must be decomposed. For Natorp, as for Kant, nothing can be decomposed unless it was previously gathered²⁷⁴. The spontaneity of thinking consists in the ability to produce its own object, without the reference to anything outside of itself. The conclusion of the Transcendental Analytic of the Kantian *Critique* is that the understanding is "the generative and regulatory source of nature (that is, of the nature of natural sciences) and not only its interpreter"²⁷⁵. Even if logicism does not make reference to a perceptual

²⁷⁴ Natorp, P., LG, p. 9.

Natorp, P., UOSB, p. 140. Cohen's position is grounded on the same thesis. As Hernan Pringe summarize: "In his Logic of Pure Knowledge, Hermann Cohen aims to carry out the Copernican turn which, according to him, Kant fails to achieve. On Cohen's reading, if objects must conform to our cognition because knowledge produces the object (Cohen 1907, 4), then this cannot just amount to the determination of the mere form of objectivity in general. On the contrary, the spontaneity of thought must also generate the matter of cognition." For this reason and in opposition to Kant, Cohen claims that in cognition thought does not face any given matter, not even a pure one (Cohen 1922, 26–27). What Kant calls given is nothing but a product: a product of thought. The Kantian distinction between thinking and cognizing an object (CPR B 146), which relies precisely on the consideration of intuition as a non-conceptual representation, is thus abandoned in favor of a doctrine of thought that is at the same time a doctrine of knowledge. Though, in Cohen's sense, thought does not depend on any receptivity that would provide it with a sensible content. According to Cohen, only in this way can the Copernican turn that Kant prescribed to metaphysics be finally executed successfully.", Pringe, H., 2020, p.137. In this sense, the Neo-Kantian method should be called a transcendental method. Christian Krijnen explains: "Das Transzendentale ist der Sache nach ein Inbegriff von Geltungsgründen, der nicht durch den Rückgang auf ein Seiendes außerhalb der Erkenntnisrelation begriffen werden kann, sondern nur durch einen Rückgang auf das Denken selbst als Grund aller Geltung. Die objektive Gültigkeit konkreter Erkenntnisleistungen des Subjekts findet ihren Grund in einem Inbegriff von Geltungsprinzipien (,Bedingungen der Möglichkeit'); die objektive Gültigkeit

element, however, it considers the contents of thought as given. Therefore, for Frege, for example, the rules of analysis are sufficient to legitimize the process of thinking. The rules of the analysis, indeed, are useful to provide intelligibility to the synthetic process but are always grounded by it. According to Natorp, the analysis represents only the 'reverse' of the synthesis. The affirmation of the preeminence of the synthesis is introduced primarily as a rejection of the idea of a purely analytical foundation of knowledge. The fundamental problem of the attempt to establish an analytical foundation is the homologous nature of the analysis. The analytical foundation of knowledge cannot express the expansion of thinking content, it cannot exhibit the progression of thinking. The analytical foundation transforms knowledge into tautology under the expression: A is A. The synthetic foundation, on the other hand, is the expression of the heterology of thinking, which no longer expresses that A is A but that A is B²⁷⁶. Affirmatively, it is established that thinking is precisely the possibility of setting the differentiated elements. Formal logic, as it is analytical, does not aim to extend our knowledge as long as the principle of non-contradiction is valid only for its clarification and has no function for its extension²⁷⁷. According to Natorp, Frege's arguments are not convincing. Even when analysis reveals new implicit content, it does not create anything new. The function of thinking remains tautological. This conception considers that mathematics, and therefore all sciences, is a closed sum of finished truths, which one day could be completely known. The analytical conception cannot ground the expansion of knowledge. For Natorp, when Frege claims that the analysis is amplificative, he is making reference to synthesis. Natorp explains:

Der Widerspruch kann aber unmöglich ein Prinzip der Fortschreitung sein, sondern allenfalls nur ein Prinzip der Auslese, wodurch sinnwidrig versuchte Fortschreitungen ausgeschaltet werden. Dessen bedürfte es gar nicht, wenn die Fortschreitung streng ihrem Gesetz gemäß geschähe. Der Widersprach schafft also nichts, erhält auch nicht das Geschaffene. Auch vernichtet er nicht logisch Geschaffenes, sondern

Geltungsprinzipien wird dadurch legitimiert, daß sie sich geltungsfunktional als Bedingungen der Erkennmis (Ietzt-)begründen lassen." Krijnen, C., 2006, p. 288.

²⁷⁶ "Also was ist Synthesis? Zunächst nur ein Ausdruck der Abwehr einer bloss analytischen Begrüngung der Erkenntnis. Der Fehler der Analysis ist, dass sie Erkenntnis bestenfalls in Tautologie verwandelt. Also scheint Synthesis vielmehr Heterologie bedeuten zu müssen: Nicht A is A sondern A ist B". Natorp, P., LGEW, p. 11.

²⁷⁷ Natorp, P., LGEW, p.20.

entlarvt nur den falschen Schein einer logischen Schöpfung, wo wirklich keine vollbracht ist; einen Schein, der beim logischen Schaffen als unlogisches Tun vielfach nebenher geht und sich mit- einschleicht. Der Satz des Widersprachs ist also wirklich, wie Kant es aufgestellt hat, allenfalls ein Prinzip der Verdeutlichung, nicht aber der Erweiterung der Erkenntnis²⁷⁸.

Analytical judgments are based on the principle of non-contradiction. The principle of non-contradiction is useful only to verify the legitimacy of a logical creation, not to create it. This principle cannot legitimize the expansion of thinking. The principle of non-contradiction cannot generate the logical content itself. Then, the principle of non-contradiction, upon which the analytical conception of knowledge rests, cannot be the foundation of the progressive character of thinking. The analysis grounded on the principle of non-contradiction must be based on a more original act that allows logical creation and thus the expansion of thinking. Indeed, the principle of non-contradiction can be used to verify the creation of thought, but it cannot be the foundation of creation itself. Rather, it requires a creative act of thought in the first instance. This is the act of synthesis. Synthesis is the expression of this possibility of thinking to create its content and not only operate over given content. Synthesis is the purest expression of the spontaneity of thinking and the only act that can ensure its amplification.

Natorp argues that the conception of logicism of the logical form leads to a separation between intuitive and conceptual content. For this conception, the universal and the particular are separated. There is a gap between the law and its object. The form is conceived here as separate from its objects. Logicism does not recognize that "the general must in all cases be conceived only as general of the particular"²⁷⁹. The law as a unifying moment cannot be thought independently of the multiplicity it contains. The concept does not subsume the manifold as something alien to itself. The unification of the multiplicity is not generated departing from certain given data, neither empirical nor abstract. Taking the given as a starting point, logicism, does not overcome the dogmatic realism that, as we saw, is rooted in the Aristotelian tradition. Logicism does not overcome the dogmatism of the given because it takes the knowledge and the object as if they were two dissociated elements. First, it considers two separated elements and then

²⁷⁸ Natorp, P., LGEW, p.19.

²⁷⁹ Natorp, P., ZLGNM, p.180.

tries to explain the relationships among them. The concept must be defined, as Frege correctly notes, in its functional character, as the unity of the multiplicity of instances. The cases, however, are determined a priori by the concept as their instances and not as something that exists independently of the concepts. Thought creates the case in concomitance with the position of the law. The object of knowledge does not exist independently of the act of thinking. The construction of the case in the law exhibits the way in which thought constructs its object and does not start from this object as a mere fact. The universal expressed in the law is productive. Logicism is as dogmatic as psychologism when it accepts the case as given to thinking. A foundation of the exact sciences requires displaying the way in which thought constructs its object. Logic must show the concomitant construction of the universal and of the particular. This is the only way in which the inseparable character between thinking and object can be shown. Logicism mistakenly conceives the relation between the universal and the particular and, therefore, affirms the preeminence of analysis.

In sum, Natorp shows the insufficiency of formal logic to be the grounding of knowledge. Formal logic takes the object of knowledge as if it were given and considers the analysis as the fundamental operation of thinking. For Natorp, knowledge demands a synthetic foundation that shows the construction of the object in and by thinking. Logicism correctly addresses the purely ideal foundation of legality. However, it does not take the creative nature of the concept to the last consequences. The amplifying nature of the judgments does not demand a reference to intuition. Judgment can be synthetic without reference to intuition. Frege evidenced the insufficiency of the principle of non-contradiction but he was not deep enough, he did not see the insufficiency of an analytical foundation of knowledge. All science must, indeed, proceed logically in the sense of avoiding contradictions. However, in addition to the task of developing the consequences from certain given conditions, the need of going back to the last achievable foundations persists. Formal logic must be grounded on the transcendental logic that exhibits the creative principles of thinking. For Frege, the logical law has an indirect application to the objects of knowledge. Arithmetic is based on logic and rules over the natural world by regulating the laws of the science of nature. The universal laws of logic are applied to objects indirectly. For Natorp, on the contrary, the foundation of knowledge requires an original act of synthesis that shows the creation of the object in thinking. There is no data to be decomposed but a creation of the object of knowledge. And according to the conception of Natorp, there is no indirect application of the law to the object of knowledge. The object of knowledge is not given facing the universality of the law. On the contrary, it must be shown how the law in its universality builds the object. The synthesis is the expression of this possibility of thinking of creating content and not only operating with given contents.

Conclusion

According to Natorp, psychologism and logicism were incapable of explaining the relation between concepts and intuitions. From an incorrect understanding of the problem of the method, psychologism and logicism are unable to explain the possible reference of thought to reality. For psychologism and logicism, the fact of experience always remains as an extrinsic moment to the legality of thinking. Psychologism confuses logic, the science of the laws of knowledge, with psychology, the science of laws that regulate the psychic life of individuals. Psychologism carries out a subjective foundation of knowledge. From this methodological error, psychology considers what is given to the perceptual intuition as the initial data of the investigation. A given data is conceived as the starting point in the formation of knowledge. The conception of psychology is based on the prejudices inherited from the Aristotelian realistic conception. The starting point from a given data would be the way to guarantee that thought can achieve objectivity. The object is conceived as fully determined, and thought must be able to display the determinations of the object. Psychologism starts from the unfounded assumption of an external element. Thinking would build its concepts departing from this first given factum. According to his position, the concepts are constructed form what is given to intuition. Against this current, Natorp argued that this definition of the notions of concepts and intuition must be reformulated. The concept cannot be conceived as a sum of marks that are abstracted from the factum given to sensation. Thought is spontaneous. This means that it is creative. Concepts must be conceived as functions. We will return to this point in the next chapter. Natorp exhibited that taking the data given to intuition as the starting point of the investigation, far from guaranteeing the reference of thought to reality, the core of the Kantian problem, cancels the concept of knowledge itself. The investigation must show how thinking creates the object. The logical foundation of knowledge has the task to show the creative power of thinking in the process of concept formation. The concept is not a sum of marks nor intuition is the element given to thought. The universality of the concept and the concreteness of the intuition do not

oppose. It must be shown how the universal is universal of a particular and the particular is only particular in relation to the universal. According to Natorp. logicism does not escape this misunderstanding. On the basis of a methodological error, logicism is unable to give an accurate account of the relation between intuition and concepts. Logicism grounds the possibility of knowledge on formal logic. The principles of cognition are conceived as given. For them, the main task of thought is analysis. The task of thinking is reduced to the analysis of the given contents. Faced with this primacy of the analytical moment of knowledge, Natorp points out the need for a synthetic foundation. As we shall see in Chapters 3 and 4, this foundation will be carried out exhibiting the general legality of knowledge, its internal law. The exposition of this procedure will display how thinking constructs objectivity. These fundamental procedures will be the categories, or, as Natorp will call them, the levels of thinking²⁸⁰. This exhibition will be the way to overcome the dualism between the intuitive and the conceptual representations. In Chapters 3 and 4, we will study how Natorp undertakes this task.

²⁸⁰ "Der Mathematiker, auch der logisch interessierte Mathematiker mag sich dabei beruhigen, solche letzten Prämissen zu, postulieren"; die Logik fordert für sie, als synthetische Sätze, wie Kant sagt, "wo nicht einen Beweis" "Voraussetzungslose", dh auf solche letzte Voraussetzungen, von denen es möglich ist, sich zu überzeugen, dass sie nicht wiederum andere, fundamentalere voraussetzen, nämlich auf die schlechthin fundamentalen Verfahrungsweisen des "Dendenkens gesetzmässigen Vorstellens der Gegenstände überhaupt, die sie in einer begrenzten Zahl reiner Grundfunktionen des Denkens (Kategorieen) festzulegen sucht." Natorp, P., ZLGNM, p. 383.

Chapter 3. The Method Required to Overcome the Heterogeneity between Intuition and Concepts.

According to Natorp, the only way to guarantee a genuine foundation of knowledge is by taking a minimal definition of thinking as a starting point. The investigation must begin by taking the concept of thinking as correlation. This is the only way in which a legitimate foundation of knowledge can be guaranteed. Taking this starting point will be the way in which the Kantian dualism between intuition and concept - which has the problems we exhibited in Chapter I- can be overcome. Overcoming the dualism between intuitions and concepts requires a new model in philosophical argumentation that is consistent with the task of philosophy. The problem must be posited from a new standpoint. This model should take the concept of thinking as a starting point.

The objective of this section is to analyze the definition of thinking as correlation and to show that this is the starting point of philosophical research in the pursuit of an internal foundation of knowledge. We will show that this immanent foundation is a necessary methodological prescription required by the very concept of philosophy. More specifically, it will be shown that taking this definition of thinking as the starting point of the investigation leads to a possibility of overcoming the distinction between intuitions and concepts. We will show that the deduction of concepts must be carried out departing from a minimal notion of thinking. This new starting point will be the clue to solve the problems raised by the Kantian distinction between intuitions and concepts. More specifically, it will be exhibited how the understanding is the source of its objects in a progression in different stages. Natorp will show how thought builds its cases from a minimal notion of thinking. This new standpoint will ultimately overcome the heterogeneity between intuitions and concepts.

In this section, we will analyze the definition of thinking as correlation. It will be exhibited that this conception is present all along Natorp's intellectual development. We will show that from his early works up to his mature presentation, Natorp claims that the principle of correlation is the starting point of the deduction of concepts. This starting point will be the first step to guarantee to overcome the heterogeneity between intuitions and concepts. Natorp will prove that the universality of the concept and the singularity of the object in its concreteness have the same root. They are both rooted in the principle of correlation. From this new standpoint, it could be explained how thinking can grasp the

object in its concreteness. The relation between the universality of the concept and the individuality of the object will be clarified. Therefore, it will be offered the grounding for a proper account of the relation between intuition and concepts. We will study how Natorp sees in Plato's theory the historical foundation of his proposal. Kant started his investigation from the Transcendental Aesthetic. He isolated sensibility and proved that intuition and concepts were two different sources of knowledge. In this part of the Critique, Kant proved that sensibility has its own sort of representations: intuitions. Only later, in the Transcendental Logic, Kant shows that the understanding is a source of representations: concepts. Kant takes the table of judgements as the leading thread to discover the concepts of the understanding. In this chapter, it will be exhibited that for Natorp, the Transcendental Aesthetic is not a proper beginning. The philosophical investigation must start from the Transcendental Logic. Natorp considers that Kant was wrong to take the table of judgments as a starting point of the deduction of concepts. We will show how Natorp corrects the Kantian proposal. We will focus on Natorp's conception of the relation between concepts and judgments. We will observe the reasons why it can be stated that thinking must be conceived as a process that involves synthesis and that, only in a derivative way, it can be considered as a mode of analysis.

3.1. The Task of Philosophy.

According to Natorp, the object of investigation determines the method that should be employed in every specific field of knowledge. It is not possible to carry out the study on a certain scope of objects without considering that each scope of objects demands a peculiar method of study. The investigation is based on a minimum of assumptions, and these assumptions determine the method that will be used in the investigation. This is the way to ensure that the method that we employ is consistent with the purpose of the investigation. For this reason, the first step of the philosophical investigation is to determine the object of philosophical research. Natorp defines the task of philosophy following the delineations of the classical tradition. For him, philosophy is the science that has the task of unfolding the fundamental principles of thinking.

٠

²⁸¹ For Natorp: "Durch die Eigenthümlichkeit des zu untersuchenden Gegenstandes muss die Eigenthümlichkeit der Untersuchungsmethode zum Theil bedingt sein; es lässt daher über die letztere sich nicht eher etwas feststellen, als das Gebiet der zu erforschenden Gegenstände mit Sicherheit bestimmt ist." Natorp, P., EP, p. 2.

²⁸² Natorp, P., PILb, p. 460.

Philosophy is the first science on which all others depend. This science is the fundamental science of thinking and knowledge²⁸³. As we shall see, this definition of philosophy as a science will be present in the entire work of the philosopher, from his early writings up to his later elaborations. His redefinition of the distinction between intuition and concepts assumes as an initial premise this peculiar conception of philosophy and philosophical method.

Sciences relate to each other and to philosophy in a specific way. Natorp uses an analogy to explain such a relationship. Natorp claims that if we consider knowledge as a circle, the particular sciences go from the centre to the periphery while philosophy goes from the periphery to the centre²⁸⁴. The multiplicity in which knowledge is branched is the field of particular sciences, which study the peculiarity of each region of knowledge. On the contrary, the task of philosophy is to find the centre that originates the periphery. Centre and periphery are two directions of the same path. However, as a centre of origin, philosophy guarantees the unity of knowledge. The logical procedure of thinking, as we will see in detail later, is expressed in a circle. Centre and periphery are co-involved. However, it is the centre that guides and determines the periphery. This centre, observes Natorp, should not be conceived as a mere empty midpoint but rather is the origin of the periphery, its law of formation²⁸⁵.

In this sense, philosophy is the most abstract and general science²⁸⁶. Special sciences study a particular domain of objects. Philosophy studies the conditions of all objectivity. It is the science of the principles of the conformation of the object in general²⁸⁷. Therefore, this general science cannot be identified with any particular field of objects, but it is the foundation of any position of objects. The general form of knowledge determines the form of each particular cognition. A particular cognition can

²⁸³ "Philosophie ist nach ihrem historischen Begriff die Grundwissenschaft, d. h. diejenige Wissenschaft, welche die Einheit der menschlichen Erkenntnisse durch den Nachweis des gemeinsamen letzten Fundaments, auf dem sie alle ruhen, sicherstellen soll." Natorp, P., PP, p. 3. As we will see later, there is no qualitative difference between thinking and knowing. The most concise development of this issue is found in the summary of its logic lessons. Natorp, P., L, §2.

Natorp, P., PIP, p. 3. Hans Schneider explains this analogy. Cf. Schneider, H., 1936, p.13.

²⁸⁵ "Dabei hat man sich aber das "Zentrum" nicht als leeren Mittelpunkt, sondern als zentrale Kraft, ganz im Ganzen und ganz in jedem Teil, lebendig schöpferisch das All durchwaltend und eben zum Ganzen zusammenschließend zu denken." Natorp, P., PILb, p. 512.

²⁸⁶ Natorp, P., TDM, pp., 49, 54.

²⁸⁷ Natorp, P., TDM, p.48. Aristotle would have recognized in his *Metaphysics* this fundamental task of philosophy. However, Aristotle identified the science of being in general with the science of substance. Plato's approach was superior in this regard. Natorp, P., TDM, p.40. This would be one of the reasons to consider Aristotle's book K of *Metaphysics* to be inauthentic. Natorp thinks that there is a contradiction in considering philosophy as a science of being in general and then identification of this science as the science of God, a particular being. Natorp, P., TDM, esp. pp.49ss. Natorp, P., AM, esp.180ss.

properly be knowledge because it obeys the rules of what in general can be legitimately called knowledge. The philosopher's task is to find this general legality that determines all particular cognitions. 288 The task of the philosopher is not to find a specific "logos" (τις λόγος) - the legality of a certain domain of knowledge - but the logos itself (αυτός ο λόγος) - the general legality of knowledge. The study of logos is the task of philosophy. This was Plato's great discovery, and this method is called transcendental philosophy. In this sense, Plato did not ground a particular philosophy but rather laid the foundations of the philosophy itself²⁸⁹. In contrast to the fundamental laws of philosophical thinking, the legality of sciences is derived because it is based on relative determinations of thinking. The particular cognitions expressed in the laws of specific sciences are conditioned positions of thinking. On the contrary, philosophy aspires to find the original legality. Philosophy does not seek a law but the law, not actual laws but the lawlikeness. This search, as we will see later in detail, is an infinite task that can never be fully accomplished. The search for the unconditioned is the ideal goal towards which the philosophical efforts must tend even when it never reaches it²⁹⁰. However, philosophical thinking seeks to find this general legality, the grounding unit of thinking²⁹¹.

Consequently, philosophy guarantees the systematicity of knowledge in general, making sciences a coherent whole, and not a mere aggregate of specific cognitions²⁹². The particular cognitions are not merely juxtaposed but coordinated as a whole. Philosophy is the articulating unit that enables this unification²⁹³. In this sense, philosophy

²⁸⁸ The fundamental task of the philosopher is the "knowledge (*Wissen*) of the form of knowledge (*Erkentnissform*)" since we generally call knowledge to a certain cognition only because it conforms to the general form of knowledge. "Aber das Wissen von der Erkenntnisform darf nicht getrennt bleiben von dem Wissen um das bestimmte Objekt, es muß in diesem zugleich liegen und zwar als es bestimmend, denn nur dem Formgesetz des Erkennens gemäß ist es überhaupt Wissen." Natorp, P., PILb, p. 28. Natorp, P., PILa, p. 26

p. 26. ²⁸⁹ "Philosophie" besagt, nach der klassischen Bedeutung dieses Wortes, die eben Plato ihm erteilt: das Streben zu jenem "Einen, allein Weisen" (έν το σοφόν μούνον), von dem schon Heraklit zu sagen weiß; zur Einheit aber des Vielen, damit auch Vielheit des Einen. (...) Auf diese Einheit muß somit alle Philosophie, die diesen Titel rechtmäßig tragen soll, hinstreben und strebt die heutige Transzendentalphilosophie entschlossen hin." Natorp, P., PILb, p. 460.

²⁹⁰ Natorp, P., PILa, p. 191. Natorp, P., PILb, p. 196.

²⁹¹ "Hier ist endlich mit einer jede Zweideutigkeit ausschließenden Bestimmtheit beantwortet, wieso die letzte Denkgrundlage nicht τις λόγος, eine (besondere) Denksetzung sein soll. Es ist nicht τiς λόγος, weil es αυτος ο λόγος, nicht eine Setzung, weil es die reine Setzung, das letzte Gesetz der Denksetzung selbst ist, aus welchem alle besonderen Setzungen des Denkens sich müssen herleiten und kraft dieser Herleitung verstehen lassen." Natorp, P., PILa, p. 189. Natorp, P., PILb, p. 194.

²⁹² Geer Edel considers that the Neo-Kantians are the last philosophers who defend the idea of systematic philosophy. He holds: ;,,Wie der Neukantianismus insgesamt, so ist in den Zwanziger Jahren des nun endenden Jahrhunderts bekanntlich auch der Systemgedanke in Mißkredit geraten." Edel, G., 2001, p. 110. ²⁹³ ,,...die Begründung aller besonderen Wissenschaften in einem Systemzusammenhang, in welchem sie in bestimmter Ordnung, nach ihrer wechselseitigen Abhängigkeit und Verwandtschaft, sich zuletzt

is a fundamental science because it determines the place that all particular cognitions occupy and the relations among them. Thus, philosophy guarantees the systematic unity of science. Thanks to this systematic unity of thought, science is "one and indivisible"²⁹⁴. Knowledge is, in general, systematic knowledge²⁹⁵. Philosophy must find a system of concepts in which the analysis of one concept refers to the rest of concepts so that the elucidation of one of them leads to the understanding of others. This analysis will be executed by logic. Logic is another name for philosophy because it is the discipline that carries out its task. Philosophy is logic because it is the science of *logos*.²⁹⁶

In this sense, philosophy can be defined also as method, as it marks the path that thinking takes to become knowledge, and "the method is precisely what makes science a science" Philosophy is a grounding science as a science of the method. Therefore, philosophy should not be defined by its content but by its method. It was Plato who discovered the "unconditional sovereignty of the laws of the method." Thus, the method becomes the fundamental principle of critical idealism. Methodical idealism is a synonym for critical idealism²⁹⁹.

According to this conception, philosophy is the first science. Philosophy, as the ultimate basis of all knowledge, cannot depend on any other science. In this sense,

zurückführen auf eine gemeinsame Grundwissenschaft, die Wissenschaft von der Methode..." Natorp, P., PILa, p. 75. Natorp, P., PILb, p. 76.

²⁹⁴ For Dufour, this is one of the ruptures of Natorp's thinking with that of Cohen, for whom the science division is a *factum* found a posteriori. Cohen takes the science division as a given fact. Natorp believes that this division is exhibited *a priori* in the foundation of science in the logical law. Dufour, É., 2003, p.104.

p.104. ²⁹⁵ "Es wird also im Begriff einer (besonderen) Wissenschaft ein erschöpfender systematischer Zusammenhang aller möglichen Besonderheiten, je innerhalb eines durch einen generellen Begriff abgegrenzten Problemgebietes, und zwar mitsamt ihren wechselseitigen Beziehungen und Verknüpfungsweisen (wie oben die "Systeme" von Intervallen), gedacht." Natorp, P., PILa, p. 303. Natorp, P., PILb, p. 319.

²⁹⁶ Christian Krijnen highlights that this aspect has not always been sufficiently emphasized: "Philosophie wird also keineswegs auf Erkenntnistheorie reduziert, sondern die Erkenntnistheorie fungiert im Neukantianismus als philosophia prima. Als solche hat sie nicht nur eine spezifische Thematik, sondern zugleich eine darüber hinausgehende Bedeutung für das System der Philosophie, dessen Methode und Grundbegrifflichkeit sie vorzeichnet. Entsprechend ausführlich und umfassend haben die Neukantianer sich mit der Erkenntnistheorie auseinandergesetzt. Erkenntnistheorie ist für sie allerdings weder bloss "Epistemologie" anderer, nicht-philosophischer Erkenntnisse, sondern bezieht ihre eigene Erkenntnis mit ein, noch ist sie bloss Wissenschaftstheorie handelt es sich bei der wissenschaftlichen Erkenntnis doch um einen spezifischen Erkenntnissinn." Krijnen, C., 2014, p. 12.

²⁹⁷ "Es bedarf allgemein der Betonung, daß für Plato wie nur je für Descartes oder Kant der Gesichtspunkt der "Methode" der oberste Gesichtspunkt der Philosophie und Wissenschaft überhaupt ist." Natorp, P., PILa, p. 62. Natorp, P., PILb, p. 63.

²⁹⁸ Natorp, P., PILa, p. 82. Natorp, P., PILb, p. 84.

²⁹⁹ "Man darf dies Prinzip, in dem der methodische Sinn der Idee rein und radikal zum Ausdruck kommt, von sonstigem, abweichendem Sprachgebrauch unbeirrt, das Prinzip des Idealismus nennen; wofern diese Vorsicht nötig ist: des kritischen oder, wie wir noch lieber sagen, des methodischen Idealismus." Natorp, 1928, p. 154. Natorp, P., PILa, p. 150. Natorp, P., PILb, p. 154.

philosophy is independent. As an independent science, philosophy must be autonomous. Autonomy is the first general methodological prescription of philosophy. This first science must be grounded on itself and provide the foundation to the other sciences³⁰⁰. Philosophy must be grounded on itself to then ground the remaining areas of knowledge. This is what Natorp calls in the *Philosophical Propaedeutics* the "formal criterion" that philosophy must satisfy to become the first science. To this formal criterion of autonomy, and consequently of self-grounding, a "material criterion" is added. This discipline, as the centre of knowledge, must delimit the regions of the remaining objects of knowledge. However, it cannot contribute to the content of the particular sciences, but philosophy can only provide the ultimate principles of thought in general and, consequently, the elementary normativity in which the other areas of human knowledge unfold. That is, philosophy will not determine the material content of science but, rather, it will give the fundamental logical principles of its procedure. To accomplish this goal, philosophy must be able to have an internal foundation. Philosophy requires immanent legitimation. The circularity of foundation is introduced from the beginning as a primary methodical prescription because it is demanded by the purpose pursued. Therefore, there is a virtuous circularity internal to the method; philosophy, as a general science of knowledge, must have immanent legitimation. This requirement arises as an initial prescription in order to guarantee the epistemic status of a first science. This is the only way to guarantee that knowledge is raised upon secure foundations.

From this analysis, we can demarcate the following fundamental features of the conception of the philosophy of Natorp. First, philosophy is the science of the first principles. Philosophy must search for the most fundamental principles of knowledge, its fundamental concepts. Its goal is not to know a particular object but the unity of knowledge in general. Secondly, and as a consequence of its peculiar goal, philosophy cannot depend on any other science. It must ground itself and thus establish the unity of the multiple fields of knowledge. Thirdly, as philosophy is the general science of knowledge, it must provide the general principles of all objective knowledge. In this sense, it is normative with respect to all regional knowledge expressed in the particular sciences, even when it only determines the general principles and not the particular laws

^{300 &}quot;Denn Philosophie will allerdings Wissenschaft sein, nicht aber besondere eines besonderes Gegenstandes: vielmehr eben, was allen Sonderungen der vielen Wissenschaften gegenüber der Einheit der Wissenschaft ausmacht und begründet: das ist ihre eigentümlich Frage und Ausgabe." Natorp, P., PIP, 1911, p. 3. Also: UOSB, p. 257.

of each particular science. Then, a primary task is to explain the relation of thinking in general with its concretions: particular sciences. Thus defined, the problem of knowledge deals with the relation of thinking to its objects. The problem is to establish how the mind can refer to what is real, departing from thinking itself, i.e., from what is purely conceptual. The problem is the relation between universality and particularity. Philosophy needs to establish how those general principles of knowledge relate to particular laws and, ultimately, to the concreteness of the object. The question is how the universality of the concept refers to the concreteness of the intuitive content.

Every particular determination of thinking is grounded on the determination of thinking in general. Philosophy must seek that unity in which the multiplicity of knowledge originates. Philosophy consists in pursuing the unity of multiplicity and concomitantly to the discovery of multiplicity in the one, while the units reached may become multiplicities that can be brought together under a new higher unit. Philosophy tends to the ultimate unity of thinking and discovers in each unit the multiple contained in it. In this search for the general laws of knowledge, it must be guaranteed the relation of thought with objects. The centre cannot be separated from the periphery. The goal of philosophy demands that philosophy has an immanent foundation that does not take as its starting point anything outside the process of thinking itself. The determination of the general laws of thinking must show its realization in what is *real*. Philosophy will reveal the path that thinking follows in the construction of knowledge³⁰¹. Therefore, the central issue of philosophical thinking is the problem of the method, because the knowledge of the method is the source of every other knowledge.

³⁰¹ We disagree with Éric Dufour in this regard. According to Dufour, Natorp begins to consider the starting point of research in the unity of the system from 1914.Cf. Dufour, É., 2010, p.154.

3.2. The Necessary Starting Point of the Deduction of Categories.

As we observed, Natorp argues that the task of philosophy is to provide the general foundations of knowledge. However, this task cannot be carried out without assumptions. A justification devoid of any assumption is, in general, nonsense. A minimum of necessary assumptions must be accepted, that make possible the pursuit of what is proposed as the task of the investigation. All legitimation requires a minimum of initial premises that are accepted as valid. This starting point is essential for all foundational research. To provide a foundation for knowledge, it must already be available a general concept of knowledge to enable the task to be undertaken. A minimum concept of thinking is essential to allow the investigation to begin. The definition of thinking as correlation will be this basic assumption necessary to provide the conditions of possibility of knowledge³⁰².

By taking the concept of thinking as a starting point, Natorp follows the guidelines introduced by Hermann Cohen. According to Cohen, the logic of pure knowledge must have the concept of thinking as a starting point. Natorp also maintains that thinking is the origin. For both Cohen and Natorp *Denken* is the *Anfang*. Thinking is the beginning and the origin, and the fundamental forms of knowledge are obtained from the analysis of the ways in which thinking operates. One of the weaknesses of Kant's thinking consists in making thinking depend on something external to itself: on intuition. This is a weakness in Kantian proposal that must be remedied. Thinking must be the starting point. Natorp fully agrees with Cohen on this point. Cohen argues:

Wir fangen mit dem Denken an. Das Denken darf keinen Ursprung haben außerhaupt seiner selbst, wenn anders seine Reinheit uneingeschränkt und ungetrübt sein muß. Das reine Denken in sich selbst und ausschließlich muß ausschließlich die reinen Erkenntnisse zur Erzeugung bringen. Mithin muß die Lehre vom Denken die Lehre von der Erkenntnis werden. Als solche Lehre vom Denken, welche an

³⁰² "Dass eine Deduction ohne alle Voraussetzungen beginne, wäre ein widersinniges Verlangen. Vorausgesetzt wird in jedem falle, ausser dem, was zum Verständniss der Aufgabe gehört, irgend ein Letztes, woraus deducirt wird. Voraussetzunglogigkeit kann nur in dem Sinne gefordert werden, dass nicht mehr als das Unerlässliche vorausgesetzt, nichts, was schon zur Lösung gehört, vorweggenommen werde. Es ist daher unser Erstes, dasjenige Minimum von Voraussetzungen festzustellen, welches zur verlagten Deduction nothwendig und hinreichend ist." [...] "Die allgemeine Aufgabe, der die unsrige such als besodere Problem unterordnet, ist: die letzten im vorher erklärten Sinne objectiven Grundlagen der Erkenntniss überhaupt festzustellen. Vorausgesetzt wird also jedenfalls ein allgemeiner Begriff von Erkenntnis." Natorp, P., QQ, p. 2.

sich Lehre von der Erkenntnis ist, suchen wir hier die Logik aufzubauen. 303

A doctrine of knowledge is obtained from a doctrine of thinking. Thinking cannot start from anything other than itself. The construction of the system must begin with thinking. It cannot introduce any doctrine of sensibility because thinking cannot take as a starting point anything alien to itself. The Kantian project of starting the system with a Transcendental Aesthetics is rejected. Kant's mistake was to start with a theory of sensibility. The research must begin with thinking and not with anything external to it³⁰⁴. The sovereignty of thinking is also expressed in the methodological aspect, thinking is the starting point of deduction. Regarding the general perspective, as we will exhibit, Natorp will follow the Cohenian prescription at this point.

Historical background: the genesis of Natorp's conception in his early writings

In his first published writing, Natorp defines thinking as correlation. The concept of correlation is one of the most important within Neo-Kantian terminology. It is interesting to note that this way of conceiving thinking, which accompanies Natorp throughout his philosophical trajectory, appears for the first time in his first published writing, *Ueber das Verhältniß des theoretischen und praktischen Erkennens zur Begründung einer nichtempirischen Realität* (1881). On the occasion of discussing the work of Wilhelm Herrmann, Natorp introduces the principle of correlation. This principle holds that the unity of the object is supported by the unity of consciousness and that, vice versa, the unity of consciousness is made explicit by establishing the unity of the object. The unity of the object only makes sense on the basis of the unity of consciousness. Natorp holds:

Die Wahrheit oder Realität dieser Art Erkenntnisse beruht darauf, daß die Objecte derselben Objecte unsrer Erkenntniß nur sind auf Grund eben der Einheit des Bewußtseyns, von deren Standpunkt allein wir über ihre Realität als Objecte urtheilen können. Die nothwendige Geltung einer so begründeten Einsicht beruht darauf, daß Object und Bewußtseyn nur in unauflöslicher Correlation zu einander bestehen, das Bewußtseyn seine Einheit nur behauptet in der Einheit des Objects und das Object nur in der Einheit des Bewußtseyns,

³⁰³ Cohen, H., 1922, p.11.

³⁰⁴ Cf. Cohen, H., 1922, p.11.

dem es Object ist. Die auf diese nothwendige Correlation gegründete Erkenntniß hat allein Anspruch auf gegenständliche Wahrheit. Wofern man also mir diese im Auge hat, wird der Realitätscharakter unsrer Erkenntniß richtig bestimmt durch diejenigen Bedingungen, von denen die Einheit des Bewußtseyns in der Vorstellung seiner Objecte in unsrer Erkenntniß abhängt.³⁰⁵

The unity of consciousness is the highest principle on which the reality of the object is grounded. The unity of the concrete object is only possible on the basis of the unity of the consciousness, and the unity of the consciousness is only possible in the unity of the object. This way of understanding the essence of thinking, the preeminence of the correlation, is maintained throughout his writings. In his inaugural lecture "Leibniz und der Materialismus" (October of 1881), published in 1985 by Helmut Holzhey, Natorp argues that the truth of the object is grounded in the peculiar form of unity constituted by consciousness. The unity of the concept, given by the unity of consciousness, is the truth of the object. Natorp maintains:

Schon Parmenides und Platon hatten erkannt, dass die Materie nichts völlig Reales sein könne, weil ihr diejenige Einheit mangle, welche zum Begriff der ousia, als des wahrhaften Seins, erfordert wird. Was diese verlangte Einheit sei, lässt sich durch nichts Sinnliches deutlich machen, hingegen versteht es sich sofort durch die Reflexion auf die Grundbeschaffenheit unsres Denkens, welches, wiewohl eine Vielheit von Objecten umfassend, doch diese stets in einer Einheit darstellt, in einer Concentration gleichsam, welche eben das ausmacht, was wir Denken oder Bewusstsein nennen. Leibniz sah ein, dass auf solcher formalen, ideellen, begrifflichen Einheit das beruht, was die Wahrheit der Phänomene, die Substanz oder das Wesen der Dinge im Unterschied von der blossen Erscheinung ausmacht. 306

The conceptual unity and the concreteness of the object are just two sides of the same problem. This idea that appears in his early writings is developed and deepened in subsequent years. The methodological starting point in the research is pure thinking understood as unity and correlation.

In his Habilitation Thesis, *Descartes' Erkenntnistheorie*, the unity of apperception, as the starting point of deduction, is identified with the unity of the 'I think'

³⁰⁵ Natorp, P., UV, p. 245.

³⁰⁶ Natorp, P., LM, p. 9.

of Descartes. Descartes recognized that objects depend on the form of conception of the understanding and, since the understanding is a perfectly articulated unity, it is possible to start a deduction of concepts from the unity of the understanding, taken as the leading thread of the knowledge of objects. For this reason, knowledge can be defined as a form of self-knowledge³⁰⁷. For Natorp, an element that brings the philosophy of Descartes closer to that of Kant, and that can lead to thinking of Descartes as a precursor of critical philosophy is precisely the definition of thinking as an original synthetic unit. In Descartes' Erkenntinstheorie, Natorp shows that Descartes would have originally introduced into the unity of the 'I think' what would later be called by Kant the unity of apperception. The cartesian unity of the Cogito is the Kantian unity of the transcendental apperception. In the Cartesian system, it can be seen the recognition of the fundamental synthesis as a condition of possibility of any specific synthesis expressed in each concept. The intuited object is the product of this conceptual synthesis, and the deduction of concepts must show how this construction is produced. The author of the *Meditations on* First Philosophy exhibited that each particular synthesis supposes the articulation in the central unity of the apperception. According to Natorp, Descartes saw that the unity of reason was the foundation of the possibility of knowing the multiplicity of objects. The very concept of reason is the name of this unity³⁰⁸. In this sense, Descartes was very close to critical idealism, more than Kant himself was willing to accept. Descartes anticipated Kantian ideas. However, the interference of metaphysical and theological prejudices prevented Descartes from taking his thinking to the last consequences³⁰⁹. Descartes saw that it is by virtue of this immanent foundation of knowledge that knowledge of objects can be achieved. Therefore, the foundation of knowledge is obtained by an analysis of the

_

³⁰⁷ "Bis ins Einzelne stimmt die Vorstellung Descartes' zu dem kantischen Begriff einer Grenzbestimmung der Vernunft durch ihre Selbsterkenntnis (Kr. d. r. V., Kehrbach, S. 5), welche mit absoluter Gewissheit und Vollständigkeit muss erreicht werden können, weil die Vernunft eine vollkommene Einheit ist (6), und weil es sich hier nicht um die Natur der Dinge, welche unerschöpflich ist, sondern um den Verstand handelt, der über die Natur der Dinge urtheilt;" Natorp, P., DE, p.4.

³⁰⁸ "Die Vernunft ist der Ausdruck der Einheit in unsrer Erkenntniss gegenüber der Mannigfaltigkeit ihrer Objecte; von ihr hängt alle Erkenntniss der Gegenstände ab, nicht sie von der Kenntniss der letzteren; und durch sie, nach dem Gesetze ihrer Einheit, giebt es allein Beziehung der Vorstellungen auf ein Object, Wahrheit und Falschheit;" Natorp, P., DE, p. 19.

³⁰⁹"Es fragt sich: hat Kant wohl ein Bewusstsein davon gehabt, dass das Ich, dessen Existenz Descartes begründen will, nothwendig nur das Ich sein kann, welches er selbst das Ich der reinen, transscendentalen Apperception nennt, nämlich jene rein intellectuelle Vorstellung, die Kant, mit Anlehnung an Descartes' Cogito, doch, wie gezeigt worden, nicht ganz in seinem Sinne, als den »Actus Ich denke« bezeichnet? — Es scheint nicht, dass Kant es so aufgefasst hat; es hätte ihm sonst nicht entgehen können, dass Descartes' Fundamentalsatz mit den tiefsten Grundlagen seiner Transscendentalphilosophie in genauer Beziehung steht, und den Grundgedanken seines "transscendentalen Idealismus" — wiewohl ohne das Bewusstein seiner entscheidenden Bedeutung — anticipirt.". Natorp, P., DE, p.37. See also: Natorp, P., DE, p. 83. Cf. Tamb. pp. 34, 42ss, 83. Natorp, P., DDE, p.24.

principles of reason, starting from this conception of thinking. Only a study of the nature and legality of reason itself can provide knowledge of objects³¹⁰. We can only have an integrated knowledge of objects taking as a starting point the path that thinking follows in the construction of its objects. According to Natorp, when Descartes claimed that the method of mathematics should serve as a model of cognition, this statement must be understood under the light of how Descartes defined mathematics: as a universal method of construction of objectivity. For this reason, the Cartesian method is the method of universal mathematics. Only starting from this highest point of view, the unity of knowledge can be guaranteed; that is, it can be ensured that a multiplicity of cognitions is brought together in a unified whole. Since the operation of thinking consists primarily in correlating, the principle of correlation is the starting point of deduction also for Descartes. For this reason, it can be argued that "the universal science of human understanding is the basis of knowledge.³¹¹" Descartes warned that the beginning of the investigation can only take place starting from this unity. Only taking as a starting point the principle of correlation, can true knowledge be achieved. Natorp holds:

Die Einheit des Fundaments alles wahren Wissens setzt Descartes durchaus voraus. Alles wirkliche Wissen muss aus Einem Geist concipirt werden können, sowie Alles, was aus Einem Geist und Plan entspringt, vollkommner ist, als woran Viele zusammengearbeitet haben. Denn das Wissen, von dem hier allein die Rede ist, das Wissen aus Principien, *a priori*, hängt von der Vernunft ab, nicht von Lehre und Ueberlieferung, vom Urtheil, nicht von blosser Erfahrung, von der selbsteignen Einsicht der Verstandes, nicht von der Autorität der Schul- und Bücherweisheit. Vernunft aber verlangt völlig unzweifelhafte Gewissheit; Alles, was dieser Forderung nicht entspricht, muss sie als falsch verwerfen³¹².

Descartes presupposes the unity of the foundation of all true knowledge. He considers that all true knowledge has its origin in the unity of the understanding. This unity is the

^{310 &}quot;Der Ausgang von der Forderung einer Wissenschaft der menschlichen Intelligenz, d. h. einer unabhängig und für sich selbst feststehenden Gewissheit über den Begriff und das Fundament aller Wahrheit der Erkenntniss, welche deswegen erreichbar sein muss, weil sie nicht ausser uns in den Objecten, sondern in der Natur und Gesetzlichkeit der Erkenntniss selbst ihren Ursprung hat..." Natorp, P., DE, p. 21. 311 "Die universelle Wissenschaft des menschlichen Verstandes liegt allem Wissen, die universelle Mathematik näher aller Erkenntniss von Maass und Ordnung, endlich die besonderen Wissenschaften des Quantitativen aller Erkenntniss der Qualitäten zu Grunde." Natorp, P., DE, p. 23. 312 Natorp, P., DE, p. 27.

unity of the foundation (*Die Einheit des Fundaments*). Without this unity, there would be a mere dispersion of separate groundless cognitions and not true knowledge. True knowledge can only be achieved from a certain form of self-knowledge, on the self-own insight of the understanding (*von der selbsteignen Einsicht der Verstandes*). The point of departure of the investigation must be the unity of the understanding on which all knowledge depends. According to Natorp:

Es braucht kaum noch darauf hingewiesen zu werden, dass diese Anschauungen mit denen der »Regeln« genau zusammenhängen und im Einklang sind. Auch dort ging Descartes aus von dem Gedanken der Einheit alles Wissens im menschlichen Verstände, als von dem alle Erkenntniss abhängt und in dem sie ihre letzte gemeinsame Wurzel hat. Auf dieser Grundlage wurde auch dort eine völlig zweifellose Gewissheit gefordert für alle Erkenntniss, die diesen Namen in Wahrheit verdient; und behauptet, es müsse erreichbar sein, sich einmal für allemal wenigstens darüber Gewissheit zu verschaffen, was von jedem Gegenstande zu erkennen möglich ist und was nicht. Auch dort wurde aus diesem Grunde alles irgend zweifelhafte, bloss wahrscheinliche Wissen für nicht viel mehr als Täuschung, alle bloss historische, auf die Autorität der Ueberlieferung gegründete Kenntniss für werthlos vor dem Anspruch der Vernunft erklärt³¹³.

The process of thinking cannot start from anything external to itself. All knowledge always depends on the unity of the understanding, and it is from that unity that it finds its foundation. The unity of the foundation guarantees the objectivity of its concretions. The study of the understanding and its principles underlies all knowledge. Therefore, the starting point of research is not the Transcendental Aesthetics with its study of the forms of sensibility -as it was for Kant- but the Transcendental Logic. The starting point of the investigation is not the point of view of the intuition in its concreteness. On the contrary, the departing point is the very operation of the concept which has its origin in the transcendental unity of the apperception. The beginning is the Logic and not the Aesthetics because the origin is the conceptual process rooted in the unity of the synthesis.

³¹³ Natorp. P., DE, p. 27.

In the *Objective and Subjective Foundation of Knowledge* (1887), one of his first systematic works, Natorp argues that the task of philosophy is to show how thinking can build objectivity. The formation of the objective takes place in the relationship between the multiple that makes up the object and the unity that determines it as an object as such. This unity is provided by the activity of thinking and can be defined in terms of synthetic unity. This unity of the understanding guarantees that each of these expressions is always in an interconnection. The unity of thinking guarantees the unity of the principles that govern each particular field³¹⁴. The multiplicity of appearances must be subjected to the unity provided by a law that establishes how the parts are articulated. The law regulates the way in which the various appearances are brought together in unity. The fundamental law of thinking is the search for this unity. For this reason, thinking can be defined as a synthetic unity, as a principle of unity of diversity³¹⁵. This synthetic unity is the law of legality. Synthetic unity is the highest principle³¹⁶.

These peculiarities of the definition of thinking will find a mature expression around 1890. Quantity and Quality (1891) is one of the first works in which Natorp presents its deduction of concepts and, thus also, a definition of the concept of thinking that begins to acquire its own characteristics, different from the notion of synthetic unity that is properly Kantian. Indeed, Kant defined thinking as synthesis as well. However, as we shall see later, Natorp's definition of the notion of synthesis is quite different from the Kantian definition. For Natorp, thinking is the starting point of the deduction of concepts. The deduction of the categories is established from the minimum assumption of thinking defined as synthetic unity or as correlation. Synthetic unity is the original form of thinking. This original form is only possible as a correlation of two moments: unity and multiplicity³¹⁷. However, Natorp highlights that these moments are not to be understood as part of a process of any faculty of the mind. This work of 1891 shows that Natorp is willing to distinguish his concept of thinking from a psychological interpretation. The synthetic unity is the origin of the deduction of categories. The categories are deduced from this original act of thinking defined as the unification of multiplicity. Natorp warns that this origin can be confused with a psychological

³¹⁴ Natorp, P., USOB, p. 257.

^{315 &}quot;Alle wissenschaftliche Erkenntis nun zielt auf's Gesetz. Die Beziehung der Erscheinung zum Gesetze (die Beziehung des "Mannigfaltigen" der Erscheinung auf die "Einheit" des Gesetzes) muss daher die in aller Erkenntnis ursprüngliche Beziehung auf den Gegenstand erklären. Die gesetzmässige Auffassung des Erscheinenden gilt als die gegenständlich wahre." Natorp, P., USOB, p. 259.

³¹⁶ Natorp, P., USOB, p. 285.

³¹⁷ Natorp, P., QQ, p.130.

beginning. In particular, because having defined thinking as an action, one could raise the question of the subject who performs the act. That is, if we define thinking as the act of linking, the question of the agent responsible for this action arises. Natorp is aware that the use of notions such as consciousness or subjectivity can lead to a psychological interpretation of his position³¹⁸. For this reason, he develops several arguments to justify that the use of notions such as consciousness or thinking does not necessarily involve a reference to the mental processes of the formation of representations. The unification of multiplicity does not require a subject to perform the action. Indeed, Natorp affirms, a minimal concept of consciousness is necessary. The content of this concept of consciousness will be an indispensable minimum assumption for the deduction of the concepts of thinking. Consciousness is the unifying point of view of multiplicity. It is the unit of the correlation that allows the connection of the multiple. The deduction of concepts departing from thinking is not an analysis of the formation of representations in the mind. Thinking should not be understood in this context as mind. Rather, the definition of thinking as correlation lies in the need to think of an articulating unity that expresses the "relation of a multiplicity to the central unity"³¹⁹.

Natorp's conception progresses in identifying the concept of thinking with the concept of law³²⁰. In the *Social Pedagogy* (1899), Natorp maintains that this articulating unity is the law itself. Therefore, the fundamental way of thinking is legality. The law is the articulating unity of multiplicity. It is the central point of view that articulates multiplicity. Thinking is the unity of those laws. The fundamental law of thinking is the demand of giving unity to the multiplicity. The general form of the law is the unity of the multiplicity. The task of thinking is to form concepts, i.e., to join a multiplicity in a unity. The total conjunction is its goal. For this reason, it can be considered that the requirement of this total unity "is a consequence of the fundamental law of unity, which is the law of thinking itself."³²¹

In *Number, Time and Space*, the concept of thinking acquires the definition of the union of multiplicity³²². Again, Natorp claims that the unity of thinking must be the starting point for the deduction of concepts. The general concept of law is contained in

³¹⁸ Natorp, P., QQ, p.4.

³¹⁹ Natorp, P., QQ, p.7.

³²⁰ In chapter 4, we will study the relationship between the notions of law and concept.

³²¹ "Diese Einzigkeit ist aber selbst eine Folge des Grundgesetzes der Einheit, welches das Gestz des Denkens selbst ist." Natorp, P., SP, p. 29.

³²² «Penser, c'est saisir le multiple dans l'unité». Natorp, P., NTE, p. 344.

the concept of thinking and, it must be shown how the concepts generate objectivity. The investigation must find the peculiar ways in which the object is constructed *in* and *by* thinking. That is to say, the intuited object is nothing but the object of thinking because it is the result of the conceptual construction, which is an operation of thinking itself.

As it is well known, the definition of thinking as a synthetic unity is proper of Kantian philosophy. However, Natorp sees in Plato the germ of this idea. The comparison of the 1903 edition of his *Plato's Theory of Ideas* with that of 1921 makes it possible to highlight that throughout its philosophical development, Natorp maintains its definition of the concept of thinking as the origin of all objectivity. The conception of thinking as synthesis is an idea that Natorp supports from his early writings up to his late writings³²³. A clear moment in which this is noticed is the comparison between the first (1903) and the second edition (1921) of his study on the theory of ideas in Plato.

Natorp claims that Plato was the first to see that the task of thinking is the search for unity and that this search for unity is carried out through concepts. The concept, as a position of thinking, is "the unity of the multiplicity of the cases that take place" This discovery makes Plato the precursor of critical idealism by noting that every being is a position of thinking that determines multiplicity through concepts. The concepts contain the multiplicity of instances, giving them a peculiar unity according to each type of relationship. The concept determines specific forms of unity of multiplicity. The task of thinking is to establish these relationships through concepts. In the generation of concepts, thinking determines special types of relationships between the unity and the multiplicity. Insofar as these relations are the necessary forms with which thinking relates multiplicity, the concept is also a law. The law determines the peculiar way in which multiplicity is put together. The law is the point of view that establishes the specific way in which the thinking relates to the multiplicity.

From the horizon of Socratic philosophy, Plato understands the object of philosophy as a form of self-knowledge. This Socratic idea that knowledge is self-knowledge should not be understood as if Plato were refereeing to the process of the individual consciousness, as if the subject involved were the specific individual. Rather, the concept of consciousness which Plato referred to herein is consciousness in general,

³²⁴, Der Begriff ist, wie seit dem Meno feststeht, die Einheit der Mannigfaltigkeit vorkommender Fälle." Natorp, P., PILa, p139. Natorp, P., PILb, p. 143.

We disagree in this regard with Éric Dufour. For Dufour, Natorp began to consider the starting point of the investigation in the totality of the system only after 1914. Cf. Dufour, É., 2010, p.154.

not one particular thinking but the thinking. However, this knowledge is not only the expression of the general consciousness but of the legality of the consciousness. This knowledge of oneself is the knowledge of the legality of consciousness³²⁵. Thinking seeks to know its own legality, and for this reason, it can be conceived as well as selfknowledge. The study of how thinking builds these relationships is the theory of concepts, and it is a form of self-knowledge too. The deduction of concepts that Natorp will undertake can be considered self-knowledge since it is knowledge of the forms of knowledge. Thinking aims to know its own legality. In this sense, the deduction of concepts is an analysis of the ways in which thinking generates its contents in the formation of concepts. For this reason, the deduction of concepts can be considered as the knowledge of the legality of thinking and, as well, as a form of self-knowledge. The search for unity is the central task of thinking. Thinking is a function of unity. This is what Plato also expressed, and the Greeks in general, with the term soul. The task of thinking is to put the unity in the diversity of multiple instances. The ways in which thinking can put these units are diverse. The deduction of concepts is the exhibition of the various ways in which the multiplicity can be reunited following certain laws. The deduction of concepts consists in the exhibition of these fundamental modes of connection³²⁶. This is the meaning of the Platonic idea. Ideas should not be construed as metaphysical entities. The concept of idea in Plato must be understood as the ways in which thinking operates to achieve unity. Ideas are the expression of the legality of thinking because they provide a determined order to the determinable. The ideas are the foundations of the unity of diversity³²⁷. Ideas are, therefore, also "methods" because they indicate the path of knowledge in the pursuit of this unity. Ideas are the relationships that thinking puts - and, in this sense, they are positions - to generate the unity of the determinable. Determinations are these specific relationships established by thinking. For this reason, it can be affirmed that all determination is a product of the concept operation

³²⁵, Daß man "aus sich selbst" die Erkenntnis hervorhole, hätte keinen Sinn, wenn nicht in dem "Selbst" etwas mehr gedacht wäre als Bewußtsein überhaupt; wenn nicht darin mitgedacht wäre die Gesetzlichkeit des Bewußtseins, gemäß welcher es das Objekt, nämlich das reine Objekt des Begriffs, selber gestaltet. Die Form der Erkenntnis überhaupt ist Gesetzlichkeit; diese Form aber ist es, welche den Inhalt, den reinen Inhalt der Erkenntnis konstituieirt; denn es ist allgemein das Gesetz, welches in der Erkenntnis und für sie den Gegenstand schafft." Natorp, P., PILa, p. 28. Natorp, P., PILb, p. 29.

³²⁶ Natorp, P., PILa, p. 150. Natorp, P., PILb, p. 154.

[&]quot;Das Gesetz als der wahre Inhalt der Wissenschaft, der Grund aller Richtigkeit und damit Güte, als das was jedem, dem Einzelnen und dem Ganzen, seine "Gestalt", sein Eidos gibt, dies und nichts andres ist das Zentrum, in dem diese ganze bei aller Knappheit der Andeutung so tiefgründige wie weit ausgreifende Betrachtung zusammenhängt. Wir stehen hier schon unmittelbar an der Schwelle der "Idee". Denn die Idee bedeutet das Gesetz, nichts andres." Natorp, P., PILa, p. 48. Natorp, P., PILb, p. 49.

(i.e., of thinking), and the sensible can be defined by opposition as that which the mind must determine, as the determinable. The determination of thinking is a determination by means of concepts and the determination of the concept is a task of thinking. To emphasize this point Natorp makes a slight modification in the second edition of his work on Plato:

Alle Bestimmung also ist vielmehr Leistung des Begriffs. Sogar nur im Hinblick auf die bestimmende Funktion des Begriffs vermochte das Sinnliche charakterisiert zu werden als das noch nicht bestimmte, erst zu bestimmende." (PILa, 107).

Alle Bestimmung also ist vielmehr Leistung des Denkens. Sogar nur im Hinblick auf die bestimmende Funktion des Denkens vermochte das Sinnliche charakterisiert zu werden als das noch nicht Bestimmte, erst zu Bestimmende. (PILb, 110).

The determination of thinking is equivalent to the determination of the concept. Natorp emphasizes that the task of thinking is to carry out the forms of connection through concepts. This discovery makes Plato the precursor of critical idealism. He established that thinking creates its own object by creating certain forms of relationships. This conception of thinking makes Platonic philosophy the origin of critical philosophy. Every being is the position of unity of a determination of thinking. Plato warns that thinking cannot go beyond thinking itself. The work of thinking consists in the analysis of its own productions. Every knowable being consists of the positions of thinking³²⁸.

Natorp finds in Plato the germ of his own philosophical proposal. The deduction of categories must be undertaken from the concept of thinking because the concept of thinking is the expression of the form of legality that underlies all particular legality. The deduction will allow showing how each particular position of thinking is carried out on

328 "Oder will man etwa sagen, Plato sei im Phaedrus von der Überschwänghchkeit der Erhebung des letzten

Staat 511B), das Grundgesetz des Logischen, welches alle besonderen Denksetzungen (λογοι) und in diesen alles besondre Sein erst begründet. Möchte das im Phaedrus vorschweben, so stände er ja damit auf dem Boden des reinen platonischen Idealismus." Natorp, P., PILa, p. 82. Natorp, P., PILb, p. 84.

Wissensobjekts über Sein und Erkenntnis wieder zurückgekommen, indem er sich auf die in der Tat von keiner Philosophie ungestraft zu überschreitende letzte Grundrelation von Sein und Erkennen wieder zurückbesonnen habe? Aber der Sinn jener viel getadelten Überschwänghchkeit ist kein anderer als der reine Idealismus Platos; die unbedingte Souveränität des Gesetzes der Methode. Auch nicht ein Logos τις λογος, Gastm. 211 A) ist die letzte Instanz der Erkenntnis, wohl aber der Logos "selbst" αυτος ο λογος,

the basis of the legality of thinking in general. All foundation is found in the law and thinking is the general expression of legality. Therefore, thinking is the foundation and beginning of deduction. It was Plato who evidenced this general law of deduction:

Der "voraussetzungsfreie Anfang", den die Idee des Guten besagt, soll erreicht werden einzig im logischen Rückgang von den relativen Grundsätzen der besonderen konkreten Wissenschaften zu den letzten, völlig reinen Denkgrundlagen, d. i. solchen ursprünglichen, ersten Setzungen des Denkens, aus denen jene, sofern sie gelten sollen, rein deduzierbar sein müssen. Für diesen Rückgang gilt schlechterdings kein andres Gesetz als das des deduktiven Zusammenhangs, desselben, der innerhalb der Wissenschaften herrscht. Hier ist endlich mit einer jede Zweideutigkeit ausschließenden Bestimmtheit beantwortet, wieso die letzte Denkgrundlage nicht $\tau\iota\zeta$ $\lambda \acute{o}\gamma o \zeta$, eine (besondere) Denksetzung sein soll. Es ist nicht $\tau\iota\zeta$ $\lambda \acute{o}\gamma o \zeta$, weil es $\alpha \acute{v}\tau o \zeta$ \acute{o} $\lambda \acute{o}\gamma o \zeta$, nicht eine Setzung, weil es die reine Setzung, das letzte Gesetz der Denksetzung selbst ist, aus welchem alle besonderen Setzungen des Denkens sich müssen herleiten und kraft dieser Herleitung verstehen lassen 329.

For Plato, the idea of the good represents the beginning, since it is the idea that contains the form of legality that supports all particular legality. The legitimacy of particular laws is guaranteed in their adjustment to the general form of the law. The analysis allows discovering in each particular legal system, i.e, the sciences, the principle of this general legality, which in Plato appears represented with the idea of the good. The idea of good is the form of the legality of thinking in general. All being is being for thinking. Therefore, this idea is also the foundation of all being in general. The idea of good represents the principle that governs all legality. Every law (*Gesetz*) is also a positing (*setzen*) but the form of legality itself is the total order required, which is never completely accomplished. This general guiding principle of every particular position of thinking is a regulative idea. This general form of legality does not end in any particular position of thinking. The complete order in a legal system is a task that is required but

³²⁹ Natorp, P., PILa, p. 189. Natorp, P., PILb, p. 194.

never achieved. This form of the law is the ultimate foundation of all laws³³⁰. The starting point of the deduction is the law itself³³¹. Natorp sees in Plato the germ of his conception of thinking. The unity of consciousness grounds how the multiplicity must be brought together. Concepts are these fundamental modes of relationships. A complete exposition of the development of these relationships cannot be achieved, since there is not a closed system of concepts but an eternal development of the operation of thinking in the search for unity. However, it is possible to find the fundamental structure of this operation. The analysis of this development of consciousness makes it possible to find the particular forms of relationships, according to which each specific object of thinking is constituted. These particular functions "are just the different expressions of the pure functions of thinking, each emphasizing a particular aspect of this function"³³².

Starting from this guiding idea, Natorp undertakes the deduction of categories from a conception of thinking conceived as a unity of synthesis. The categories are the expression of the general principle of legality represented in the concept of thinking. They express the structure of being in general as a correlate of the positions of thinking. As we exhibited, this idea is presented *in extenso* for the first time in his study of Platonic philosophy. In the light of these considerations, one must understand Natorp's assertion in the *Philosophical Propaedeutic*, that the fundamental law of knowledge is the law of synthetic unity. The law of synthetic unity is the foundation of objectivity³³³. The development of thinking occurs in accordance with this form of legality. This starting point guarantees to overcome the heterogeneity between intuition and concepts, as it shows that universal and the particular have the same origin, as they are rooted in the principle of correlation. In what follows, we shall analyze the elements of the process. In the next chapter, we will explain in detail how this construction of objectivity takes place.

2

³³³ Natorp, P., PP, p.24.

³³⁰ Natorp, P., PILa, p. 189. Natorp, P., PILb, p. 196.

 $^{^{331}}$ "Nichts andres ist man demnach unter der Idee des Guten zu denken berechtigt, als: nicht ein letztes logisches Prinzip, sondern das Prinzip des Logischen selbst und überhaupt, in welchem alle besondere Denksetzung und damit alles besondere Sein — Sein besagt ja nur Setzung des Denkens — zuletzt zu begründen ist; zu begründen nicht als in einem letzten, dem Denken vorausliegenden, vorgedanklichen Sein — nichts ist bündiger abgelehnt als dies —, sondern einzig als in seinem eigenen letzten Gesetz. Denn den "Anfang" einer Deduktion nennt man ein Gesetz. Das Gesetz ist es allgemein, welches den Gegenstand konstituiert; dieses Gesetz selbst, daß im Gesetz der Gegenstand zu begründen, ist somit übergegenständlich, aucti über allem besonderen Gesetz (λ ó γ o ς) , nicht ein, sondern das Gesetz; woraus zugleich klar wird, inwiefern dies letzte Prinzip sogar über die Erkenntnis der Wissenschaft hinaus ist." Natorp, P., PILa, p. 184. Natorp, P., PILb, p. 194.

^{332 &}quot;Die reinen Denkfunktionen sind sämtlich nur verschiedene Ausdrücke der reinen Denkfunktion, welche je eine besondere Seite an dieser herausheben." Natorp, P., PILa, p. 238. Natorp, P., PILb, p. 245.

3.3. The Elements of the Definition of Thought as Correlation.

3.3.1. The Point of View of the Unity.

As we explained, thinking can be defined as the search for the interconnection. Thinking consists in the search for unity in diversity. Thinking is the activity of "uniting multiplicity in a unity"³³⁴. The main task of thinking is to achieve total unity. In the search for unification, thinking generates partial units from multiplicity. Concepts are these functions of unifying. The task of thinking is the total unity, and the fundamental functions of thinking are the ways in which thinking generates partial units in the search for this complete unity. Thinking determines all the different ways in which multiplicity can come together as it is the general form of gathering. The form of the unifying is the union between unity and multiplicity; that is, the synthesis. Thinking is synthesis and the levels of thinking are the expressions in which synthesis takes place. Thinking is this central point of view of articulation. This unifying pole is the perspective of the reunion. The point of view of the reunion determines the way in which the multiplicity is reunited. Each particular function is only an expression of the general function of thinking. The foundation of knowledge in this unity allows each of the particular expressions of thinking to come together in an articulated whole. The sciences, the factual expressions of thinking, can form a unity thanks to the foundation in this central unity. Scientific thinking can form an articulated whole because it is grounded on the articulating unity of thinking. Each particular science is a specific expression of this central unity. For this reason, each science looks for the specific unity in a determined region, establishing partial units. Total unity is a task for thinking. This task is partially accomplished by each particular science that constitutes the specific concretions of this procedure. The central unity of thinking allows the sciences not to be disintegrated as isolated units but to be articulated in a coherent whole. The laws of each particular area are linked together thanks to thinking as a principle of total unity, as a form of legality. The factual division of the sciences must find its foundation in their common logical origin³³⁵.

³³⁴ «Penser, c'est saisir le multiple dans l'unité». Natorp, P., NTE, p. 344.

³³⁵ Éric Dufour finds in this aspect a difference with respect to Cohen's approach, for whom the division of the sciences into its different branches would be a fact only accessible a posteriori. Cf. Dufour, E., 2003, p.104.

The law is the concept that gives expression to this unity. In the relationship between unity and multiplicity, the law is the concept that represents the form of articulation. The law is the expression of each specific form of unity. The mode of unity determines the necessary connections between the parts of the multiplicity. The conceptual articulation in the law allows the multiplicity to be organized in certain ways. The determinations of thinking are the ways in which a multiplicity comes together in a peculiar way. The concepts unify the multiplicity of appearances in a unity. For this reason, the concept is the form of both the universal and the particular, since it represents the form of the universal and determines the mode of construction of the particular³³⁶. The concept constructs the universal and the particular, as it is the function for the construction of the individuality. The law is the form that articulates the multiple in each creation of thinking. Synthesis is the general form of legality and each specific knowledge expressed in science seeks the law that governs a particular field; that is, the specific synthesis with which the object of the field in question is constructed³³⁷. For this reason, the understanding is the law of laws, insofar as it is the general form of articulation. An exhibition of the ways in which legality is constructed exhibits both the general form of thinking and the ways in which objectivity is constructed. As a general form of legality, thinking represents the unifying point of laws, and each particular law is the specific articulation between a unity and its multiplicity. Although the law represents the perspective of the union, it also contains multiplicity. In the concept of law, both the unity and the multiplicity are included.

The concepts - the rules of the unifying - determine the way in which the multiplicity is ordered. The concepts represent these laws of unifying that make each of the moments of multiplicity to be united in a certain way. Concepts are the specific modes of the order of the multiplicity that, from the perspective of the unity, is given to thinking. Thinking is generating concepts that, by establishing a peculiar form of unity, also generate the multiplicity that this unity contains. As we shall explain later in detail, the multiplicity of the cases that the concept contains is established at the same time of the generation of the unity. The multiplicity can only be conceived as the multiplicity of certain unity. The concept itself contains the form of the reunion and the cases it

Natorp, P., PILa, p.48; Natorp, P., PILb, p.49.Natorp, P., USOB, p. 257, 259.

includes. The multiplicity of cases and their unity are joined in the unity of the concept. The concept is the unity of the multiplicity of cases³³⁸.

The assumption of the search for unity is the starting point of the deduction of the concepts of thinking, since it is the necessary presupposition for any conception of thinking in general. The search for a total unity is the minimum concept of thinking, and it is the starting point of the investigation. For this reason, although the synthesis consists of both the unity and the multiplicity, the most paradigmatic expression of thinking is the central unity. The task of philosophical inquiry is to exhibit the ways in which this unity is generated. The path of investigation should show the different fundamental forms of connection with which thinking builds objectivity. The particular sciences will be in charge of showing the specific modes of articulation in units. Philosophy will investigate the fundamental operations with which thinking seeks unity in multiplicity. The total whole will always be the searched unity. However, the central unity of thinking must be assumed as the minimum necessary assumption to think the modes of articulation. As a general form of unification, thinking is the foundation of all unity. In this sense, thinking is the general form of an articulation: the correlation between unity and multiplicity. This ultimate unity from which research must start is not a substrate but an act, the activity of gathering. The understanding is the unity of all unity because it is the form of all forms. The deduction of the levels of thinking will take the form of the unity as a starting point with the aim to exhibit the general laws of unifying, the form of the unity³³⁹.

3.3.2. The Point of View of the Multiplicity.

In contrast to this central unity, multiplicity is the expression of the diversity that must be unified. This multiplicity has a negative and a positive meaning. Positively, multiplicity is what must be brought together to form an objective unity. In this sense, the multiplicity can be considered as given. The multiplicity is given as it is what thinking must unify. Thinking also requires the parts that make up the whole. The multiplicity will be the point of view of these parts that must be related in a unity. Multiplicity is a necessary moment for the operation of thinking. However, multiplicity cannot be thought as merely given. One should explain how it is constructed. The diversity given is actually originally

³³⁸, Der Begriff ist, wie seit dem Meno feststeht, die Einheit der Mannigfaltigkeit vorkommender Fälle." Natorp, P., PILa, p. 139 . Natorp, P., PILb, p. 143.

³³⁹ Natorp, P., PILa, p. 150. Natorp, P., PILb, p. 154.

produced by thinking. This multiplicity is also a product of thinking. At a certain stage, it is considered as an element ready to be reunited in a superior unity. The new units can be taken as elements that together with other units make up new multiplicities that must be brought together. To consider multiplicity as given means to take it as a relative moment in the process of thinking. This is the positive sense of the multiplicity. However, the multiplicity can be negatively considered. Negatively, multiplicity represents the limitation of thinking. For this reason, the sensible is defined as the ultimate multiplicity, since it represents 'the other' with respect to the unity required by thinking. Multiplicity is the negation of the absolute character of thinking As long as multiplicity is the negative moment with respect to the unity required by thinking, it can be said that thinking has an alterity. Regarding the unity demanded by thinking, multiplicity is presented as that which must be brought together. In this sense, multiplicity is constituted in the negation of the absolute character of thinking. Multiplicity is the expression that this unity is always sought but never actually achieved. This point represents a manifest distance from absolute idealism. For critical idealism, the multiplicity requires a form of reunion that only thinking can provide. In this context, the parts of multiplicity are not isolated elements that are brought together by a subsequent action. Moments of multiplicity arise from a creative act of thinking and concomitantly with their peculiar modes of connection. There is not a juxtaposition but a legal concatenation that unites each of these moments in certain forms. Therefore, the relationship between each of the elements of the multiplicity is always determined by the concept, which is the determined form of the unifying.

Multiplicity is that which must be determined by thinking, which is still indeterminate, and which thinking demands to determine in the search for unity. In this sense, multiplicity can be defined as the determinable. From the point of view of the 'periphery', the multiplicity must be brought together in a common unity. The moment of conception of multiplicity as given will be overcome when this instance is proved to be a result of the process of thinking. Multiplicity turns out to be another determination of the concept, whose constitution is also the result of a task of thinking. The moment of externalization of thinking in multiplicity can be taken separately in a conceptual analysis. However, it should be noted that this moment of diversity is also a product of thinking. The multiplicity considered as given is a relative moment that must be overcome. It must be shown the origin of this multiplicity in thinking itself. Multiplicity has its origin in thinking. Although Kant was able to identify in this synthetic unity the

most fundamental mark of thinking, he did not take this characterization of thinking radically enough because he presupposed a multiplicity that thinking only had to gather but could never produce. For Kant, the multiplicity is given in intuition. For this reason, in the Kantian conception, the last elements seem to be given in advance, before knowledge. According to the Kantian conception, thinking can produce the forms of the connection, but the multiplicity must be provided by a faculty independent from thinking. The understanding is capable of creating concepts, but it depends on intuition because the intuition provides the multiplicity to be united. For thinking to be able to refer to objects, it requires a capacity that it does not have: to provide the terms of the connection. In the Kantian proposal, the terms are given independently of the relation.

Kant argues that to refer to objects, thinking must have a multiplicity that must be given by intuition. The multiplicity is the element provided by the sensibility. In contrast, the concept is the function of connection. The concept can never be given but is always spontaneously produced. The multiplicity is given in intuition, the function of unity is a product of the understanding. Unity is the contribution of understanding, the multiplicity of sensibility. As we observed in Chapter I, according to the Kantian conception, thinking cannot by itself produce the multiplicity that must unify. Intuition must provide the multiplicity. The multiplicity is always given to the understanding. For Natorp, there can be no extrinsic starting point to thinking. Multiplicity requires a determination to become a set of elements that must be brought together, and its production cannot occur independently of the act of thinking³⁴⁰. The production of the multiplicity requires a conceptual determination and cannot take place without it. Natorp argues:

Allerdings stumpft Kant selbst die Schärfe dieser radikal idealistischen Einsicht wieder ab, wenn er den Urakt der Synthesis beschreibt als die "Handlung, verschiedene Vorstellungen zueinander hinzuzutun" und "ihr Mannigfaltiges" zu einer Erkenntnis zu begreifen. Danach scheinen die letzten Elemente, in der fragwürdigen Gestalt von "Vorstellungen", doch wieder vor der Erkenntnis, selbst vor dem Urakt des Erkennens, dem Akte der Synthesis, voraus gegeben sein zu sollen. Aber hier ist nun Kant sehr leicht aus seinen eigenen Voraussetzungen zu korrigieren. Man braucht nur

³⁴⁰ "Allzu unbefangen sprach Kant von einem Mannigfaltigen der Sinnlichkeit a priori, welches die transzendentale Logik als Stoff "vor sich liegen habe, das aber noch vom Denken "auf gewisse Weise durchgegangen, aufgenommen und verbunden zu werden nötig habe, wenn daraus Erkenntnis werden solle" Natorp, P., LGEW, p. 40.

zu fragen: sollen diese Elemente vor dem Grundakt der Synthesis voraus einen "gewissen Inhalt" schon haben oder nicht? Aber die Synthesis soll ja vielmehr das sein, was sie "zu einem gewissen Inhalte erst vereinigt". Also waren sie vordem — Vorstellungen zwar, aber ohne gewissen Inhalt? Vorstellungen, in denen — nichts Bestimmtes vorgestellt war? In der Tat darin liegt: nichts Bestimmtes. Die Bestimmtheit des "Was", das ist genau, was der Urakt der Erkenntnis als Akt des Bestimmens erst zu erbringen hat³⁴¹.

The risk of the Kantian proposal consists in assuming that there is certain multiplicity that is there ready to be gathered into a unity. It could seem as if there were representations preexisting the action of synthesis. On the contrary, neither the unity of the concept preexists the multiplicity, nor the multiplicity exists before the generated unity of the concept. The unity of the concept and the multiplicity arise at the same time.

3.3.3. The Point of View of the Relationship between Unity and Multiplicity.

The process of thinking includes both moments, unity and multiplicity. Therefore, although thinking consists in the search for unity, it must be defined as synthesis or correlation. The synthesis involves both the articulating unit and the multiplicity that is articulated. Unity and multiplicity are two moments of the same act.

To express the relationship between unity and multiplicity, Natorp uses the analogy of a circle. Unity is the point of view of the center, while multiplicity is the periphery. Just as the circle is composed of both moments, thinking also requires for its formation a central unity that determines the shape of the periphery, and the periphery that gives expression to the unity. The origin and the originated arise concomitantly. As in the circle, the center cannot exist without the periphery, and the periphery cannot exist without the center. However, the particular shape that the circle takes is determined by the primitive force of the central unity³⁴². Multiplicity is the expression of the

³⁴¹ Natorp, P., LGEW, p. 46.

³⁴² "Fragt man auch, ob im Kreis das Zentrum für die Peripherie oder die Peripherie für das Zentrum sei? Für Plato ist das Letzte gewiß nicht das "All", in seiner Zerstreuung in die "Andersheit", sondern die Einheit der vom Zentrum her lenkenden und bestimmenden Urkraft. Dabei hat man sich aber das "Zentrum" nicht als leeren Mittelpunkt, sondern als zentrale Kraft, ganz im Ganzen und ganz in jedem Teil, lebendig schöpferisch das All durchwaltend und eben zum Ganzen zusammenschließend zu denken." Natorp, P., PILb, p.512.

disgregation of thinking in the process of concept formation. Thinking is instantiated in particular ways. Understanding the origin of this multiplicity implies a task of redirecting it to the origin that created it. This mode of relationship between the center and the periphery determines the path of philosophical inquiry. Returning to the analogy of the circle, Natorp observes that if we consider the whole of knowledge as a circle, the particular sciences go from the center to the periphery while philosophy goes from the periphery to the center³⁴³. The multiplicity in which knowledge branches out is the proper field of particular sciences, which studies the peculiarity of each region of knowledge. On the contrary, philosophy takes as its starting point the clarification of knowledge, but its objective is to find the center that originates the periphery. Natorp observes, center and periphery, even when expressed as opposite directions, culminate corresponding, since they are only two directions of the same path. However, as a center of origin, philosophy guarantees the unity of knowledge. Thus, philosophy expands as much as the region of scientific knowledge expands. The logical procedure of thinking is expressed in a circle in which the center and the periphery are co-implicated. However, it is the center that guides and determines the periphery. This center should not be conceived as a mere empty midpoint but rather is the origin of the periphery, its law of formation³⁴⁴. Certainly, the center cannot exist as an independent part. On the contrary, in the connection between unity and multiplicity, there is no pre-eminence of one moment over the other. The relationship is original, while thinking about the moment of unity and the moment of diversity is ulterior, and only a result of a process of abstraction. What unites and what is reunited are only two poles of the correlation that is the origin. Neither the multiplicity can be conceived if it is not from the perspective of the unity nor the unity can be thought without the multiplicity that it brings together, since to be a unity is to be the reunion of a multiplicity, and to be a multiplicity is to be the plurality that is reunited in a unity. There are no independent parts that can subsist one without the other but a whole whose parts can be isolated to analyze them separately. In fact, Natorp claims that the whole-part relationship is derived from correlation. Thinking includes both moments: multiplicity and unity since thinking in general is the method of uniting a multiplicity³⁴⁵. Thinking is the path of reunion and separation that contains both the part and the whole.

³⁴³ Natorp, P., PIP, p.3.

^{344 &}quot;Dabei hat man sich aber das "Zentrum" nicht als leeren Mittelpunkt, sondern als zentrale Kraft, ganz im Ganzen und ganz in jedem Teil, lebendig schöpferisch das All durchwaltend und eben zum Ganzen zusammenschließend zu denken." Natorp, P., PILb, p. 512.

³⁴⁵ Natorp, PILb, p. 238.

The moments of correlation are relative. A multiplicity can be considered as a unity in a later instance of thinking, and a unity can be considered as a component of a multiplicity. The unity brings together the multiplicity, but the units can be considered from a higher point of view as new multiplicities that must be reunited in a unity. Multiplicity and unity are relative moments in the process of thinking. Kant's mistake was to take these relative moments as absolute moments³⁴⁶. Kant considered this instance of thinking as an isolated part and not as what it is: a point of view of the operation of thinking. However, it is also a mistake to make the moment of unity an absolute instance. Unity is also a perspective on which it is possible to pause to consider and analyze each moment separately. This unity is possible only as a unity of multiplicity and this unity itself may also be considered as part of a multiplicity later in the development of thinking. For this reason, the thesis of the Transcendental Aesthetic must be abandoned. In a genuine idealism there is no place for a given matter³⁴⁷. If Kant's mistake in the Aesthetic was to absolutize the moment of multiplicity, Cohen's mistake was to make unity an absolute moment³⁴⁸. Neither Kant nor Cohen realized that the only absolute moment is the process, each moment of rest is relative. The elements of the correlation arise simultaneously. They are two necessary moments. There is no priority of one over the other. Neither the multiplicity can be thought without unity, nor the unity without multiplicity. Against Kant, Natorp argues that multiplicity is not an independent part of unity. The multiplicity arises concomitantly with the unity. Conversely, against Cohen, Natorp argues that multiplicity cannot be completely reduced to unity of thinking, Multiplicity is a necessary moment in the operation of thinking. As reflected in the analogy of the circle, the center and the periphery arise concomitantly.

In this relationship between unity and multiplicity, the object is created. The modes of unity of the multiplicity determine the possible objects of experience. The object is the particularity that thinking achieves in the process of concept formation by combining multiplicity. Therefore, thinking, as a general form of the connection, is also

³⁴⁶ Natorp, P., LGEW, p. 46.

³⁴⁷ Natorp considers that Kant recognized this problem, so he reformulated the theory of the sensibility of the Transcendental Aesthetic in the Transcendental Logic. In the Transcendental Logic, Kant corrected the results of the Aesthetic. For this reason, this required correction of the Kantian system is at certain point self-correction. Natorp states: "Das allein ist reiner Idealismus. Es hieße gerade das Tiefste der Vernunftkritik preisgeben, wenn man diese radikale Berichtigung, die im Kern nach als Selbstberichtigung in KANT bereits vorliegt, nicht aufnehmen und rein durchführen würde, bloß um die längst unhaltbar gewordenen, aus der Inaugural-Dissertation von 1770 (d. h. einer noch halb, ja mehr als halb dogmatischen Position) stehen gebliebenen Bestimmungen der transzendentalen Ästhetik um jeden Preis zu retten". Natorp, P., KMS, pp. 204 ss.

³⁴⁸ Cf. Natorp, LGEW, pp. 28ss.

the foundation of the object as the foundation of all objectivity. The object is formed in the connection between the unity and the multiplicity. The object is the concretion of this relation. For this reason, thinking, defined as synthesis or as correlation, involves both moments: unity and multiplicity. Thinking expresses the general relationship of multiplicity with a unity, it is the form of the connection between the unity and the multiplicity. The definition of thinking as correlation emphasizes the necessity to think of an articulating unity that expresses the relation of multiplicity to the central unity ³⁴⁹. In this relationship, thinking builds objectivity in peculiar ways of connection. The concept determines the form of the unification and thus generates a peculiar mode of order. Multiplicity is determined by the concept, but it also allows the concept to have expression. Thinking is primarily oriented towards concepts because it seeks to discover the peculiar modes of connections. Therefore, the moment of the concept formation and the moment of intuition are just relative instances in the process of thinking.

3.3.4. The Preeminence of Correlation.

Natorp called this principle of correlation the principle of synthetic unity to show the connection of this definition of thinking with Kant's concept of synthetic unity. Similarly, Natorp relates his correlation principle to Cohen's principle of origin³⁵⁰. Against Cohen, Natorp argues that the structural elements of the judgment of origin must be revealed by analyzing each of the logical moments in which the judgment unfolds. These structural elements are the functions of judgment, the specificity of each particular function. The correlation is the origin. The origin is the foundation of every relationship and, also, the total reunion of all the logical moments of thinking. The origin structures the relation of the moments of the totality, but this origin can only be exhibited once these logical moments have been revealed. This process of revelation of thinking in its expressions is an infinite task. As a structuring unity, the judgment of origin determines the general structure of all judgment, therefore it is the *judgment of the judgment*. To argue in this direction, Natorp once again takes the analogy of the center and the periphery. The origin would be the center of the circle, which determines the order of the periphery. However, the periphery, the originated, must not be separated from the origin, but it arises

³⁴⁹ Natorp, P., QQ, p.7.

³⁵⁰ Natorp, P., LGEW, p. 28ss.

concomitantly. The origin cannot be separated from what it originates. The originated refers to and points to the origin. In this sense, it also constitutes a point of reference for thinking. The thinking process is an infinite progress that surpasses every partial stop. The order of the fundamental elements of thinking must be understood in a 'concentric' way, in such a way that the starting position of the original judgment becomes a center position. This relationship between center and periphery, which allows expressing the correlation of orientations, contains the totality of the principles of judgment, which represent the whole of logic. In this way, Natorp corrects Cohen's proposal in the same way that he corrected the Kantian. Both Kant's principle of synthetic unity and Cohen's judgment of origin are replaced by the definition of thinking as correlation. It is not convenient, Natorp observes, to introduce the notion of judgment together with the definition of the concept of thinking, as it could lead to the confusion that judgment can be understood independently of the act of thinking. On the contrary, the judgment of origin is already a judgment. The construction of the judgment can only take place through the act of thinking, that enables the connection between union and separation. This original action of thinking is the condition of possibility for all judgment. This conception of thinking as correlation is the 'presupposition free of presuppositions' which, under the established restrictions, can take the name of 'synthetic unit' or 'origin'.

The act of thinking must provide both the mode of unification and the unified terms. It contains both the law of the connection and the multiplicity. Natorp will show, as we shall see, that thinking produces the unity and the multiplicity by its own means. It generates both: the cases and the law that produces them. The act of correlation contains the original unity of the laws that, each one in its specificity, unifies multiplicity. For this reason, thinking is defined as a correlation of those two moments required by all determination: separation and union. Every particular rule of the connection will be regulated by this highest form of union. Thinking is both separation and union. The essence of thinking is determined by the relation between these two moments. Nothing can be separated if it was not united. Conversely, the possibility of all reunion requires some sort of separation. Therefore, correlation is required as the highest form of connection. Natorp argues:

Darum hat es jedoch mit der "Einheit eines Mannigfaltigen" als der Urform der Bestimmung übrigens seine volle Richtigkeit; Denken ist Vereinigung, sagten wir; dann aber zugleich Sonderung; denn wo nicht ein Mehreres, also die Möglichkeit einer Sonderung, da bestände auch nicht die Möglichkeit einer Vereinigung. Aber dies darf nun nicht so verstanden werden, daß das Mannigfaltige als solches gegeben und nur die Einheit dieses Mannigfaltigen durchs Denken erst hineinzubringen wäre; sondern vielmehr so, daß in jedem Urakte des Denkens, als Akt der Bestimmung, ein X sich bestinmt als Eines und doch Mannigfaltiges, Einheit eines Mannigfaltigen, Mannigfaltiges einer Einheit. Denn diese, wie überhaupt alle — unter diesen noch sehr unbestimmten, unsicheren, allem Folgenden eigentlich vorgreifen- den Hauptbenennungen des Einen und Mannigfaltigen sich ergenden — Grundmomente des Denkens werden sich in gleicher Weise zueinander streng korrelativ erweisen³⁵¹.

The analysis of this fundamental correlation of separation and unification will show that the multiplicity that thinking faces is not given in the Kantian sense. Multiplicity is not provided by an independent faculty. It is not given to the understanding. The moment of multiplicity is not indebted to a principle other than thinking itself. In a single act, both the unity and the separation are originated. This act is the correlation. The course of the investigation must show that multiplicity and unity are different aspects of a single act. Natorp says:

Damit aber entfallt nun ganz die Frage nach einem dem Denken und zu denken "Gegebenen". Es kann überhaupt nicht mit Sinn gefragt werden, was das Nichtgedachte, Nichterkannte vor seinem Gedacht- oder Erkanntwerden sei. Es gibt für das Denken kein Sein, das nicht im Denken selbst gesetzt würde. Denken heißt nichts anders als: setzen, daß etwas sei; und was außerdem und vordem dies Sein — sei, ist eine Frage, die überhaupt keinen angebaren Sinn hat³⁵².

³⁵¹ Natorp, P., LGEW, p. 47.

³⁵² Natorp, P., LGEW, p. 48.

Thus, the definition of thinking as correlation results in the clarified sense of Kant's synthetic unity. This definition of thinking as correlation is the most appropriate definition of Kant's concept of synthetic unity. It is the exhibition that thinking can produce the multiplicity and the unity at the same time. The conceptual unity does not depend on a multiplicity externally given. On the contrary, the exhibition of the operation of the correlation shows that the conceptual unity is produced with the multiplicity. We will analyze the specificity of this process in chapter 4.

The process of knowledge can be originally conceived as this development of the correlation. Analysis can only take place once the synthesis has been accomplished. The objective is to analyze each moment in particular without losing sight of the systematic interconnection. The analysis abstracts the synthetical process to take each creation of thinking as if it were a separate element, as a product. This is not a problem for pragmatical purposes, if we are to study a particular field of knowledge. However, it must be taken into account that each field of knowledge is the product of the spontaneity of thinking. The analysis reveals the process involved in the creation of the field of knowledge that is under consideration. Only in this sense the analysis is amplifying. The moments of synthesis can only be understood and revealed through analysis. The analysis is possible by an abstraction in which each element is taken in isolation. Abstraction is a detachment from correlation. In the analytical instance, each moment is conceived in isolation through a process of abstraction. However, it must be borne in mind that the analysis of a stage of knowledge is only possible because this stage is founded on an original act of synthesis. The analytical moment is possible by virtue of an abstraction of thinking that allows each logical moment to be considered separately. Abstraction is a process of reflection in which each moment is considered in isolation with the aim of exposing new interconnections. In this sense, the analysis also represents progression. Natorp uses an analogy introduced by Moses Mendelssohn to explain this point. The analysis operates like the microscope. The microscope enlarges the images allowing the observer a closer view. The microscope does not introduce anything new. In this sense, the analysis reveals something, but it does not incorporate new features. Natorp explains:

> Analyse, Abstraktion wird dann Herauslösung aus dem korrelativen Zusammenhang, der damit aber nicht etwa zinichte gemacht, sondern nur zum Zweck der Schritt um Schritt vorgehenden Betrachtung der

Einzelmomente beiseite gesetzt wird, schließlich nur, damit immer neue Zusammenhänge auch innerhalb jedes Einzelgliedes des für diesmal außer Betracht gelassenen zutage treten. So erklärt sich die Meinung, daß gerade die Analyse erweiternd sei. Mendelssohns Gleichnis ließe sich auch so deuten: das Mikroskop gerade erweitert und gibt Neues zu erkennen. So ist also wirklich alles Zusammenhang, also Synthese, und verbleibt doch und eben damit der Analyse ihr nicht minder umfassendes Recht, aber nur als einem Momente der Synthese selbst, die in Wahrheit die Analyse vollständig mitumfaßt³⁵³.

The analysis is a moment dependent on the synthesis. The analysis reveals elements that are already synthetically constructed. In this sense, the synthesis is the *ratio essendi* and the analysis the *ratio cognoscendi*. The synthesis is the *ratio essendi* because it is the process that creates the objectivity. The analysis is the *ratio cognoscendi* because one should take the objectivity as a point of departure to reveal the synthesis involved in the creation of the considered field of knowledge. For this reason, the correlation involves both moments: synthesis and analysis. However, it must be taken into account that the division is always based on the unity of origin, the unit of the correlation. The function of analysis is to discover the synthesis that serves as its foundation. Each of the logical moments must be revealed in order to discover the process of synthesis that created them. The exposition of each logical constituent in isolation aims to discover the grounding unity.

3.4. Concept and Judgment.

According to Natorp, concepts and judgments emerge concomitantly as expressions of the synthetic unity. Correlation implies the necessity to think the unity and multiplicity contained in a single moment. The general expression of this requirement is the concept. The concept is the point of view from which it is possible to consider a multiplicity in a unitary way. The concept is the point of view of the unity. Natorp states:

³⁵³ Natorp, P., LGEW, p. 27.

Begriff bedeutet eine Einheit der Betrachtung, welche besteht für eine Vielheit zu betrachtender Objekte, mithin eine Einheit des Mannigfaltigen, eine Identität des zugleich zu unterscheidenden, d. i. synthetische Einheit.³⁵⁴.

The concept is the expression of the synthetic unity of thinking. Each particular specification of thinking has a conceptual form. Thinking is 'conceptualizing'. The concept is not only the perspective of unity, but it contains both the one and the multiplicity. The extension of the concept is the perspective of multiplicity while its intention is the perspective of unity. The concept can be defined as unity of multiplicity. The concept is a synonym for synthetic unity. The general form of the concept is: $x_1, x_2, x_3, ... = a$. The same point of view (a) contains the multiplicity of moments (x_1, x_2, x_3) . The multiplicity is distinguished -distinction expressed with the subscripts- while being at the same time identified by a common element, the identification under the mark 'a'. Multiplicity is only determined in view of the identity established in the concept that determines it. The concept limits the perspective from which to consider multiplicity. Multiplicity is not considered arbitrarily, but from a perspective that restricts the point of view of consideration. Each element of the multiplicity is differentiated since each of them is identified with each other only by this unity of perspective. This multiplicity is identified as a multiplicity thanks to the unity established by the concept. The components of multiplicity are identical by virtue of their belonging to a common perspective, and they are at the same time distinguished as differentiated components of this point of view, which is the concept.

The concepts are connected through judgments. In the judgment "x is A", an element of the multiplicity is the subject of the judgment, and the unity of the multiplicity its predicate. The predicate determines the common elements of certain multiplicity. The copula establishes the relationship between the multiplicity and a certain concept that makes multiplicity a multiplicity. Through the copula, the indeterminate multiplicity turns into a determined multiplicity. Therefore, the relationship expressed in the judgment is a determination of the indeterminate. The copula is the expression of the connection between the unity of the point of view delimited by the concept and the multiplicity that the concept contains by virtue of this delimitation. The 'is' expresses the relationship

³⁵⁴ Natorp, P., PP., p. 13.

between the unity of the concept and the multiplicity conceived under it. This relationship established through the copula is not an identifying relationship. The judgment "a is P" does not generate an identification between the subject (a) and its predicate (P). The copula expresses the relationship between the unity of the concept and the multiplicity contained in it. The copula indicates the generation of a determination. The copula does not establish an identification between the components of the multiplicity with the unity that articulates it, but rather affirms the effectuation of a determination. The judgment connects the terms of a relationship with an articulating unity. In this relationship, the terms are united and distinguished at the same time. Identification and separation are the conditions of possibility for thinking to generate the link 's is P'. This connection implies the possibility of distinguishing one term from the other and unifying them in a unity. The judgment 's is P' is only possible through this correlation between unity and separation. The terms are distinguished because stating that s is P means stating that s is different from P. At the same time, the copula unites one element with the other in a peculiar way. Unification takes place through the copula ('is'), through the specific reunion between the subject and the predicate. The fundamental correlation is this action of separating and unifying. The moments in the concept and in the judgment are the particularization of this general characterization.

The concept and the judgment have the same form since both are expressions of the synthetic unit. The concept is not a primitive element with respect to the judgment. The judgment is not a unification of concepts given before the connection. The concept is not the simple element of the judgment; that is to say, "the concept cannot be put as a foundation, as the last element, which precedes the judgment" Judgment and concept arise concomitantly. Through the judgment the relationship between the unity of the concept and the multiplicity that the concept determines is affirmed. The analysis of the judgment exhibits the form of connection of the concept. The judgment cannot be characterized as a connection of two concepts that are prior to it. Natorp insists on the preeminence of the relationship over the *relata*. Thinking is the articulating unity and the terms that it puts together are nothing outside this relationship. This mode of relationship also applies to judgment. In the judgment, both the form of the relationship and the concepts that the relationship contains take place together; that is, the concept arises in the judgment. The act of correlating produces both the concept and the judgment

³⁵⁵ Natorp, P, L., p. 12.

simultaneously. This action of thinking allows concepts and judgments to emerge simultaneously. Natorp will show that the table of concepts and the table of judgments arise simultaneously from a single principle. As we observed previously, this moment is methodologically different from Kant's proposal, since in the Kantian system, the discovery of the categories is carried out from the analysis of the logical functions of the judgment. On the contrary, Natorp showed that judgment and concept arise at the same time. The forms of concepts and judgments arise from an analysis of the essence of thinking as a correlation. This operation will be the only way to satisfy the demand for systematicity. Natorp argues against Kant:

Aber indem nun Kant sich dieser Wegleitung anvertraute, erwies sich, daß er die überlieferten Einteilungen sich erst mannigfach zurechtrücken mußte, um das System der Grundleistungen der synthetischen Einheit (denn das sollten seine Kategorien und Grundsätze sein) daraus zu erhalten. Damit wird aber dieser ganze Weg schlüpfrig und ungewiß. Wir können daher selbst einem Kant auf diesen Weg nicht folgen, zumal das Vertrauen zur logischen Überlieferung der Jahrtausende, das in Kants Zeit noch leidlich feststand, seitdem mehr und mehr erschüttert ist, und vor allem das jetzt offen zutage liegt, daß die wirkliche, schöpferische Logik, nämlich die der Wissenschaften, eine weit andere ist und in die überlieferten Formen sich schon längst nicht mehr pressen läßt³⁵⁶.

In Natorp's deduction, the starting point is this definition of thinking as correlation. Unlike the Kantian conception in which the table of concepts results from the table of judgments, here the leading thread is the study of the structure of thinking itself, which will be revealed as the foundation of the table of concepts and of the

Natorp, P., LGEW, p. 43. And Natorp argues in the same line two years later: "Nach dem allen aber bleibt eine gewichtige Forderung noch zu stellen, ohne deren Erfüllung die transzendentale Methode noch immer nicht zu ihrer vollen Konsequenz gelangen würde. Bei Kant scheinen neben den Anschauungsformen auch die reinen Denkfunktionen» in Gestalt der, mindestens starkem Anschein nach, nur historisch aufgenommenen Urteils- und Kategorieentafel, als starre Gegebenkeiten, obgleich des Denkens, stehen zu bleiben. Zwar hat Kant das "System" der Kategorieen "nach einem Prinzip", ausdrücklich mit dem Anspruch der Vollständigkeit, zu bestimmen geglaubt; aber er verlässt sich dabei, wie heute wohl von keiner Seite mehr bestritten wird, allzu unbedenklich auf die "fertige Arbeit" der Logiker, an der er nur einzelne Mängel auszubessern nötig findet; während schon die gänzlich neue Rolle, die er den Kategorieen zuweist, eine radikale Neubegründung, statt solcher blossen Flickarbeit, gefordert hätte." Natorp, P., KMS, p. 209.

judgments. The table of judgments and the table of concepts arise simultaneously. The starting point in the concept of thinking as correlation is the only way to guarantee the systematic construction of the deduction. As we previously exhibited, Kant and Natorp share this methodological prescription. In the *Critique of Pure Reason*, Kant defines the concept of system as follows:

Unter der Regierung der Vernunft dürfen unsere Erkenntnisse überhaupt keine Rhapsodie, sondern sie müssen ein System ausmachen, in welchem sie allein die wesentlichen Zwecke derselben unterstützen und befördern können. Ich verstehe aber unter einem Systeme die Einheit der mannigfaltigen Erkenntnisse unter einer Idee. Diese ist der Vernunftbegriff von der Form eines Ganzen, so fern durch denselben der Umfang des Mannigfaltigen sowohl, als die Stelle der Theile untereinander a priori bestimmt wird. Der scientifische Vernunftbegriff enthält also den Zweck und die Form des Ganzen, das mit demselben congruirt. Die Einheit des Zwecks, worauf sich alle Theile und in der Idee desselben auch unter einander beziehen, macht, daß ein jeder Theil bei der Kenntniß der übrigen vermißt werden kann, und keine zufällige Hinzusetzung, oder unbestimmte Größe der Vollkommenheit, die nicht ihre a priori bestimmte Grenzen habe, stattfindet³⁵⁷.

Our knowledge constitutes a system. To discover its fundamental laws, one must depart from the fact that knowledge operates as an articulated whole. The fundamental laws of knowledge must be discovered from a guiding principle, which reflects the fundamental operation of thinking. Kant aimed to develop such a task when he investigated the definition of thinking as synthesis. However, Natorp considers that Kant was not consistent enough in this direction. In the Kantian system, the table of concepts is discovered by reference to the table of judgments. Kant presupposes this table of judgments to obtain from there the table of concepts. According to Natorp, Kant would have recognized the idea of totality as the starting point of the metaphysical deduction. In particular, in the development of the concept of apperception, Kant saw the necessity to take the concept of thinking as a starting point to unfold the forms of the

³⁵⁷ Kant, I., KrV, B 860.

connection. However, he used as a leading threat the table of judgments. For Natorp, the study of the basic notion of the correlation is the foundation of both tables, the table of concepts and the table of judgments. Natorp rejects that thinking can take a moment extrinsic to itself to analyze its rationality. The table of the judgments of the logic of the time is an unsatisfactory starting point. Thinking cannot start from anything other than itself to ground knowledge. The place of Kant's table of judgments is occupied by the analysis of the fundamental form of thinking. This analysis will reveal the functions of both the table of concepts and the table of judgments. As we have seen, Kant started from given 'data' from which he deduced the table of categories. On the contrary, for Natorp, the only way to guarantee the systematic nature of the deduction is to start from the definition of thinking as correlation. Starting from an extrinsic element limits the possibility of building thinking as a system. Even considered the same Kantian definition of system, an extrinsic starting point prevents thinking from reaching this peculiar mode of integration. According to Natorp, the Kantian table of categories is not a proper system because it is not a whole ordered according to an idea. The synthetic unity would be the idea that orders the whole of the categories in a system. The synthetic unity determines a priori both the extension of the multiple -in this case, it is an infinite extension- and the respective place of the parts. The only way in which thinking can maintain its systematic character is not by starting from anything extrinsic to itself but by analyzing its essence. As we exhibited, Natorp takes Kant's own concept of system. Kant uncritically takes the table of judgments accepted at the time. Therefore, he cannot satisfy the requirement of a system that allows to determine a priori the way in which each moment of thinking is connected to all others. These forms will determine the path that thinking follows to pass from one determination to another determination in a systematic way. That is to say, the foundation of the forms of thinking in the concept of synthetic unity allows satisfying the Kantian system requirement. This foundation will not be properly a deduction, since a specific form of thinking is not determined but rather an operation that indicates the way in which thinking develops and thus builds the path of knowledge.

Conclusion

As we studied in Chapter 2, Natorp highlighted the inadequacy of the conception of psychologism and logicism to ground knowledge. He showed that in these currents, there

underlies a dogmatic approach, as they accept an element that thought cannot produce but that must be given to it. None of the philosophical systems of the time were capable of solving the problem raised by Kant: how can thought legitimately represent the object? This problem, as we exhibited in Chapter 1, can be translated in terms of the relation between intuitive and conceptual representations. Psychologism and logicism misunderstood the relationship between intuitions and concepts based on methodological errors. After showing that the philosophical systems were incapable of explaining the relation between intuition and concepts, Natorp has to present his own proposal. He must exhibit the way in which affirmatively knowledge can be grounded. More specifically, he must expose how thought is capable of creating its object by its own means. In other words, Natorp argues that the conceptual representation and the intuitive do not oppose but they are different aspects of the same process.

As we studied in this chapter, for Natorp, the only way to answer the central question of knowledge - on what foundation rests the relationship of our representations with the objects - is to study the structural elements of thinking. So as to argue in this direction, we exhibited, Natorp starts with a reconsideration of the task of philosophy. According to him, an analysis of the essence of thinking allows: a) to ensure the systematic nature of thought and, then, b) to overcome the dualism between intuition and concepts. The deduction of categories will be the exhibition of the systematic character of thought in the construction of objectivity. Natorp considers that the first germ of the idea of a systematic deduction of the categories can be found in the philosophy of Rene Descartes. The author of the *Meditations* was the first to notice that a system of the fundamental functions of thinking should be developed. These functions will be the different expressions of the unity of thought. The unity of thought is manifested in a coherent system. As we exhibited, the requirement to find such a system is rooted in the essence of philosophy. The task of philosophy demands seeking this system.

In Chapter 2, we showed that psychologism and logicism misunderstood the method that philosophy should employ. Due to these methodological mistakes, these currents could not give a satisfactory answer to the problem raised by Kant. In this chapter, we exhibited that Natorp's proposal is grounded on his definition of the method of philosophy. Natorp argues that the task of philosophy demands a peculiar method. Following this method will guarantee to overcome the separation between intuition and conceptual representations. We exhibited that Natorp proceeds from an unclear definition of thinking to the understanding of the structural elements of objectivity. We analyzed

the arguments that lead Natorp to conclude that thought defined as correlation must be the starting point of philosophical research. The internal coherence of the system can only be guaranteed by the deductive method. The starting point in the definition of thought as correlation is the way to ensure the overcoming of the dualism between intuition and concepts. We exhibited that thought is able to build its cases as instantiations of its own acts. This will be the first step to prove that the universality of the concept does not oppose to the concreteness of the object. On the contrary, the universality of the concept and the concreteness of the intuition are two aspects of the same process. In the following chapter, we expect to reconstruct the specific nature of this process. In chapter 4, we will analyse the Natorpian deduction of the categories, which constitutes the specific way this construction of the thinking takes place.

Chapter 4. Overcoming the Heterogeneity between Intuition and Concepts. The Deduction of Categories.

In the deduction of categories³⁵⁸, we find the clue to understanding the way in which Natorp conceives the relationship between intuitive and conceptual representations. The main task of his project is to explain how the totality of experience can be constructed by thinking. As we explained in the introduction, Natorp considers that it is necessary to reformulate the Kantian distinction between intuition and concepts. The crucial moment in order to overcome the heterogeneity between intuitions and concepts takes place in this deduction. We will show what we anticipated in the previous section (III.1). Thinking can build objectivity in all its determinations by its own means. As we studied, philosophy must show the way in which the object is constructed in and by thinking. This was introduced as a necessary task. Now, the specific development of this process will be exhibited. Natorp must show how concepts, as ways of organizing the multiplicity, can constitute the object completely determined. He must explain how thinking can concomitantly generate both unity and multiplicity. Natorp must clarify the way in which the processes of thinking can generate the object in all its determination. In the deduction of categories, it must be shown that thinking can constitute the object as it is presented in the experience without any reference to intuition. Natorp will show that the construction of objectivity is developed on four levels: quantity, quality, relation, and modality.

As we anticipated, in the deduction of categories, we find the core of the rupture with the Kantian proposal. Kant considers that the concepts have their origin in the understanding. Intuitions have their origin in sensibility. Concepts and intuitions are types of representation that arise from heterogeneous sources. As we explained in the first chapter, for Kant, the process of knowledge requires the contribution of concepts and intuitions. The reference of concepts to objects can only take place through intuition. The concepts provide the unity, while the intuition provides the multiplicity that must be gathered by the concept. Intuition cannot be the source of unity just as the concept cannot provide the multiplicity. As the object of experience is composed of the unity and

³⁵⁸ More precisely, Natorp develops a metaphysical deduction of categories. The question is to identify the categories. As there is no heterogeneity between sensibility and understanding, a transcendental deduction is not necessary at all. Hernán Pringe explains: "Without the distinction between a passive capacity (sensibility) and an active faculty (understanding), there is no *quid juris* question and therefore no necessity for a transcendental deduction". Pringe, H., 2011, p. 210.

multiplicity, its conformation requires both: concepts and intuitions. Furthermore, as we studied in Chapter 1, pure multiplicity can be provided by pure intuition. However, the matter for the construction of experience must always be given *a posteriori*. The multiplicity that must be gathered is always provided by intuition, which in the case of man has the forms of space and time. The multiplicity can never be provided by the understanding. Therefore, in the Kantian proposal, the understanding has a relationship with objects only on the basis of this reference to intuition. As we explained in the first chapter, completely determined objects can only be given to intuition. Concepts cannot construct the object of knowledge. Knowledge always depends on a factor external to the understanding, i.e., intuition. For Natorp the understanding is the source of the totality of the determinations of the object. The understanding can provide both: the multiplicity and the unit. For this reason, Natorp's deduction of categories will be one of the fundamental points of disagreement with the Kantian system. However, as we studied in Chapter 3, Natorp considers that by showing how the understanding is the source and architect of nature, he is understanding Kant better than Kant understood himself.

The goal of this section is to carry out an analysis of Natorp's deduction of categories. We must show how thinking is constitutive of objectivity without any reference to intuition. The question that Natorp must answer is how thinking can constitute the object of experience. It will be exhibited how thinking can construct both the unity and the multiple that this unity contains, which means that thinking produces both the conceptual and intuitive aspects of the object. Natorp will show that thinking is synthetic, unifies a multiplicity, even if it has no reference to intuition. This is the only way in which philosophy can guarantee the overcoming of the dualism between intuitive and conceptual representations. In the first section of this chapter, we explained that objectivity must be constructed in and by thinking. Now, it will be exhibited how this process takes place. We will divide this section into two parts. First, we will analyze the categories of quantity and quality. Second, we will study the levels of relation and modality. In chapter five, we will draw the consequences that can be extracted from this deduction. We will study how the deduction of categories leads to the reformulation of the definition of intuitive and conceptual representations.

4.1. Main Features of the Deduction

The deduction of the categories will show that intuitions and concepts must be considered as moments of the process of thinking. As we explained in chapter 3, this guarantees the systematicity required by the very concept of knowledge, and the possibility of overcoming the Kantian dualism between intuitions and concepts. Natorp will show that the complete determination of the object of knowledge does not require a factor external to thinking. The deduction will show that all the determinations of the object are posited by thinking itself, proving that thinking can have reference to objects without any referee to intuition whatsoever, as the object is completely based on purely conceptual determinations. The question of how thinking can have access to the object in its singularity is now resolved in a brand-new way: concepts do not need a reference to intuition to have a reference to objects. The first step of this deduction is to exhibit the mathematical determinations of the object, the quantitative and qualitative determinations of objectivity. It will be shown that, on the one hand, the logical foundation of mathematics is grounded on thinking and, on the other, that logic as the science of thinking is expressed primarily in the legality of mathematics³⁵⁹. The deduction of the categories of quantity and quality will show that the most general determinations of objectivity, the mathematical determinations, are grounded on thinking. The laws of mathematic are deduced from the laws of thinking.

According to Natorp, the first germ of the idea of a systematic deduction of the categories can be found in Descartes. The author of the *Meditations* was the first to notice that a system of the fundamental functions of thinking should be developed. The unity of thinking is manifested in a coherent system, and this course is possible on the grounds of certain fundamental principles. These principles will be the expression of the synthetic unity of thinking, of its deductive chain. Under this systematic unity of thinking, science is one and indivisible ³⁶⁰.

Descartes and Leibniz followed this conception initiated by Plato, and whose greatest exponent is Kant. The Kantian system of categories is the culminating point of

^{359 &}quot;Es schwebt also unmittelbar eine Mathematik der Qualitäten vor, wie sie Leibniz gefordert hat, und wie die jüngste Entwicklung der Mathematik sie der Verwirklichung näher zu führen scheint, wenn sie, allerdings nicht eine Arithmetik, aber wohl eine Algebra ohne Quantitätsbegriffe zu entwickeln wagt, ausdrücklich in dem Sinne, daß Mathematik ist nicht notwendig mit Quantität zu tun habe, sondern sich (wie einer der entschlossensten Vorkämpfer dieser Richtung, AN Whitehead, *Universal Algebra*, I, Cambridge, 1898, sagt) auf Alles erstrecke worin, "die Folge der Gedanken oder der Ereignisse in bestimmter Weise ausgemacht und präzis festgesetzt werden kann (Preface, pg. VIII). "According to Natorp, in this way, the ideal of Leibnizian philosophy would be concreted. Cf. Natorp, P, PILb, p. 439. ³⁶⁰ For Dufour, this is one of the ruptures of Natorp with Cohen, For Cohen, the science division is a *factum* a *posteriori* grounded. Cohen considers the division of sciences as a given fact. Natorp believes that this division is exhibited *a priori* in the foundation of science in the logical law. Dufour, É., 2003, p.104.

this task initiated by Platonic philosophy³⁶¹. The proposal of Cartesian idealism has pointed in the direction of overcoming the dualism between intuition and concepts. Kant has followed this tendency. However, neither Descartes nor Kant were deep enough in their approaches. According to Natorp, Descartes begun the path of overcoming the heterogeneity between intuition and concepts, "but after all Descartes fell into a gross dualism" ³⁶². Even in the Cartesian idealist philosophy, there is always an element in the experience that thinking cannot provide from its own source. The object is an external element for knowledge. Cartesian idealism recovers the task begun by Plato. However, in the system of Descartes "the naive belief in the existence of the object, given in itself before all knowledge, and to be grasped by knowledge remains unchanged" ³⁶³. The overcoming of dualism between the given and what is thought is not carried out either by Cartesian or by Kant's proposal. Kantian idealism failed to show the way in which thinking is the producer of objectivity. The task of constructing a coherent idealism remains. This is the task that must be accomplished by the deduction of categories, where it is shown how thinking can truly be the source of objectivity.

³⁶¹ "Der wesentliche, rein objektive Sinn der Einheit des Intellects ist die deductive Verkettung der gesamten menschlichen Erkenntnis, kraft deren sie von den ersten, einfachsten Elementen an in,, continuirlicher, nirgends unterbrochener Gedankenbewegung" (Reg. III) gewonnen werden kann. Die Forschung nach jenen, in Grundbegriffen und Grundsätzen zu definirenden Elementen des deductiven Zusammenhanges der einen unteilbaren Wissenschaft hat eigentlich Descartes zu Ehren gebracht. Leibniz ist darin sein Nachfolger; die Kategorien und Grundsätze Kants sind die spät gereifte Frucht dieser langen Vorarbeit." Natorp, P., DED, p. 16.

^{362 &}quot;Aber schließlich fällt Descartes in den groben Dualismus zurück." Natorp, P., DED, p. 18.

³⁶³ "Vielleicht wendet man ein, dass selbst bei Kant dieser Dualismus keineswegs ganz überwunden sei, dass neben dem in der Erkenntnis rein erzeugten doch noch etwas wie ein "gegebener" Gegenstand übrig bleibe. Allein das ist bei ihm blos ein rudimentärer Rest einer in der Hauptsache verlassenen Stufe des Philosophirens; ein Rest, der auf der Höhe des Systems verschwindet. Bei Descartes im Gegenteil ist der erste Ansatz rein und folgerecht, aber daneben wuchert das naive Vorurteil des an sich vor aller Erkenntnis vorhandenen und nun zu erfassenden Gegenstandes ungestört weiter, m endlich auch jenen richtigeren Ansatz zu überwuchern und sich auf der Höhe der Entwickelung des Philosophen, in seiner Metaphysik, zum System zu verhärten." Natorp, P., DED, p.19.

Natorp introduces the deduction of the categories³⁶⁴ for the first time in *Quantität* und Oualität³⁶⁵. The core of the deduction was focused on the categories of quantity and quality. The deduction reappeared in 1900 in Nombre, temps, espace, dans leurs rapports avec les fonctions primitives de la pensé. The point of departure was the definition of thinking as synthetic unity. After briefly considering the problems of quantity and quality, Natorp focused on the core of his presentation: the problems of arithmetic and geometry. The explanation of the categories of relation and modality is only briefly sketched. The reference to the quantitative and qualitative procedure of thinking is required to clarify the three main issues of his presentations: number, space, and time. According to Natorp, the explanation of the procedure of quantification and qualification sheds light on the operation of thinking that constitutes numbers and the fundamental relations among them. The process of quantity and quality grounds the mathematical determinations of objectivity. From the process of quantity and quality, Natorp draws the properties of numbers and, from the properties of numbers he obtains the determinations of space and time. His article of 1901, Zu den logischen Grundlagen der neueren Mathematik follows the same line. Thus, the first deduction that Natorp proposed is focused on the categories of quantity and quality. In his *Philosophical Propaedeutics* (1903), Natorp incorporated a more detailed account of the categories of relation and modality. From 1904 on, the deduction already had the full development that it will exhibit in LGEW, his most systematic work. For his reason, we will take this text as a point of reference.

Before studying the process of the deduction of categories, we can already highlight some interesting aspects in relation to how Natorp introduced the deduction of categories in his philosophical development. Regarding the modifications of his

³⁶⁴ In ZLGM, Natorp defines categories as the fundamental procedures of thinking. The categories are the way in which the legality of objectivity can be conceived. Natorp defines the concept of categories as follows: "Der Mathematiker, auch der logisch interessierte Mathematiker mag sich dabei beruhigen, solche letzten Prämissen zu "postulieren"; die Logik fordert für sie, als synthetische Sätze, wie Kant sagt, "wo nicht einen Beweis" (der hier in der That ausgeschlossen ist) "doch wenigstens eine Deduktion der Rechtmässigkeit ihrer Behauptung", sie fordert, nach Plato, den Rückgang auf "voraussetzungslose", d. h. auf solche letzte Voraussetzungen, von denen es möglich ist, sich zu überzeugen, dass sie nicht wiederum andere, fundamentalere voraussetzen, nämlich auf die schlechthin fundamentalen Verfahrungsweisen des "Denkens", d. i. gesetzmässigen Vorstellens der Gegenstände überhaupt, die sie in einer begrenzten Zahl reiner Grundfunktionen des Denkens (Kategorieen) festzulegen sucht." Natorp, P. ZLGM, p. 383. Natorp uses both, the concept of levels (*Stufen*) and of categories (*Kategorieen*). For example: In NTE, L, LGEW, we find the concept of *Stufen*, But in ZLGM, EGM, Natorp talks about categories. As Holzhey, explains Natorp prefers to talk about logical functions rather than of categories. We will analyze the problem of the definition of categories in chapter 4.

³⁶⁵ Quantität und Qualität in Begriff, Urtheil und gegenständlicher Erkenntniss. Ein Kapitel der transcendentalen Logik. Helmut Holzhey explains the development of Natorp's position. He considers that the main differences are to be found between 1903 and the Logik of 1904. Cf. Holzhey, H., 1986 p. 107 ss.

presentation, we consider that two significant issues must be stressed. In the first place, when Natorp introduced the problem for the first time in the article of the *Philosophische Monatshefte*, he focused on the deduction of quantity and quality. The full development of the system was presented by 1900. This highlights the crucial importance that deduction of categories has in relation to the problem of the mathematical determinations of the object. As we saw, when Natorp first introduced the problem, he was focused on the mathematical determinations of objectivity. Second, Natorp differentiates two possible ways of how the deduction could take place. We could carry out the deduction in two ways. Both would lead to the same point. In the *Erkenntnistheoretische Grundlagen der Mathematik*, Natorp maintains:

Es handelt sich um die letzten gemeinsamen Grundlagen der Arithmetik und Geometrie, deren Bloslegung nichts geringeres bedeuten würde, als eine rein logische Deduktion des Raumes wie auch der Zeit. Die bezüglichen Untersuchungen sind niedergelegt in zwei Abhandlungen, die eine aus Anlass des internationalen philosophischen Kongresses bei der Pariser Weltausstellung, daher in französischer Sprache veröffentlicht: Nombre, temps et espace; die andere "Zu den logischen Grundlagen der neueren Mathematik", im "Archiv für systematische Philosophie". Ich werde aber hier einen etwas anderen Weg einschlagen, da ich glaube, dass auf diesem neuen Wege der Beweisgang logisch strenger wird, obgleich er zu keinem anderen Ergebnis führt." [..] "Ich ging dort so zu Werke, dass ich zu nächst die Gesetze der Zahl herleitete aus den Grundgesetzen der "quantitativ – qualitativen Synthesis", d. h. aus den beiden, überhaupt fundamentalsten, von einander untrennbaren Denkverfahren, durch die wir, einerseits ein Mannigfaltiges als solches, andererseits jene Einheit eines Mannigfaltigen, die einen Denkinhalt konstituiert, gedanklich erzeugen.366

In this remark, we appreciate these two interconnected aspects. In the first place, the problem that Natorp had in mind when he introduced the problem of the categories of quantity and quality. The core of the problem here is the mathematical determinations of objectivity. He wants to show that both the properties of numbers and of space and time,

³⁶⁶ Natorp, P., EGM, p.2.

can be drawn from the fundamental forms of thinking. That is to say: arithmetical and geometrical determinations of objectivity have the same root, and the nature of the properties of space and time can be derived from the nature of numbers³⁶⁷. It would prove that mathematics is purely grounded on thinking. Second, Natorp claims that the determinations of number and time can be obtained in two ways. On the one hand, the most general laws for the conformation of objectivity, the categories of quantity and quality, can be taken as a starting point. However, he affirms that another possibility consists in starting directly from the very concept of synthetic thinking. This is the path that he will take, says Natorp, in EGM. In contrast, both in ZGNM and in NTE, the properties of numbers and arithmetic relationships were derived from the categories of quantity and quality. However, this deduction of the properties of numbers could have been carried out directly, starting from the very concept of thinking. Kant calls this way of proceeding synthetic method. This is the method of the Critique of pure reason³⁶⁸. Thus, on the one hand, it can be clearly seen the crucial importance of deduction of categories for the development of the concept of number. On the other hand, it is evident how the problem of the deduction grows until it occupies the heart of Natorp's proposal.

The deduction of categories shows the stages in the constitution of objectivity. However, for some commentators, the deduction of the categories of quantity and quality does not represent a relevant element within the Natorp system. Morris Cohen argues that this moment is only part of a modern category deduction that does not affect the core of

³⁶⁷ This question is briefly and clearly exposed particularly in NTE and EGM. The shortness of the exposition makes it much easier to see the relation between the deduction of categories and the problem of the mathematical determinations of objectivity.

³⁶⁸ The method is synthetic or progressive. The synthetic method is the method that Kant follows in *Critique* of Pure Reason (Proleg., AA 4: 274 ss). The synthetic method is progressive. It starts from a first representation that is conceived as confuse and unclear and seeks to gain clarity and distinction. In this process, the elucidation of the elements that constitute each part of the representation leads to an elucidation of the other parts. Knowledge is organic. The way in which each part of knowledge operates determines the operation of the remaining areas. This allows the application of the progressive method. This organic conception of knowledge demands a synthetic method of exposition. The synthetic method allows exhibiting this organic structure of knowledge (Proleg., AA 4: 263 ss.). This procedure is that it has no empirical assumptions (Proleg., AA 4: 275.). The only assumption is the possibility of the very concept of thinking. We only depart from confused representation of what thought is. For this reason, the synthetic method is progressive, it advances by gaining determinations as they are required by the investigation, i.e., by the elucidation of the representation that is being analyzed. Kant explains: "In der Kritik der reinen Vernunft bin ich in Absicht auf diese Frage synthetisch zu Werke gegangen, nämlich so, dass ich in der reinen Vernunft selbst forschte und in dieser Quelle selbst die Elemente sowohl, als auch die Gesetze ihres reinen Gebrauchs nach Principien zu bestimmen suchte. Diese Arbeit ist schwer und erfordert einen entschlossenen Leser, sich nach und nach in ein System hinein zu denken, was noch nichts als gegeben zum Grande legt ausser die Vernunft selbst und also, ohne sich auf irgend ein Factum zu stützen, die Erkenntnis aus ihren ursprünglichen Keimen zu entwickeln sucht." Kant, I., Proleg, AA, pp. 275ss.

the system³⁶⁹. Helmut Holzhey, in this same direction, affirms that the very concept of "category" has a merely historiographical function to refer to the Kantian system³⁷⁰. However, for others, the quantitative-qualitative synthesis is the most important step in Natorp's philosophical system. For some commentators, such as André Laks and Éric Dufour, the deduction of the categories of quantity and quality represents the philosopher's rupture with the Cohenian proposal. Dufour focuses his analysis on the double front of the debate: against Kantianism and Cohenianism.³⁷¹ In fact, Natorp would find himself, at this point, distant from Cohen and close to Cassirer.³⁷²

For Natorp, within the primary categories of thought, quantity and quality have traditionally been accepted as the most essential. This is due to the fact that the conceptualizing functions of quantity and quality "... represent the original process of the synthetic unity of a multiplicity in general ..."³⁷³. Natorp will show that these functions allow the logical progression. The relationship between these two functions (quantitative and qualitative) is so close that the separation is only an abstraction of thinking that allows

³⁶⁹ In his review to the LGEW Morris Cohen holds: "In the second chapter, we have a modernized deduction of the categories. The dry bones of the Kantian framework receive a great deal of flesh and blood. In the end, however, they turn out to be our old friends the Twelve, marching in four groups of three each. If it were not for the fact that students at our colleges do not read German, this chapter could profitably be recommended to those who are reading Kant for the first time and who generally cannot grasp what these categories are about." Cohen, M., 1911, p. 694.

³⁷⁰ "In his book *Die logischen Grundlagen der ecxakten Wissenschaften* of 1910, Paul Natorp employed the concept of 'category' only in a historical sense when referring to Kant." Holzhey, H., 2005, p. 70.

³⁷¹ "Cependant, en critiquant la thèse de Cohen et en plaidant pour un retour à Kant, donc en affirmant que toute synthèse est synthèse de la diversité, Natorp n'est-il pas obligé d'admettre ce à quoi Cohen voulait précisément échapper, à savoir la présupposition d'un divers qui relève d'autre chose que de la pensée? Ce n'est pourtant pas le cas (...) Il ne s'agit pas pour Natorp de réhabiliter, contre Cohen, une passivité primordiale qui équivaudrait à la donation d'un divers que l'activité de la pensée aurait ensuite à penser. Car c'est la pensée elle-même qui pose, dans sa propre activité, un divers qu'elle a pour tâche d'unifier. Dès lors, contre Kant, le divers relève bien de la pensée et non de la sensibilité, mais, contre Cohen, la synthèse est bien synthèse du divers et non de l'unité." Dufour, É., 2002, p. 337.

³⁷² "Il faut remarquer combien Natorp est proche de Cassirer et combien tous deux s'éloignent de Cohen". Dufour, É., 2002, p. 338. André Laks focuses on the dispute with the Cohenian system. He highlights:" Cohen claims to follow the Kantian principle of the division of the forms of judgment; yet this is not all the case. The Kantian table of judgments places at its head the judgments of quantity and quality ... For Cohen, on the contrary, the table of judgments is entirely related to the analytic – i.e. purely logical- use of judgment, with no reference to content at all. (...) But Natorp while accepting this programme (which is the programme of neo-Kantianism) nonetheless rejects the way in which Cohen in fact puts it into action. Instead of subsuming the (analytic) principles of traditional logic under the (synthetic) principles of objective cognition, one must recognize that the two series of principles correspond to each other (...) for this reason it is necessary to start, like Kant, from the judgments of quantity and quality, i.e. from mathematical judgments, and not form a purely formal principle like the principle of contradiction. For the commentator, this divergence represents the break within Marburg neo-Kantianism. Laks, A., 2004, p. 481, 482. Also, for Dufour, this is Natorp's most important criticism of Cohen's system. Dufour, É., 2002, p. 338 n. 62.

³⁷³ Natorp, P., LGEW, p. 52.

us to delineate each of the moments that are part of the whole. Although they are characterized separately, they are part of a unique process.

Throughout his works, Natorp identifies synthesis as the most essential operation of thinking. The first element of the logical is defined by the synthetic unity. Thinking is synthesis. The task of thinking consists in generating relationships. The parts of the relationship are the terms of the relation. To overcome the separation between intuition and thinking, it must be exhibited how thinking by its own means generates multiplicity and the unity that unites it³⁷⁴. The deduction of the levels of quantity and quality will arise by means of a synthetic procedure³⁷⁵. The analysis of one of the parts will lead to revealing another necessary element required by the concept. The primitive notion is the very concept of thinking. Thinking must be defined as synthesis, as an activity whose main task is generating relationships. To think is to establish relationships between the whole and the parts, between multiplicity and unity. The first step is to define what a multiplicity consists of. The concept of multiplicity necessarily implies a plurality of differentiable moments. The quantity and quality levels will emerge as the necessary moments implied by the definition of thought. Thinking consists of producing relationships. The possibility of the position of relations implies the union of multiplicity in a unity. Multiplicity is a plurality of differentiable moments. Then, the conformation of the plurality requires the position of units. Thus, the function of quantitative synthesis depends on the correlation of two fundamental logical moments: unity and multiplicity. The multiplicity leads to the concept of unity because it needs a unity to conform a multiplicity. Without the unity, it would be a mere rhapsody of elements. The unity requires the multiplicity to have a content. The deduction of categories does not need more than this because, as we pointed out, the development of the levels of thinking follows the synthetic method. It is this definition of thought that 'pulls' the deduction of the categories of quantity and quality. Thinking consists of the link between unity and multiplicity. Multiplicity, by representing a plurality of differentiable moments requires first a quantitative unity.³⁷⁶

4.2 The Level of Quantity.

 $^{^{374}}$ Natorp considers that all the relations of thinking can be grounded on this fundamental relation of the unity and the multiplicity. Natorp, P., L, $\S9$.

³⁷⁵ Cf. Kant, I., *Proleg*, AA, pp. 275ss.

³⁷⁶ Natorp, P., LGEW, p. 53.

The fundamental act of thinking consists in correlating. The action of the synthetic unity requires three fundamental moments: unity, plurality and totality. When we claim that the concept A contains the elements x_1 , x_2 , x_3 , etc., we assume the three moments: the units, the plurality and the unity in a totality. The concept A is the unity that brings together an undetermined plurality. In the expression $A = (x_1, x_2, x_3, ...)$, it is included the unity (A), the plurality of elements that this unity contains $(x_1, x_2, x_3, ...)$ and the elements that compose the plurality: (x_1) , (x_2) , (x_3) ,.... The relation of these moments requires the position of the plurality and an articulating unity. The moment of the unity (A) is empty per se. Being empty means that it is meaningless as such without the multiplicity it contains. It makes no sense to state A is a unity of a plurality of differentiated moments if one does not think in concomitance with it the terms that this unity contains $(x_1, x_2, x_3, \dots, x_n)$...). Conversely, the elements gathered can only be posited as such in the relationship that constitutes them. The constitution of the plurality of elements of A can only take place as these units are differentiated in relation to each other. The analysis of the concept of thinking as a correlation leads to the discovery of three fundamental actions of thinking in the position of quantity: 1) position of the one, 2) Repetition of the position, 3) Totality. The function of quantitative synthesis depends on the correlation of three fundamental logical moments: units (Einheiten), the plurality (Mehrheit), and the quantitative totality (Ganze). In this case, the multiplicity represents a plurality of differentiable moments. Natorp explains:

Eine Mehrheit ist als solche notwendig Mehrheit aus Einheiten. Die Einheit im Sinne des numerisch Einen, des Einzelnen der Zahl nach, ist also der unvermeidliche Ausgang, das unerläßliche Fundament jeder quantitativen Setzung. Es bedeutet den Einsatz des quantitativen Verfahrens selbst, als des Verfahrens der Diskretion. Was in jedem Falle als Eines gelte, ist hierfür gleichgültig. Eine (der Zahl nach) ist die Welt, eins das Atom, oder was sonst man als Letztes (der Teilung) oder Erstes (der Zusammensetzung — auch das gilt hier gleichviel), als letzte Eins gleichsam, mit der die Natur zähle, ansetzen mag. Stellt man ein solches auf, so ist solche Hypothesis selbst diktiert durch das Gesetz jenes Denkverfahrens, welches vorschreibt, von irgendetwas als Erstem zu beginnen, einen Anfang überhaupt zu setzen, d. h. aber in

quantitativer Hinsicht: ein letztes Eines, etwas, dem unser Gedanke diesen Charakter der Einsheit erteilt³⁷⁷.

The concept of unity is the indispensable logical requirement for the characterization of plurality. This is the unavoidable origin of the quantitative process: the establishment of the unitary. The establishment of the numerically one is the beginning of this function, and what is considered as one is completely indifferent. 'One' is both the atom and the triangle and, for quantitative judgment, the determined content of what is established as unitary is indistinct. This establishes the discretion as the first moment of the quantitative synthesis. At this level, the unit is the point of departure. The possibility of a plurality (Mehrheit) requires the position of the units (Einheiten). It is posited an indeterminate "x" that must be conceptualized under a general concept, for example: A. The first judgment we obtain is: "This particular x is A". A second moment is required necessarily. Each element differentiated as a unit is only relative to something else. The distinction of an x₁ requires an x₂ to constitute a distinct unity. However, this x₂ is nothing considered independently. The x_2 is always in relation to an x_1 . The concept of plurality starts from the unity and generates a plurality as a repeated one-to-one position. The position of x₂ can only be repeated (a second position) if x_1 is retained as already posited. This is the way in which an indeterminate plurality is conceived as a multiplicity. In this way, the open series expressed in the judgment is obtained: "These (individuals) x_1, x_2, x_3 ... are A". The plurality is the mediating element between individuality and totality as it represents the possibility of repetition one by one infinitely. This second stage consists in the repetition of the units. Thus, we obtained pluralities and units as correlated moments, the units are units of a plurality, and the plurality is a plurality of differentiated units. However, in this second moment the series is still undetermined, a third articulating form that constitutes the unity of the series is required. This is given by the third moment: the conformation of a totality, unity as the unity of many. This third moment is expressed in the judgment: "Every x is A". This judgment contains the previous two moments as its condition. In the third moment, we obtain the totality of the units³⁷⁸. The beginning of the position is always a relative beginning. The element that is posited as the initial moment

__

³⁷⁷ Natorp, P., LGEW, p. 54.

³⁷⁸ "auch der letzte notwendige Schritt des quantitativen Verfahrens: die Zusammennehmung allemal einer bestimmten, durch diesen neuen Akt eben sich bestimmenden Folge von Einzelsetzungen zu einem Ganzen, d. h. wiederum einer Einheit, aber im neuen Sinn der Einheit aus den Mehreren, ihrer Vereinigung in einem Totale." Natorp. P., LGEW, p.55.

may contain within it a multiplicity. Likewise, the whole can be placed as a unit in relation to a superior synthesis of thinking. This process of thought enables the development of the progression. It is possible to conform more comprehensive units every time. This possibility of thinking to determine more and more its object to reach higher units allows progression. The symbolic representation of the quantity levels would be³⁷⁹:

I II III ... (I) (II) (III) ...

The number is the scientific expression of this natural operation of thinking that includes these three moments: the setting of the numerical one; the establishment of the unlimited plurality, and the generation of the determined plurality of the totality. From the point of view of the concepts, the category of quantity results in these three stages:

one (this one) a, several (these many) a, all (these all) a,

And it is introduced the possibility of these three types if judgments:

this (one) x is a, these (several) x1, x2, ... are a, all of these ... are a

4.3. The Level of Quality.

The categories of quantity are insufficient in themselves to guarantee a differentiated objectification. The functions of the quality categories that objectify the sensation are required to conform the object. This function is intended to distinguish one thing from another in order to understand it from a higher point of view (from a comprehensive

³⁷⁹ Natorp. P, NTE, pp. 345ss.

unity). Quality is the synthetic function of unity that provides a central understanding, an original unity. This synthetic function has, as in quantitative synthesis, three differentiable moments. First, a plurality of differentiation must be put on the basis of a qualitative identity. In the same way as with the numerical unity, in this case, the identity is the first basis, regardless of what is considered as the identically one. However, an allusion to an alterity is inevitably found in every identity judgment. The "this" something can only be defined in relation to an "other" something, and the "other" something can only be defined in relation to a "this". Both terms are required by the comparison itself. In this qualitative relationship, the one is set as qualitative opposite of the other. There must be at least one differentiating characteristic that establishes the one with respect to the other. This is the basis of the identity position series. However, there must be a third moment where what was separated is reunified from a point of view, under a higher unity of understanding. This point of view is required by thought as that from which it is compared. In this way, the qualitative function represents the synthetic unity of diversity on which a genus is grounded. Genus (Genos) is the logical name for this new qualitative unity of uniformity of diversity (Einerleiheit des Mehrerlei). The quality, as a production of the diverse from the unity, sets the condition for the exercise of the quantitative function; this is: homogeneity. The establishment of something liable to numbering occurs thanks to the quality function that gives something differentiable that can be measured by number. Only the procedure of enumeration of elements allows to define 'the what', while allowing not only a mere description of its attributes but the differentiation of one entity from others. In this way, qualitative synthesis constitutes a unity of understanding that differs from mere composition, allowing the identity to be constituted in diversity. This comprehensive totality based on qualitative synthesis should not be confused with quantitative totality, which is a composition. The qualitative unity is the unity of understanding, an original unity. The synthetic-qualitative function constitutes unity as identity. Natorp concludes:

Denn das entscheidende Moment im Begriff der Gattung ist nicht die äusere Umfassung (Die Einheit des Begriffumfangs), sondern die innere, zentrale Vereinigung unter einem gemeinsamen Gesichtspunkt des Denkens (dem "Gattungsmerkmal", als der Einheit des

Begriffsinhalts), das heißt in einer neuen, man pflegt zu sagen, höheren Identität³⁸⁰.

Unlike the quantitative unity that establishes a purely compositional totality (*Allheit*); qualitative synthesis constitutes a comprehensive (*komprehensive*) whole (*Ganzheit*). However, if the number was the scientific expression for quantity, the quality does not have a mathematical expression that reflects its function. Only the expression of the number can serve as a basis for expressing the quality. Therefore, quality is measured by quantity. The degree is defined as the number applied to the quality that can indicate the intensity of something qualitatively characterized. Thus, it is *a priori* established that all content of thinking will have a degree expressed in a *quantum*.

The constitution of the object in general is made in the correlation between qualitative and quantitative synthesis. The synthesis of quantity and quality together represent the two fundamental forms of logical development of thought. The function of synthesis is characterized by the type of judgment A is B, unlike the mere tautology A is A. The judgment A is B, expresses the progression of thinking. For this type of judgment, it is necessary, firstly, the introduction of an A, identical to itself (A = A) and, also, a B identical to itself (B = B); finally, a general point of view from which to postulate a superior identity³⁸¹.

However, the understanding does not stop at the position of universality. The universality cannot take place without the category of infinity that includes the three stages of quantitative synthesis. This infinity should not be understood as a mere absence of an end, that is, in a purely negative sense. If so, it could be conceived as the mere denial of the end, through the mere non-thinking (*Nichtdenken*) of the end. On the contrary, it is the concept of finitude that expresses something merely negative as long as "something has an end means that somewhere the being-nothing (*Nichtsein*) takes place" ³⁸². In contrast, it is the absence of the end that states that something is continually. This sense of relative negativity, expressed through quantitative infinity, is the origin of the thinking

³⁸⁰ Natorp, P., LGEW, p. 62

³⁸¹ "Bevor ich sagen kann: A ist (identisch mit) B, muss ich ein Identisches = A und ein Identisches = B haben. Schon der Gebrauch die Begriffszeichen ist ja bedingt durch eine im Gedanke gesetze Identität, welche durch die des Symbols vertreten wird." Natorp, P., QQ, p. 9. This is: Identity is established through synthesis and in no way given. Natorp, P., QQ, p. 8

process as it represents the possibility of its unlimited progression³⁸³. Infinity is constituted in a purely positive concept by representing the continuing possibility of a quantitative position. Natorp concludes:

Unendlichkeit bedeutet nicht blosse Unbestimmtheit des Endes; sie ist nichts rein Negatives, in dem Sinne, dass sie schon gedacht wäre durch das blosse Nichtdenken des Endes. Es ist tausendmal gesagt und noch immer wahr: nicht der Begriff des Unendlichen ist negativ, sondern der des Endes. Etwas hat ein Ende, heisst: es macht irgendwo dem Nichtsein Platz; es hat kein Ende, heisst: es ist immerfort. Also wird die Unendlichkeit der Quantität die immer fortbestehende Möglichkeit quantitativer Setzung bedeuten müssen³⁸⁴.

Then, the category of infinity is an indispensable condition to ensure the continuation of the task of thinking. The category of infinity expresses the totality of the three categories of quantity. Qualitative infinity, as an inclusive condition of the three moments of quantitative synthesis, is the condition of existence of thought in general and, through its scientific expression, that reflects its most proper legality. The category of infinity allows the continuous limitation of the unlimited that is the proper task of thinking. It also guarantees the very existence of thinking given that "the limiting process itself must be applicable without limitation"³⁸⁵. Quantitative synthesis, in the pursuit of unity and progression of thought, are the true origin of it. The infinity category, which allows the perennial search for unity and progression, is the ultimate legal origin of thought in general as the origin (*Ursprung*) is the ultimate unity pursued³⁸⁶.

In this way, quantitative-qualitative synthesis establishes a transcendental logical concept of infinite progression, which moves away from both a generative-psychological and logical-formalistic conception. The infinite progression in thinking does not consist

³⁸³ For André Laks, the concept of "nothing" is another point of rupture with Cohen. The commenter notes: "Nothingness (the *Nichts*) does not have the absolute sense implied by the original Cohenian concept, but must be interpreted, within Natorp's conceptual Framework of synthetic unity, as alterity." Laks, André; (2004), p.483. In this same direction, Éric Dufour observes that Natorp's central criticism of Cohen at this point is that denial establishes an alterity as a relative negativity and not as an absolute negativity. Cf. Éric, Dufour, 2009, p.41. Coinciding on this point with Laks and Dufour, we believe it is necessary to emphasize that the notion of relative negativity as an infinite possibility of progression in the determination of thought is, strictly speaking, attributed by Natorp to Cohen. Cf. Natorp, P., LGEW, p. 25.

³⁸⁴ Natorp, P., QQ, p. 19.

³⁸⁵ Natorp, P., QQ, p. 20.

³⁸⁶ Natorp, P., LGEW, p. 27.

of empirical progression nor does it establish a logical series concept merely applicable to a special region of objects, as mathematical objects. On the contrary, by means of qualitative and quantitative synthesis, a logical-transcendental notion of progression in knowledge is established on which the empirical genesis and the formation of the series of numbers depend³⁸⁷. The logical-transcendental synthesis is a condition of legitimacy and regulations regarding the empirical experience. Thus, the first laws of thinking, which determine the essentially relational character of it, are progressively grounded through the deduction of the categories of quantity and quality.³⁸⁸The laws of number are derived from the logical process of quantity and quality³⁸⁹. These logical functions build the numerical series. Therefore, the numerical series holds the properties of the logical position. The fundamental operations of numbers are also obtained by virtue of this mode of thinking. Regarding the fundamental series of numbers, Natorp points out:

Comme pure expression du procédé pris en lui-même et généralement, elle est — nécessaire et universelle — unique et la même dans toutes ses applications—indéfiniment prolongeant, car le procédé qui la fonde a des ressources illimitées (6) — homogène, car ses termes d'après leur détermination même ne sont que des numérateurs et tous produits également par juxtaposition, ils ne se distinguent donc pour la pensée par aucun autre signe que leur rang dans la série ; de plus chaque terme de la série enveloppe cette série tout entière, puisque le procédé de la numération est déterminé dès le commencement et conséquemment à partir de n'importe quel terme donné par une régression vers le commencement ou une progression à l'infini. 390

³⁸⁷ This is also stated by Young: "The author regards as a primitive faculty of the mind the power of conceiving any mental act to be repeated indefinitely. He thus obtains essentially what mathematicians would call the abstract form of an unlimited sequence". Young, JW, 1913, p. 370.

³⁸⁸ Faced with the correlativity of qualitative and quantitative moments, in some instances, Natorp seems to grant a preeminence to the level of quality. Natorp states in 1900: "Thus, the two fundamental processes of quantity and quality correspond in all their evolution and that they raise in a narrow interdependence the fundamental signs of the progress of knowledge, its peripheral extension and its central deepening; that is, its unification. But what is first in itself is the primitive unity of the third level of quality. For in thought, the content determines the continent, the height of the point of view establishes the extent of the horizon." Natorp, P., NTE, p.347.

³⁸⁹ Cf. Natorp, P., EGM, p.2; NTE, p.355; LGEW, p.98ss.

³⁹⁰ Natorp, P., NTE, p. 355.

Number is the purest expression of thinking³⁹¹. The series of numbers is built thanks to the processes of quantity and quality. The relationship of the series with its members is determined based on these fundamental logical processes. What distinguishes the members of the series is their place. The distinctive feature of each term is defined by the place they occupy in the series. To be a particular number is to occupy a certain place in the series. The number is the term of the relationship. The number cannot be defined separately from the relationship it establishes with the other members of the series. The determined number is the term in the set of relationships.

The series is generated in the iteration of the quantitative and qualitative process in which each term placed is considered as a counter-term in relation to a previous position. First of all, there is the position of the one, the position of an individual as the first element to form multiplicity. Second, a repetition of this initial position is necessary. This repetition must ensure that the previous moments are preserved, a repetition of the initial position is generated while retaining the previous positions. The second term is presented as a counter-term with respect to the previous one. The third moment generates the possibility of taking the terms as new initial moments. That which was put as 'the other' with respect to an initial position, can be considered as a new position in relation to another position. The unity of one and the other can be considered as a totality with respect to a later moment. The unity of the one and the other can also be a unity. Each of the terms can be either first term or a counter-term. This process is infinitely iterable. That which was a whole with respect to its parts can also be considered as a unit for the conformation of higher order totalities. There is no absolute beginning of the position of one, but there is an iterable structure where nothing is itself a unit or a totality in an absolute sense. This possibility of positing the terms in different relationships can lead to the mistaken conviction that the terms can subsist regardless of the relationship itself. This possibility of putting the terms of the relationship in one or another relationship gives the appearance of independence. Each of the terms has the appearance of independence by the possibility of being put into different relationships. The possibility of positing each of the terms in different relationships can generate the appearance that these terms can exist in itself and that only in one instance further are placed on relationships. On the contrary, the terms can be posited in different relationships because their determination only consists in being a term

³⁹¹ Natorp, P., LGEW, p. 98.

of a relationship. The terms do not have any other determination beyond these positions in which they are placed. An element can be term or counter-term depending on the relationship considered. This possibility gives the appearance of independence of the terms. The possibility of entering into multiple relationships generates the appearance of the independence of the terms, from the possibility that they have of entering into a relationship in general. However, this apparent independence is nothing more than the possibility that the terms have to establish different relationships. Each member of the series is defined by virtue of the position it occupies. The iterated position from term to term generates the series along with the possibility of directing the plus and minus as oppositional relations. The quantity and quality allow a positional relationship according to the before and after³⁹².

The properties of the numerical series are derived from the operation of these fundamental logical processes. The properties of the series are concrete expressions of the general operation of thinking. As an expression of pure thought, this series is: necessary and universally valid, unique, infinite, homogeneous and continuous. It is universally valid because it is grounded on the pure process of thinking. It is unique because the permutation of values only alters the position in relation to the same set of relationships. The exchange of values does not generate a new series as long as the determination of the value is only based on its position in the series. The function of each value is always interchangeable³⁹³. As the process is always iterable, the series is infinite and open. This iteration results in the open infinite series on both sides, from the plus side and the minus side. In the series, each fundamental member of a first relationship can turn into a counter-member, and each counter – member can become a fundamental member in relation to another counter - member in a new relationship ³⁹⁴. This iteration allows a term to always be considered as counter-term and vice versa. No term can constitute an absolute beginning, but every term can adopt the function of beginning. The series is homogeneous because its values are equivalent. The direction of the plus and minus can be reproduced anywhere in the series by taking a moment as 0 and expressing in relation to it, again, a term and counter-term relation. The difference of

³⁹² "Die Beziehung der Position oder der Ordnung des Vor und Nach erwies sich als das letzte Gattungsmerkmal der Zahl, welches aller Maßbedeutung derselben logisch vorhergeht Sein mathematischer Ausdruck ist das Plus und Minus, welches eine immer gleiche Art der Relation von Glied zu Glied unserer Urreihe" Natorp, P., LGEW, p. 225.

³⁹³ Natorp, P., LGEW, p. 113.

³⁹⁴ Natorp, P., L, p. 31.

the members of the series is only in relation to the place they occupy, so the series is homogeneous. Each determination of a value is relative to the function it occupies.

The quantity allowed indefinite positions and guaranteed the possibility of considering the plurality of differentiated positions in a total quantitative unity. Meanwhile, the quality allows the continuous transit from one magnitude to another. The possibility of this continuity is not given but is based on the fundamental operation of thought³⁹⁵. Each unity will be a moment of provisional rest. The relationship of a term with another will be the place where a term is positioned in relation to a position selected as instance '0'. The l is the zero point of the numbering. The position of before and after is, in relation to this initial position, always relative. The definition of the numbers in the series is done by virtue of this term-to-term relationship in the fundamental series with respect to the function assigned as '0'. In this way, the 'absolute' need for an initial position and the relative character of what is set as the beginning is recognized. The 0 can be defined as "a reference point or comparison point in view of the position of a given value" ³⁹⁶.

Concomitantly, the fundamental operations of numbers can be defined by virtue of the position of quantity and quality. Just like the moment 0, the position of 1 is a necessary moment for the beginning of numbering. The 1 will be defined as the addition of a unit to what has been considered as moment 0. Thus, operation 1 + 1 is logically equivalent to operation $0 + 2^{397}$. In this way, the fundamental series finds its foundation in the operative of thinking. The process of quantity and quality are expressed in the numerical relationship.

4. 4. The Level of Relation.

The categories of quantity and quality determined the object as a magnitude³⁹⁸. Natorp exhibited that the object, as an object of thinking, is already completely created by

³⁹⁵ Natorp, P., NTE, p. 365. LGEW, p. 180. At the foundation of continuity, Natorp seems to give a preeminence to the category of quality over that of quantity. He states: "Kontinuität ist ein so ursprüngliches, unverbrüchliches Gesetz des Denkens, dass überhaupt irgendwelche Diskretion sich nur als Diskretion eines Kontinuums will denken lassen. Also gibt es für das reine Denken das Kontinuum der Beziehungssine oder Richtungen ebenso wie das Kontinuum der Werte." Natorp, P., LGEW, p.237.

³⁹⁶ Natorp, P., LGEW, p.181.

³⁹⁷ Natorp, P., LGEW, p.135.

³⁹⁸ ³⁹⁸ "Durch die beiden eng Verbunden Verfahren der Quantität und Qualität ist für die mögliche Bestimmung eine Gegenstand eine erste gesetzmäßige Grundlage gegeben. Sie enthalten das zureichende

thinking itself by the categories of quantity and quality. The quantitative and qualitative determinations were capable of constructing objectivity in its most general determinations. Natorp argues that one could be easily led to conclude that a further moment is not necessary, as it was already exhibited how the object of thinking can be created by thinking alone. However, the task is the constitution of experience, and experience is more than a single object. For this reason, the constitution of the isolated object is not enough for our purposes, i.e., to explain how objectivity can be constituted purely conceptually. The determination of the object in general is insufficient. Natorp explains:

Wir haben vielleicht den Gegenstand, aber noch nicht die Gegenstände, nämlich nach ihren gegenseitigen Verhältnissen der Abhängigkeit, dass heißt nach der Art, wie sie nicht bloß jeder für sich als bestimmt überhaupt gedacht, sondern als in einer Erfahrug sich untereinander bestimmend erkannt werden. Erfahrung mag noch so sehr auf dem Denken beruhen, sie ist doch etwas mehr als nur überhaupt Denken; jedenfalls mehr als das Denken, welches nur Denken der Quantität und Qualität wäre³⁹⁹

Experience exceeds the determinations of quantity and quality. To construct the experience, it is necessary not only to determine the object but also to connect the objects. Concomitantly, to know is, precisely, to put objects in relation. To know is not to determine an object but to establish the relations among objects. For this reason, it is necessary to have another level of determination. The object created by thinking must be related to other objects. It must be shown how the object can be put into relationships with other objects. Now, it is necessary to establish a system of objects. It must be guaranteed the possibility of an interconnection of objects. For this reason, it is necessary to have another level, the level of relation. In the level of relation, it must be established the conditions of possibility of the interconnection among objects. At this point, the constitution of the plurality of objects and the relations among them is required. Natorp

logische Fundament für denn Gegenstand als Größe, und dies Fundament reicht aus für grenzenlos verschiedene Setzungen von Größen, unter denen auch wieder Beziehungen denkbar und, sofern sie in einer Erkenntins zusammen bestehen sollen, zu denken notwendig sind." Natorp, P., L, p. 24. ³⁹⁹ Natorp, P., LGEW, p. 65.

must show how thinking can determine not only objects but the system of objects. It must guarantee the condition of the possibility of the relation of objects among themselves. Thus, thinking seeks every time higher levels of relations to accomplish what was posited as a task: the constitution of experience. The method demanded that the determinations cannot be considered as if they were given. On the contrary, it must be shown how they are produced⁴⁰⁰. On these grounds, a further level is required. Thinking must generate the system that enables the interconnection of these systems, generated by quantity and quality, among themselves. The synthesis of quantity and quality gave us a multiplicity of series. The simple synthesis generated by the first levels is insufficient. This secondorder of relation will not be just a synthesis but a 'synthesis of synthesis' 401. Indeed, every act of thinking can be conceived as a way of relating. To think is to relate. To relate is the fundamental action of thinking. However, the general mode of relating, present in every act of thinking, must be distinguished from the determined action of relating that enables the interconnection of the series of the magnitudes generated by the process of quantity and quality. This is a specific mode of relation that generates higher levels of determination. The mathematical determination of the object must be completed by the dynamical determination.

Thinking takes as its starting point the moments that it generated. It does not depart from any given factum to conform its objects. Rather, it departs from its own creations. This determination can only come from thinking. The relation present in the interconnection of the objects is the interconnection according to law, the determined connection. The act of determination according to the relation is expressed in the law. The law is the expression of the determination of the connection⁴⁰². Natorp remarks:

> Dieses Verfahren wird beruhen müssen nicht auf einer einfachen Synthesis eines Mannigfaltigen (diese ist in dem Doppelverfahren

⁴⁰⁰ "Nicht als hätte die Logik hier nun doch zum "gegebenen" Wirklichen zu fluchten; das hieße den Weg des reinen Denkens schlechthin verlassen. Sondern was Erfahrung mehr enthält gegenüber dem Denk^i, wie wir es bis dahin kennen lernten, dem Denken der Quantität und Qualität, muß in seiner Möglichkeit doch wiederum als Denken, als eine höhere Stufe, gleichsam eine höhere Potenz des Denkens sich verstehen lassen. Wirklichkeit selbst, Gegebenheit ist Denkbestimmung, und zuletzt Leistung reinen Denkens. Aber auch noch nicht diese (die erst die Modalität zu vertreten hat) steht hier in Frage, wohl aber die bisher noch nicht erbrachten methodischen Vorbedingungen dazu." Natorp, P., LGEW, p.66.

^{401 &}quot;Diese können nur gesucht werden, nicht in der einfachen, sondern in der gegenseitigen Bestimmung; in Wechselseitigen Abhängigkeitsbeziehungen also, gemäß der ersten Stufe welchen Gegenstände (d.h. quantitativ-qualitativ bestimmte) sich gegenseitig-bestimmenr. Also nicht mehr in einer einfachen Synthese, sondern einer neuen in Synthesen von Synthesis, synthetischen oder synthetischer Einheit Einheiten." Natorp, P., LGEW, p. 66.

⁴⁰² Natorp, P. LGEW, p. 66.

der Quantität und Qualität erschöpft), sondern auf einer Synthesis von Syntheses. Dies ist es, was Kant als das Verfahren der Relation bezeichnet. 403

Quantitative-qualitative synthesis left a system of order. The order of the determined series is now required to generate a total order. The result of this total order will be nature. Nature is this system of interconnected laws. This interconnection can be called function (mathematically) or law (in connection with nature). The connection according to a law will allow articulating each separate series in a system of series. It is a demand of thinking that each generated series is not left indeterminate. It must be connected in a higher order. Each quantitative-qualitative series must be incorporated into the total order, under its subsumption to the law expressed in the function. The process of thinking itself demands that nothing remains undetermined and then, that every series can be connected with every other. The law expresses the necessary connection of any term with any other by conditionally indicating that if one condition is met, another necessarily follows ("if A, then B"). Through this connection, the function generates a new superior concatenation. The requirement is a total ordering, a requirement that remains as a regulatory idea for thinking in each of the progressions of its actions. As Natorp already showed, it belongs to the essence of thinking that this process can never be exhausted⁴⁰⁴. For a limited cognition, the accomplishment of the complete determination of experience is a demand that can never be fully accomplished.

The final objective of this act of relating is a totality in which each of the particular series can relate to another in the conformation of a system. The law is the expression of these connections. What is sought is the total connection of each of the particular laws in a total system. This is the guiding idea of thinking, articulating a coherent order

⁴⁰³ Natorp, P., L, p. 25.

^{404 &}quot;Aufgabe ist: Ordnung des Einen nach (d.h. gemäß) dem Anderen, wodurch ein System von Ordnungen, das heißt eine Gesamtordnung entstehe. Eine solche ist, in der Sprache der Mathematik: die Funktion, in der Sprache der Naturwissenschaft: das Gesetz. Die Glieder, unter denen solche Ordnung herzustellen, sind, wie gesagt, Ergebnisse einfacher, quantitativ-qualitativer Synthesen, also Größenreihen, je far sich aufgebaut nach den Gesetzen der quantitativ-qualitativen Synthesis. Die Ordnung dieser Reihen aber, gemäß welcher sie sich untereinander bestimmen, wird dann bestehen müssen in solchen Beziehungen unter ihnen« welche eine gesetzmäßige Verknüpfung von Glied zu Glied der verglichenen Reihen herstellen. Man kann es foglich bezeichnen als Ordnungs Synthese, wobei zu denken ist nicht bloß an eine irgendwie geordnete Fortschreitung von Glied zu Glied in jeder Einzelreihe; dazu würde die quantitativ-qualitative Synthesis für sich ausreichen; sondern vielmehr daran, daß die Art der Ordnung, die an sich auf vielfache Weise möglich ist, für jede Einzelreihe sich bestimme durch eine gesetzmäßige Beziehung zu irgendwelchen, schließlich allen parallelen Reihen; das heißt, es wird die Ordnung in jeder Einzelreihe determinierbar, indem sie an die Bedingung einer bestimmten gesetzmäßigen Beziehung zu den Parallelreihen gebunden wird." Natorp, P., LGEW, p. 69.

system. This is the requirement of thinking in its demand for unity. In this progression, thinking aspires to generate higher-order units. This demand for unity is the guiding idea of the progression of thinking, i.e., searching for every time higher stages of unity. This context of the total order will be the way in which the coherent total order of the various series systems is established. The requirement consists in the prosecution of an order of the multiplicity of laws in an articulated system. This total articulation will be the reality, the goal that thinking proposes as a task. Thinking seeks a unique order that can contain all the laws in an articulated system. This search is the eternal task of thinking, which can only be approached in its progression. However, this idea operates as a guiding thread. Thinking operates with this requirement as a goal. The consummation of this requirement, as we will see in detail in the next section, can never be accomplished 405.

The starting point of the moments of the relation is those that thinking created itself. The terms of the relation are generated by thinking. In this case, the starting point is the object generated by the process of quantity and quality. The relationship will also have three moments: an initial position, a continuation of the position, and a closure. As in the previous levels, there will be an initial unit, a multiplicity, and a whole as a temporary total closure. Natorp claims that he will arrive at the same result that Kant did. There will be three levels: a) substantiality, b), causality c) community.

The initial act consists in the position of a *Grundreihe* as the first term established by thinking⁴⁰⁶. This is the equivalent to the Kantian category of substance. As the unit was the first moment of quantity and the identity of the quality, the *Grundreihe* is the first moment, as a 'unit' to be connected. It is a ground for every possible relation. Natorp calls it a *fundamentum relationis*. This first moment consists of the establishment of a reference system that allows the series to be articulated with each other. This will be the fundamental series.

The various relationships series will be possible under this common reference to this basic series. The first requirement is the establishment of a system of positions, of a scale, in which the course of interconnected series of variables can be articulated. Natorp recognizes a coincidence with the Aristotelian position. Indeed, as Aristotle noted, it is necessary to establish a subject that is invariant in relation to its determinations. A subject must be established with respect to which these determinations are taken as accidents. The

⁴⁰⁵ Natorp, P. LGEW, p.69.

⁴⁰⁶ "Die Möglichkeit einer Reihenordnung der verlangten Art erfordert als Erstes eine feste Grundreihe, als Fundament der ganzen Reihenordnung." Natorp, P. LGEW, p.70.

possibility of a dynamic system of connections demands the establishment of something constant. Any possibility of determining the change would be lost if nothing could be established as constant⁴⁰⁷. Thinking demands the invariance in respect of which every change can be determined. However, this is only a logical requirement, not something real permanent given to thinking. The fundamental series is not a thing but a process⁴⁰⁸. Aristotelian naive realism confuses a logical requirement with the postulation of an entity: the substance. This is the mistake of naive realism. Thinking requires taking something as invariant, but that invariant is only provisionally determined by virtue of this requirement. It is not something given to thinking but a logical demand. This demand can only be satisfied in provisional stays in which thinking takes a certain stage as invariant in relation to a series of changes. Naive realism confuses the requirement with its satisfaction and transforms this satisfaction into a metaphysical entity. This absolute instance of the fulfilment of the requirement of an invariant is hypostasized in a metaphysical entity: the substance. This is the mistake of the Aristotelian position. The substance is not an entity that remains invariant in contrast to changes. The substance consists, rather, in the procedure of taking as invariant a certain moment to think from there the series of changes. It is an action and not a thing. This distinction between the variant and the invariant is always relative, as long as something is taken to be an invariant only by virtue of articulating certain determinations at a certain stage. The substance is not a metaphysical entity but a way of establishing a relationship, a logical condition. This is the first logical condition to establish the whole system required by the very concept of nature. As we pointed out, the very establishment of nature demands to assume this stage of invariance as a logical requirement. But this demand must remain a requirement. It may vary what is considered invariable. Only the requirement remains. It must remain as a logical requirement of thinking. The substance will be this requirement of thinking of establishing an invariant to put in relation the quantitative-qualitative series.

As we exhibited in Chapter 3, the leading thread of the task is the definition of thinking as correlation. Relating is the most proper task of thinking⁴⁰⁹. To think is to relate. As we saw, the general function will always be to distinguish and relate what was distinguished. The second moment will consist in the generation of an act that allows each

⁴⁰⁷ Natorp, P. LGEW, p.71.

^{408 &}quot;Unsere Reihe bedeutet ja nicht ein Ding, sondern ein Verfahren". Natorp, P., EGM, p.6.

⁴⁰⁹ "Aber der wahrhaft letzte Grundbegriff des mathematischen und alles strengen Denkens überhaupt ist viel mehr die Relation." Natorp, P., EGM, p.6.

term of the series to be ordered in the relation among themselves. The quantity and quality levels showed the series with ordered members: $(x_1 x_2 ...) (y_1 y_2 ...)$. Now, the series (x₁x₂...) must be able to be connected to the series (y₁y₂...), so that a unitary system can be generated between the heterogeneous. The question that must be answered is how can the indexes (1,2,3,...) be connected when they belong to different series (x,y,..). Time will be the form that allows the series to order each other and thus the ordering of events. The series formed by the level of quantity and quality had an internal order. Now it is necessary to connect a series with another in search of the total order. The first step would require the possibility of establishing a link between each member of one of the series with the members of the other. The members of the series of the x must be able to be posited in relation with the members of the series of the y. An order must be arranged that articulates the x_1 , x_2 , x_3 , with y_1 , y_2 , y_3 , etc., so that each first member of one of the series is connected to the first member of the other series. Time will be the universal scale that allows this arrangement. In every moment of time, everything that is the first member of each series will be contained. Thus, it will be possible to claim in time 1 we find x_1 , y_1 , etc; in time 2, x_2 , y_2 , in time 3, x_3 , y_3 , etc. Time is the condition of the possibility of establishing this order. Thinking requires the articulation of the series, and time will be the condition that allows a first answer to this demand. Time is the most basic order of occurrences. It is the first requirement for the series to be articulated in a common higherordered system. This scale will allow the order of the simultaneous. However, it is also necessary that what is contained in each moment can be distinguished. An order is necessary in this 'being together' (Miteinandersein). One must establish the order of these elements of each of the series contained in each moment. This second condition will be the spatial order. The space will be the order of the successive because it will be what allows us to distinguish what it is at the same time. The expansion of the simultaneous generates the spatial places. The possibility of this establishment of places allows the 'expansion' of what is contained in the instant. If the function of thinking is to distinguish and gather the distinguished, the specific expression of this function is now the distribution in places of what is contained in each moment. The establishment of spatiality allows to conserve the multiplicity contained in each moment of time. Space and time together enable an order that posits the series in a common system. Space and time are specifications of the function of thinking as correlation. They are functions of thinking; more precisely, functions of the relation. The movement establishes the relation between the instants and the places of space. The movement will allow coordinating the space

points with the time points⁴¹⁰. In this way, it is established a system of variance and invariance. Time and space are connected in the concept of movement. For this reason, the movement expresses the basic form of change in nature. The variant will always be in relation to what is considered to be permanent. Conversely, everything permanent will always refer to something that varies. Each moment necessarily demands the other. Change can only be understood in relation to permanence and vice versa. That something changes implies that there is also something that remains the same in relation to what changes. This is the relation between the substance and its accidents. It is the product of this logical requirement of thinking, the relationship between the invariant and what varies. Time could allow an order between the series. However, "the state of something that changes at a given time can only be defined through its dynamic relations with the whole with which it is disposed according to a law"⁴¹¹. The task of the function of causality will be to gather the moments that were considered at first independently. In this level, the relationship of the change from moment 1 to moment 2 is indicated. The final goal is ordering in a total system⁴¹².

As long as a systematic interconnection is required as a whole, the principle of causality will be insufficient unless each series is connected with every other. This is only possible thanks to the determination of the reciprocal action. Natorp argues that the relation among series would be insufficiently grounded if it is not established a more fundamental law according to which the relations between series can be made. Thinking cannot create anything fortuitous but a lawful integrated system. Natorp holds:

In der Tat würde die gesetzliche Beziehung von Reihe zu Reihe so lange zufällig, d.h. unzureichend begründet sein, als nur auf eine beliebige, zufällige Mehrheit, nicht auf eine durch eine gemeinsame und zwar fundamentale Beziehung begründete Allheit paralleler Veränderungsreichen die Forderung der gesetzmäßigen Entsprechung von Glied zu Glied sich erstrecken würde. 413

⁴¹⁰ "Die bestimmte Zuordnung aber bestimmter Raumpunkte zu bestimmten Zeitpunkten im Verlauf einer Veränderung ist es zugleich, welche den Verlauf dieser Änderung selbst, fundamental also als Bewegung (räumliche Änderung in der Zeit)." Natorp, P. LGEW, p.74.

^{411 &}quot;... der Zustand eines Veränderlichen zu gegebener Zeit läßt sich nur define durch seine dynamischen Beziehungen zum Ganzen, dem es sich gesetzmäßig einordne." Natorp, P. LGEW p.79

⁴¹² "Die Synthesis der Erscheinungen nach dem Verhältnis der Ursache und Wirkung beruht darauf, daß eine Mehrheit von Vorgängen zu einander im Denken in eine solche Beziehung gesetzt werden, daß allemal der Wechsel in einer Reihe des Geschehens nach dem Wechsel in einer andern Reihe von Moment zu Moment sich bestimmend gedacht wird. Nur dadurch können die verschiedenen parallelen Yeränderungsreihen sich in die eine gemeinschaftliche Zeitfolge bestimmt ordnen und also in der einen Erfahrung zusammenstehen." Natorp, P., PP, p. 29.

⁴¹³ Natorp, P., L, p. 26.

The modes of relationship between the series are established in a necessary way thanks to the generation of a total lawful system that contains the particular interconnections. It will be a task for thinking to establish the whole of these possible relationships, the totality of relations of a series of changes with every other. This is the task of thinking. This third level enables us to comprehend all events in nature in a coherent integrity. The series is determinable only in relation to the total system. The system as the global coordination of series is the final condition of the determination of each series as such. Each series is thought of as jointly determining and determined by the others. For this reason, the complete system of the coordinated series must be presupposed. Natorp presents the example of how the sun heats a stone, the same example that Kant took in his *Prolegomena*⁴¹⁴. Suppose that the sun illuminates the stone and heats it. At time 1 the stone has a certain temperature and at time 2 the temperature increases. The subject remains identical and, however, its state varies. To substantiate how the sun heats the stone, the causal relationship establishes that given condition a, consequence b occurs. However, for the sun to heat the stone, a whole series of conditions not specified in the judgment must be met. The level of causality is insufficient to explain the circumstances that led to the heating of the stone. For this reason, the problem of the relationships to other parallel changes inevitably arises⁴¹⁵. In this third level, which unites the three moments of the relation, we reach the highest point of the synthetic unity⁴¹⁶. In this way, the determination of experience required three interconnected processes. First, the

_

⁴¹⁴ Kant, I., *Proleg*, AA 4, p. 301.

⁴¹⁵ Natorp explains: "Der Stein wurde eine gewisse Zeit von der Sonne beschienen, davon wurde er warm; d. h.: im Momente 1 zeigte er einen bestimmten Wärmegrad, im Momente 2 einen anderen, höheren; woher kam diese Änderung des Zustandes in dem übrigens der Voraussetzung nach identisch bleibenden Subjekt; d. h. rein methodisch gesprochen: wonach ist diese Änderung auf gesetzmäßige Weise bestimmt? Das Gesetz der Kausalität antwortet hierauf nur, dass eine Ursache dafür sein mußte, d. h. etwas, irgendein Umstand oder eine Summe von Umständen (Bedingungen) im Zeitpunkt 1, welche diese Änderung bis zum Zeitpunkt zum Ergebnis haben mußten, d. h. aus welchen dieses Ergebnis für den Zeitpunkt 2 nach einem Gesetze bestimmt ist Fragt es sich dann aber weiter, welcher Art solche bestimmenden Momente seien, so kommen, wie das Beispiel klar zeigt, unumgänglich die Beziehungen zu anderen parallelen Veränderungen in Frage. Die Sonne traf vorher den Stein nicht, sei es weil die Achsendrehung der Erde noch nicht die dazu erforderliche Lage der Sonne gegen den Stein herbeigeführt hatte, oder eine Wolke den Zutritt der Sonne zum Stein hinderte oder dergleichen. Kurz es mußte etwas, nicht im Stein für sich genommen, sondern in sonstigen, aber ihn irgendwie mitberührenden Umständen sich geändert haben. Das Gesetz der Kausalität sagt nur: Unter gleichen Bedingungen im Zeitpunkt 1 gleiches Ergebnis im Zeitpunkt 2; es sagt für sich nichts darüber, welche und welcher Art diese Bedingungen seien; es behauptet nur eine Gesetzmäßigkeit der Zuordnung überhaupt eines Consequens zu einem Antecedens, eine Gesetzmäßigkeit also, die als solche .und unmittelbar nur die Ordnung der Sukzession betrifft." Natorp, P., LGEW, p. 80.

⁴¹⁶ "Damit ist die Einheit des Gesamtgegenstandes der Erfahrung, und also die Einheit der Erfahrung selbst oder der "Natur" in abschließender Weist methodisch ermöglicht, und so der höchste Punkt erreicht, zu dem alle einzelnen Leistungen der synthetischen Funktion zusammenstreben." Natorp, P., PP, p. 30.

establishment of an invariance, the moment of substantiality as a logical requirement. Second, the causality, as a condition of the establishment of a relation among objects. Third, to search a total articulated system of relations. The complete determination is a task for thinking, something that must be achieved, not a datum⁴¹⁷.

4. 5. The Level of the Modality.

According to Natorp, as it was for Kant, the level of modality does not add a new determination to objectivity. There is no direct determination of the object but an indication of the ways in which the object can be related to thinking⁴¹⁸. The categories of quantity, quality, and relation determine objectivity. However, it is necessary to establish the link between the object and thought. It has not been determined whether the object is merely a possible object, an actual object, or a necessary object. It has to be decided whether this object is possible, real, or necessary. The fact that the object is possible, real, or necessary does not determine the object itself. The object does not change its determinations being possible, real, or necessary. Only the statutes in relation to knowledge changes but the determinations remain. At the level of the modality, there is added a consideration regarding the way in which the object is considered with respect to the faculty of knowledge. It is possible to claim that the object is possible, real, or necessary without adding any determination. This problem could not be raised at the levels of the determination of objectivity because it is not a question that refers only to the constructed object, but it is a problem that concerns the relation between the object and thinking. Therefore, following Kant, Natorp states that the levels of the modality do not add any determination to the concept of the object⁴¹⁹.

⁴¹⁷ "Der logische Grund dieser Supposition ist zuletzt kein anderer als die Notwendigkeit, das Wirkliche auf einzige Art bestimmt zu denken; also muß is jedenfalls bezogen werden auf eine in einziger Art bestimmte Ordnung der miteinander in einer Natur Zusammenstehenden Erscheinungsreihen. Daß eine solche empirisch gegeben weder ist noch je werden könnte, macht es nur um so fühlbarer, daß diese Ansetzung eine reine Denkleistung ist und kein Datum." Natorp, P., LGEW, p. 72.

⁴¹⁸ "Dies nun betont gerade Kant: die Modalitätsstufen betreffen direkt nicht den Gegenstand, wohl aber sein Verhältnis zur Erkenntnis, ihre Gegenständlichkeit Nachdem wir uns aber überzeugt haben, wie sehr der Gegenstand überhaupt nur in der Gesetzlichkeit des Denkens, des Erkennens wurzelt, hat es wohl Sinn, dies Verhältnis auch noch besonders ins Auge zu fassen." As Éric Dufour observes, in what has been considered as the second period of his production, Natorp profoundly modifies his conception regarding the status of the modality category. The modality has in the *Philosophische Systematic* constitutive role of objectivity. Cf., Dufour, É, 2010, p. 181.

⁴¹⁹ "Es sind also nicht neue Leistungen der synthetischen Einheit, die in den Modalitätsstufen formuliert werden, sondern es ist die Gesamtleistung des synthetischen Prozesses der Gegenstandserkenntnis, wie er in der Quantität, der Qualität und der Relation nach seinen Grundrichtungen sich auseinanderlegte." Natorp P., LGEW, p. 86.

For this reason, unlike the levels of quantity, quality, and relation, the level of modality cannot be derived purely and exclusively from the principle of synthetic unity. Like Kant, Natorp considers that the determinations of the object are exhausted with the categories of quantity, quality, and relation. The modality does not add anything to the construction of the object but defines how the objects are related to thinking. The level of the modality characterizes how thinking relates to the object constituted in the previous levels. The level of the modality receives this name because, precisely, it 'moulds' the experience. It establishes how thinking is related to the object. In a judgment of modality, what is shaped is not the object itself but the way in which thinking conceives the object. The modality is not a necessary feature for the construction of the objectivity itself⁴²⁰. Therefore, it cannot be derived directly from the concept of synthetic unity.

For this reason, the levels of the modality are the clearest expression of the ideality of the object. The levels of the modality show in a paradigmatic way the dependence of the object on thinking. The qualification of possible, real or necessary is completely meaningless without its relation to the process of thinking. The modality determinations only make sense when considering the relationship of the object with the act of knowledge. These determinations cannot be attributed to the object regardless of the consideration of the way in which the object is known. Certainly, the categories of quantity, quality, and relation are also determinations that arise from pure thinking. However, in the modality this aspect is seen more clearly. In the consideration of the modality of the object, the possibility of thinking the determinations of the object independently of the act of thinking is banned from the beginning. Therefore, Natorp argues that the ideality of the object is most clearly seen in the consideration of the modality. In the modality, thinking can establish those determinations of objectivity that cannot be defined independently of the thinking process. The determination of the objects as possible, real or necessary only arises as a problem with regard to the relation of the process of thinking to the objects.

A further peculiarity of the modality is that it describes the general structure of the three category levels, quantity, quality and relation. This structure that we were emphasizing as common for the three categories - starting from a first position, the generation of a multiplicity, and the unity of this multiplicity- is the basic structure that describes the modality. The level of quantity, quality, and relation will have the tripartite

⁴²⁰ "Gehören sie dem Gegenstande direkt vielleicht nicht an, dann um so mehr dem Denken, dem Erkennen - immerhin des Gegenstandes." Natorp. P., LGEW, p.82.

structure characterized by the levels of the modality: a first arbitrary position as an initial hypothesis, a second moment of repetition of the initial position preserving the previous moments, and a provisional closure in the form of a totality. Thus, for example, the first moment of the quantity is the establishment of a unit that will operate as a reference point in the measurement, a quantitative unit in this case. This arbitrarily established measurement pattern is the initial hypothesis indicated by the modality. The second moment is the repetition of that unit a certain number of times. This is the moment of multiplicity: the repetition of the measurement pattern. Thirdly, a provisional closure is made in which the entire measure is indicated. At this moment, that multiplicity is thought as a unity. This structure of the modality is also repeated for the levels of the relation and the quality. In the case of the relation, the first moment is the adoption of one of the series as a hypothetical unit. The association of one of the series with other series is the moment of multiplicity, the construction of a series system in the union of one series with another. Finally, the third moment consists of the union in a system with the final goal of connecting that set of series in one total unity. Thus, each of the moments of the modality corresponds to each of the moments of the relation, as well as with each level of the rest of the categories. In this case, it can be clearly appreciated that the modality does not propose new determinations of objectivity. It does not describe new specificities of the synthetic unity. The modality indicates the path of the synthetic unity in general. It shows how the procedure of the synthetic unity is expressed in each of the fundamental directions of thinking: quantity, quality, and relation. In the modality, the path of the synthesis is reflected. While describing the relation of thinking with the object, the modality establishes the ways in which thinking created objectivity. Therefore, it is also present in each construction of the experience because it describes the path that thinking follows in the construction of its objects. The modality describes the universal course of synthesis levels.

Natorp argues that the problem of modality is introduced with the concept of nature. The distinction between the possible, the real, and the necessary is not present in mathematics. In mathematics, the construction of the object itself affirms its existence and, with it, its possibility and its necessity. Every object of mathematics is real, possible, and necessary. The object that the mathematician names as existing does not distinguish a real entity from a possible or a necessary one. In mathematics, the demonstration that an object is possible guarantees its existence, and this existence is never merely contingent but always a necessary existence. With the assumption of the existence of

a mathematical object, it is affirmed that the object has been constructed mathematically, i.e., with the means of mathematics. Mathematically created objects all have a logical necessity. Therefore, the object thus grounded is immediately possible, real, and necessary. In mathematics, the thought of the object does not require the distinction between possibility, existence, and necessity. Natorp notes:

Es ist sehr bemerkenswert, daß es innerhalb der bloßen Mathematik diesen Unterschied der Modalität nicht gibt. Zwar reden die Mathematiker von einer Existenz ihrer Begriffe (etwa des Irrationalen, des Imaginären), aber diese Existenz unterscheidet sich in nichts von der Möglichkeit und der Notwendigkeit Was als mathematischer Begriff möglich, ist damit für die Mathematik sofort auch existent und sofort auch notwendig. Ist der Begriff erwiesen als in den Methoden der Mathematik begründet, so ist er damit gesichert nicht als bloß möglich, sondern mit dieser Möglichkeit für die Mathematik auch existierend, und mit dieser Existenz für sie zugleich notwendig. Dagegen in der Naturwissenschaft ist es wahrlich ein Unterschied,...⁴²¹

Thus, for example, the existence of a number implies that this number is a possible mathematical object. The number, as an object derived from mathematical laws, is also a necessary object of thought. A triangle constructed from the laws of formation of geometric figures is a possible, real, and necessary object. In mathematics, no additional tests are required to show the necessity of objects constructed by thought. In the modality, the transition from pure mathematics to physical-mathematical science is carried out, since the distinction between possibility, reality and necessity occurs only in the science of nature. Therefore, by virtue of the levels of the modality, the object of mathematical thought can be distinguished from the object of nature. In the modality, the distinction between mathematics and physics can be made. This step could not take place in the previous levels as long as the quantity determinations, the quality determinations, and the relation determinations concern both the mathematical object and the object of nature. Only in the modality are the conditions of possibility defined to think of an object as an object of experience. In mathematics, the distinction between the possible, the real, and the necessary is not relevant at all. On the contrary, the knowledge of nature requires

⁴²¹ Natorp. P, LGEW, p. 84.

that it can be decided whether the object is a possible fact, a real or necessary one. The affirmation of a hypothesis requires an additional test to show that the object thus considered is also a fact of experience. The fact of experience is considered as necessary if it can be proved that it is the case of a law. In the experience, the possibility, the reality, and the necessity are well distinguished as different ways in which the object is related to thinking, The levels of quantity, relation, and modality are insufficient to show how thinking posits the object. The synthesis of the modality will be in charge of this task. The modality will indicate how the object is thought, and the way in which the object is thought shows how a mathematically constructed object can be thought as an object of experience. The modality thus contributes to the true transition from mathematics to physics.

The levels of the modality

The possibility expresses the first moment of the modality. This moment will open the way to the rest of the modal moments. The possibility is a first initial estimation (*Ansatz*). This starting point considers the object as a possible object. In the first instance, the object is constructed as a possible object for thinking. This starting point is provisional, since the course of the investigation may show that this object is only possible, a real one or a necessary one. This initial estimation is always provisional, and it is this provisional nature of the estimation that allows the rest of the moments of the modality. Natorp compares the stage of the possibility with the formulation of a question in which the formulation itself sets the beginning of the investigation. The question regarding what this something unknown is can only be posited by thinking itself. For something to be an object of thinking it must, first of all, be something for thinking. As Natorp explains:

Die Möglichkeit steht sehr nahe der Frage, aber sie geht über diese hinaus, indem sie den Prozeß zur Entscheidung der Frage wenigstens einleitet. Was als möglich angesehen wird, wird damit allerdings zur Frage gestellt, aber es wird zugleich schon der erste Schritt zur Beantwortung der Frage getan. Dieser besteht darin, daß man setzt, es

sei so; so muß dann dieser Ansatz in der Durchführung sich bewähren, oder aber seine Undurchführbarkeit sich herausstellen⁴²²

Each determination that thinking establishes is the answer that the thinking itself gives to a question raised. The determination of that something can only be given by thinking itself. The reference to something that is not the object of thinking makes no sense. As it was already established, nothing is beyond thinking. Natorp argues that, in general, every question of knowledge is based on this type of relationship. The question about the object of knowledge is based on the fundamental structure of thinking, on the possibility of thinking of something. Each determination that the thought introduces is the answer that the thought itself gives to a question raised. The idea of knowledge as a task is reflected in the fundamental structure that has the same function of inquiring. One question has three dimensions. On the one hand, it has a prospective moment in a "not knowing". What is unknown is what is investigated. The goal of the question is to provide content to what is defined, in the first instance, as the 'no' of knowledge. It is the moment of indeterminacy. The question sets the task. This is the condition that makes it possible to ask about something in general. What is asked about is the unknown, it is an x that must be determined. However, there must be elements that allow, at least, to understand what one is asking for. Without this first step, the necessary presuppositions for the question itself would be missed: the establishment of the task. Three moments can be distinguished. First, forwards, the presentation of what is undetermined, what is to be determined. Second, backward, are those conditions that allow a primary identification of the object that is investigated. The object of investigation cannot be a pure nothing. It must be something for thinking so as to be something to be determined. In the middle, there is the knowledge of not knowing, where the previous two moments are combined. This is the basic structure of thinking. The source of the process is this possibility of the understanding to generate new determinations in its object. Every new determination is a provisional answer. The purpose of the process of determination is to progressively determine the object of experience. This is the first step in the path of knowledge, the establishment of an estimation. This estimation expressed in a hypothesis can be corroborated in successive levels of thinking. The possibility represents a first initial hypothesis with which the investigation begins. As we exhibited, in the quantity

⁴²² Natorp, P., LGEW, p. 87.

category, this initial estimation corresponds to the unit. This unit is a provisional support point to begin the process. This unit is taken as an initial proposal to form the scale that will operate as a measure of multiplicity. It must be checked that the initial selection of the scale is suitable for the measurement of the object. The initial moment is this provisional generation of a tentative scale whose effectiveness must be verified. The same process also takes place in the quality and in the relation, where the initial estimation is a first question that opens a horizon of possible answers. The question itself represents the level of possibility. The moment to verify the answer to the initial question is the level of reality. The beginning at the level of possibility shows that the process is always ongoing and that there is no absolute beginning. The process is infinite because it does not start with a first data whose origin is unknown. There is no initial data for thinking, but the path of thinking always begins with a question that opens a horizon of possible answers. For Natorp, there is no beginning in a pre-logical data that operates as an absolute beginning in the construction of the experience. On the contrary, at the stage of possibility, it becomes clear that the beginning is always relative. Reality will be a continuation of the process initiated by the position of possibility that is always a position of thinking.

The second level consists of the accreditation of what was initially set as merely possible. It must be confirmed that the tentative answer to the question does indeed take place. This 'taking place' that is expressed in the accreditation of the initial estimation constitutes the fact. It represents the moment of reality. The reality requires 'making a decision'. It is decided that what was initially considered as only possible is real. The problematic judgment is replaced by the assertive judgment. It is argued that what in the first instance may or may not be the case actually takes place. That is why Natorp calls existence, the complementum possibilitatis. The existence allows determining what the possibility left undetermined. The moment of the reality consists in the accreditation of the determination. At this moment, it is shown that what was set as possible actually takes place and that it is, consequently, a fact. The proof of experience is the proof of the fact. Proving that something exists means that what was considered as possible is now part of the experience. Therefore, the hypothesis test is also the proof that something is the subject of experience; that is, it exists. It is shown that what was raised as merely possible is a fact. This process is the path of thinking. The complete determination of experience is an infinite process. Experience can be defined as this process that has no closure because the path of determination continues infinitely. As in the case of possibility, reality as a moment of the modality is also exhibited in each of the other categories. In the case of the quantity, the arbitrarily unit must be suitable for counting. At the level of possibility, an arbitrarily selected unit was taken to measure a multiplicity. Now it must be proven that the unit chosen can effectively operate as a measurement standard. In the multiplication of the unit, it is evidenced whether this arbitrarily selected unit is suitable or not to operate as a scale of what is measured. In the second moment of the quantity, that of multiplicity, it is tested whether this estimation of the unit is satisfactory or not. The initial estimation process is successful when it is proven that the unit is suitable for measurement. The moment reality is tested by showing that the unit functions as a unit of measurement. In the quality, the second moment of the modality corresponds to the comparison. The objectivity considered can be compared with another from the finished point of view of the genus⁴²³. In the case of the relation, this is particularly evident. In the relation, a first order is taken as tentative. It must now be shown that this order can operate as a pattern for the rest of the systems. This second moment of reality consists in showing that the selected pattern is indeed suitable. Thus, the initial estimation for the relation is the proposal of a fundamental order, of an order that operates as the basis for the rest of the systems. In the search for a complete system of order, the possibility leaves the way open for the selection to be corrected or not, and the initial proposal becomes real if it is verified. What was established as merely possible was that which should be determined in some way. Reality is the continuation of the process that the possibility left open. It is proved that the objectivity already determined in a quantitative and qualitative process is real. The objectivity test is performed when it is shown that the series can be integrated under a common series. This evidence establishes a fact on a provisional basis since in the course of the investigation this fact can be posited again as a question. The initial estimation becomes a hypothesis that must be tested. This test is the proof of reality.

This course is exhibited in scientific research in which after the formation of a hypothesis, it is shown that the initial approach was correct through an experiment. The initial hypothesis is thus corroborated. The experiment is always oriented according to a specific question - which left the level of possibility. The experiment seeks to verify the hypothesis. The demonstration gives a solution whose necessity must be proved. The evidence constitutes only a provisional moment of detention. Therefore, the path of the

⁴²³ Cf. Natorp, P., L, p. 29.

experiment, the *fiat experimentum*, is the clearest testimony of the second level of the modality⁴²⁴. Even in the induction, the objective is the gain of a provisional conclusion that can operate as a major premise in a deduction. The initial test allows a provisional answer whose need must be accredited later. The deductive proof will allow passing from the real thing to the necessary when it is shown that the accredited is the case of a law.

The necessity represents a provisional closure of the process in the recognition that what was considered a fact is the case of a law. What was held to be contingently becomes necessary when it is considered the result of a deductive process. What is necessary is what is taken as a consequence of the process of deduction. To maintain that a fact is necessary means that this fact is considered as having been established by a law. Ultimately, this fact is incorporated in a system of laws through which it can be considered as the conclusion of a deduction 425. The establishment of a fact implies the possibility of finding the law from which it can be considered an instance. It is concluded that a fact is necessary when it is presented as the case of a law. Indeed, the premises are held provisionally and may themselves be subject to revision. The need for the conclusion is always relative as the premises themselves can be tested. They are considered necessary if they are the result of another deductive process. It can be requested that the premises of the deduction be subjected to a new revision, and a new initial estimation is required to show the necessity for the premise that operated to ground what was held as necessary. Certain fact that is considered as a necessary fact may become hypothetical by questioning the premises on which it was sustained. Thus, knowledge affirmed as necessary is only provisionally. The point reached at this stage can also operate as a provisional starting point in the search for a further conclusion. This moment can also be the starting point for the beginning of a new three-level cycle. The point reached may turn into a new beginning. Natorp argues:

Die dritte Stufe eines jeden synthetischen Prozesses aber betraf allemal den *Abschluß* des durch die erste nur eingeleiteten, auf der zweiten Schritt um Schritt weiter verfolgten Verfahrens, sozusagen den Rechnungsabschluß, der aber nur sicheren Grundlagen dienen soll für neue Prozesse von gleichem allgemeinem Stufengang. Die zweite und

⁴²⁴ "Der Weg des Experiments, das *Fiat experimentum*, das ist daher das deutlichste Zeugnis des allgemeinen Sinns der zweiten Modalitätsstufe." Natorp. P., LGEW, p. 89.

⁴²⁵ "Die Notwendigkeit der Tatsache bedeutet nichts anderes als ihre Feststellung im Gesetz." Natorp, P., LGEW, p. 91.

dritte Stufe unterscheiden sich also als der Weg, insofern man im Gange ist, ihn zu verfolgen, und der vorläufig erreichte Haltpunkt, auf dem man stillsteht, nicht um darauf stehen zu bleiben, sondern des Gewonnenen sich zu versichern und auf der soweit gesicherten Grundlage dann weiterzuschreiten.⁴²⁶

This provisional closure is also expressed in each of the categories. In quantity, the determined totality can always operate again as a unit for the formation of another plurality. The necessity corresponds to the consummation of the function in the process of counting, this is the totality determined as a provisional total closure. This particular unity, the whole, can be a relative unity for a new counting process. In the quality, this third level represents the possibility of establishing a subsequent genus of a higher order. The first genus becomes a species of this higher-order genus. The totality that represented the genus thus becomes a new unity of a multiplicity. The genus is now a qualitative unity in the multiplicity of species now subsumed in a superior genus. In the relation, it is sought the total concatenation in a series system.

The starting point is always provisional and conditional. The initial moment, which was considered as a hypothesis, is presented as a result of a demonstration. The path of thinking consists of the pursuit of this task, to prove that the original estimations are necessary facts. The function of a first data is to be a first step for new questions. This process is always relative. Indeed, the establishment of the fact as a case of a law proves this fact to be necessary. However, this need is always relative because new variables can always be introduced. The emergence of a new hypothesis introduces a new process. The whole path of knowledge expressed in science is oriented in this direction. The purpose of science is to seek universal laws that explain the particular facts based on them. The general goal of the science is the creation of laws through this process that involves induction and deduction. The purpose of induction is the possibility of a deduction. Induction seeks the establishment of a general law that can operate as a major premise. The induction seeks the general laws that allow to deduce the fact and show it as necessary. Therefore, induction and deduction are two poles of the same procedure. The knowledge process is based on this tripartite structure of the modality, which puts a hypothesis as possible, takes the fact as a tentative response, and, finally,

⁴²⁶ Natorp, P., LGEW, p. 90.

shows its necessary character through the deductive process. This end of the investigation, establishing the fact as necessary, is always provisional as this need may be the starting point of a new start.

Conclusion

In this deduction of categories, Natorp shows how thinking can by its own means construct objectivity. The concepts do not need anything given. On the contrary, the deduction of the categories exhibited how thinking is capable of producing the object without reference to intuition. For Natorp, thinking is the source of the totality of the determinations of the object. It provides the multiplicity and the unity. In the deduction of quantity and quality, it was exhibited that the act of thinking can concomitantly generate both unity and multiplicity. In this way, it was proved that thinking can constitute the object without any reference to intuition. As we exhibited, this construction was developed in four levels: quantity, quality, relation, and modality. In this way, Natorp explains how thinking can constitute the object of experience. In this way, it can be overcome the dualism between intuitive and conceptual representations as it is exhibited that there is no gap between the rules of objectivity and the concreteness of the object.

5.1 The Reformulation of the Notion of Intuition

As we explained in Chapter 3, the act of thinking consists in determining. The process of knowledge seeks to give a value to every variable in such a way as to reduce the scope of possibilities to a single possibility. The determination indicates what value corresponds to a certain variable. Determination fixes the value of a variable. The act of determination consists in claiming that the variable x corresponds to the value a. The complete determination takes place in the assignment of a value to all the variables. The reality is the determination, and the determination is reached when nothing is indeterminate. The variable could be determined in many ways. Assigning it a value establishes that the variable is determined in one way and not another. Among the many ways in which the variable could be determined, a single value is selected. With this, it is argued that of the multiple ways in which the experience could occur, it occurs in this way and not in another. The successive performance in this determination aims to determine the experience in a unique way in its entirety.

As we observed, the introduction of a hypothesis raises the possibility of a certain determination. It is proposed that a certain value can be attributed to a variable. Reality affirms that this value is a fact, and the third level confirms it as necessary. The goal will be total determination so that nothing is indeterminate. However, the third level is always relative since it can become a starting point for new hypotheses, and thus for a new beginning of the process. The requirement remains conditional as, as we have seen, the establishment of a necessity can again open a universe of new hypotheses. Complete determination is not fully satisfied in any of the three stages of the modality. Reality does not satisfy the demand for complete determination because it is itself the infinite process of relative assignments of values to variables that is always open. Then, the need for a further element that satisfies this requirement could be raised. Natorp introduces the question of whether the requirement of complete determination does not require an additional instance. The question that arises is whether it is not a problem that the complete determination is never absolutely satisfied. Complete determination means that what is determined in a unique way, that it is so and not some other way. What may be

⁴²⁷ "Wirklichkeit bedeutet eine Bestimmtheit, so daß nichts unbestinunt bleibt. Unbestimmtheit ist eben bloße Möglichkeit." Natorp, P., LGEW, p. 92.

otherwise is somehow undetermined. The possibility of being otherwise means that some of its variables have not been assigned any value. The requirement of a complete determination means that one aims to determine what in the possibility was indeterminate. In this case, the assignment of a value is not justified by any given intuition but by the possibility of its interconnection in a coherent system through the inductive-deductive process explained in the level of the modality. The guarantee of truth is given in the compatibility of the set of judgments with another set of judgments. There is no need for any extra-logical instance, as the intuitive representation. The process starts from an initial estimation (Ansatz). One claims that a certain variable has a certain value. If when testing it, one verifies that assuming those values the set of our beliefs becomes incoherent, then that assignment of the variable should be rejected. The estimation is abandoned because it cannot be coherently incorporated into my set of beliefs. The rejection in the assignment of a value to a variable is carried out by means of the criterion of coherence. There is nothing external with which the estimation of thinking can be legitimized. There is no instance in which thought can check the estimation with that outside itself, such as an intuitive representation. The experiment, as we have seen, only allows us to show whether what is determined in this way can be introduced into a coherent system. For Kant, the concepts are insufficient to determine the object in a unique way. The complete determination of the object requires intuition⁴²⁸. At a specific stage, the entire universe of variables related to this stage of knowledge can be determined. Complete determination is always proposed as a task. The determination is hypothetically adopted and then accepted if it satisfies the condition of a fully coherent interconnection. However, that satisfaction can never be ultimately verified because the facts that we claim that exist are always relative. The search for determinacy demands the concept of complete determination as a regulative idea. The demand posed by possibility is never satisfied. This does not eliminate the demand as such. On the contrary, the requirement is justified as an idea to which the process tends. The requirement is justified as a goal to which the process aims. This is the positive meaning of the concept of intuition. Complete determination is thus required by all determinacy. Determinity as the assignment of values to all variables necessarily requires complete determination. Progressive determination assumes complete determination as its purpose. Partial determination always has complete determination as its goal. The complete determination

⁴²⁸ The complete determination is not going to be given in actual experience but is a regulative idea, as a postulate of reason. Natorp. P., KMS, p.204.

is a condition of all determination insofar as it guides the process of each partial determination. Natorp maintains that the essential impossibility of accomplishment of this requirement does not mean that this requirement is less well founded. The complete determination remains as a task. Natorp maintains that the legitimacy of the requirement is sustained even when its fulfillment is essentially unachievable⁴²⁹. The conception of a progressive determination is not opposed to the requirement of a determination in a unique way. Kant needs to incorporate the intuitive element because he has to satisfy the requirement that remains unachievable for our understanding. Kant introduces intuition as a factor that achieves the demand placed by thought. The problem consists in the misunderstanding that this requirement itself is essentially unattainable. For Kant, complete determination is achieved by reference to intuition. As we will analyze in the next section, the problem is grounded on the definition of the notion of concept. The Kantian notion of the concept leads to the conclusion that complete determination is never achieved by concepts. According to Kant, complete determination is only achieved by intuition. For Natorp, on the contrary, intuition will be this always distant goal, not the first given data. The Kantian concept of intuition is the always distant goal and not something given.

As we introduce in Chapter 1, space and time are for Kant forms of intuition. The complete determination of the object demands the introduction of a factor external to thought. The determinations of thinking are insufficient to provide a full analysis of the object because although its notes can be analyzed in exclusively conceptual terms, certain determinations still remain to be established. Its location at a certain moment and at a certain time demand the introduction of intuition as an external element to thought. For Kant, the determination of a unique way can never be conceptual, it requires the individualization of space and time and that is never achieved by means of concepts.⁴³⁰

From Natorp's point of view, as we have seen, spatio-temporal determinations are also determinations of thought that correspond to the laws of relation. In the category of

⁴²⁹ In both Kant and Natorp there is a regulative use of the requirement of complete determination. Natorp agrees with Kant that complete determination is a regulative idea. However, in Natorp the typically Kantian distinction between the regulative and the constitutive does not hold. For Natorp, the requirement is part of the constitution of objectivity. The requirement of complete determination and the determination itself are on the same level, even when Natorp accepts that the levels of the modality do not constitute the object but rather determine its link with knowledge. Only in this sense is the requirement regulatory and not constitutive.

⁴³⁰ "Anschauung heißt ihm "die Vorstellung, die nur durch einen einzigen Gegenstand gegeben werden kann. Zeit und Raum sind in solchem Sinne "wesentlich einige" Vorstellungen, darum Anschauungen; es gibt nur eine Zeit, nur einen Raum, so wie es nur eine Erfahrung gibt, "in welcher alle Wahrnehmungen als in durchgängigem und gesetzmäßigem Zusammenhange vorgestellt werden"." Natorp, P., LGEW, p.92.

relation, thought seeks to bring together a plurality of quantitative-qualitative syntheses. This task is performed in the establishment of a second-order synthesis, as a synthesis of the synthesis. Thought thus generates a system of series. Space and time will be the indexing parameters that allow the establishment of this order, allowing each term of one of the series $(x_1 \ x_2 \ x_3 \ ...)$ to be ordered in relation to another series system $(y_1 \ y_2 \ y_3 \ ...)$. Space and time are not principles of determination independent of the synthetic unity laws, but rather they are incorporated in the orientation of the relationship, in the task of thought to provide a unity between syntheses, in the synthesis of synthesis. For Natorp, also complete determination is never reached but can only be sought by means of mere concepts.

As we studied in Chapter 1, from the Kantian perspective, data given to intuition is the beginning of the investigation. The big bang of experience, as Mario Caimi calls it, begins with something given to intuition whose origin is extrinsic to thought. This given matter is passively received in the intuition. For Natorp, on the contrary, intuition is not a first given factor, but the result sought by the determination process. The fully determined datum is a task. However, it must be recognized that this process is an infinite path of progressive determinations. Reality is thought as determined but in a provisional way because, as we exhibited in Chapter 4, it can always be subjected to further analysis. The complete determination sought will be fully achieved in the complete ordering of the series of changes. However, this is a demand that is never fulfilled. The requirement of a univocal order that allows the full identification of the phenomenon makes full sense only as a requirement. Knowledge is not capable of fully satisfying this requirement, but always only conditionally. The results achieved, that which is provisionally considered as a proven fact, can then be submitted for review. As we studied in Chapter 4, being given to intuition is a moment of modality. This is the consummation of idealism in the recognition that data can never give a definitive answer to the question but, on the contrary, always opens up new questions for investigation. There is no definitive or absolute proof of experience because the path of investigation is infinite. Kant's problem is that he is not satisfied with the demand, but he seeks to satisfy it. Unable to satisfy this requirement by means of concepts, Kant introduces the intuition. A consistent idealist accepts the requirement as such. What is illusory is the claim to satisfy the demand. This is the illusion of naive realism. In this sense, what is truly speculative is the ultimate datum as illegitimately introduced. The illusion consists in the belief that this demand could be satisfied. In this way, Natorp explains not only what must be considered as the

effectively real, but also the reason for the Kantian error. There is nothing given as the ultimate datum.

For Kant, the singularity is given in intuition. The relations of thought will be insufficient to determine the object in the individuality. For Natorp, the establishment of the singularity of the object is a contribution of thought that puts the search for unequivocal identification as an object. The singularity in Kant is a given singularity while the singularity in Natorp is a constructed singularity. Natorp accepts the need for an intuitive moment as a demand. Kant, who bases his error on the assumptions of naive realism, transforms the demand into an accomplishment of the demand. The mere requirement is transformed into the accomplishment of it. For Natorp, intuition will not verify existence. Intuition, in the critical system, is the ultimate guarantee of the confirmation of experience. For Natorp, on the contrary, what is given to intuition cannot operate as a criterion of existence. The search for determination of the indeterminate is the right path. The error consists in the conviction that this determination of what seems indeterminate can be achieved through an intuition. On the contrary, as we pointed out, this notion of intuition must be reinterpreted from an idealistic perspective, according to the Copernican turn. Possibility demands a determination that is satisfied with actual reality and fully consummated with necessity. Effective reality demands the determination of the indeterminate. However, the reality check is always provisional. The determinations provided by intuition are always themselves a conceptual element, since perception answers the question posed by a concept. There is no dismissal of the concept of intuition but a reformulation of its function. The fact is not a datum of intuition but, rather, the intuitive factor is a response to a construction of thought. Regarding its content, perception also consists of conceptual determinations. The content of perception is found dissolved in the process of determination according to the different orientations of thought that construct objectivity, i.e., according to the laws of quantity, quality and relationship, which are always conceptual determinations that derive from the synthetic unity.

This is the consummation of idealism. A consistent idealism is realized by stating that the absolute fact is never achieved, and that the satisfaction of this requirement is not required either. Indeed, a total legal construction is a goal that must be achieved. The complete determination is a task. The consummation of idealism consists in this conception, in this establishment of the fact given to intuition as a task, as the ultimate determination to which thought aspires but never reaches. The ultimate legal order is the always distant goal for an imperfect thought. Kant's mistake was to transform the demand

into a satisfaction instead of leaving the demand as such. Trying to satisfy this requirement, Kant introduced the distinction between intuitions and concepts. The consummation of idealism consists in this recognition. The first step in the consummation of idealism is the recognition of this new approach to the problem of intuition. The second step consists in the reformulation of the notion of concept.

5.2. The Reformulation of the Notion of Concept

In the Transcendental Aesthetic, Kant presents the distinction between intuition and concept. Intuitions and concepts are two ways in which thinking can refer to objects. Intuition is a form of representation in which the object is given immediately. Intuitions have their origin in sensibility and concepts in the understanding. Man is not capable of intellectual intuition, since humans only know through concepts. Human beings know the object through its marks. Concepts are mediated representations of objects. The human intellect can only know the object through its common marks. The understanding refers to the object indirectly, by way of the common marks of the object. Concepts are representations that have their origin in the understanding. Since they refer to the object by their marks, the concepts are mediated representations of the object. In conceptual representations, the parts always precede the whole. The entire representation is constituted from the synthesis of the component parts. The concept is a representation that contains the notes of the object that operate as characteristics common to many things. The notes of the objects are predicable of multiple objects. For this reason, "...every concept must be thought of as a representation which is contained in an infinite number of different possible representations (as their common marks), and that therefore contains them under itself. 431" In the process of the concept formation, the parts precede the whole. This is the premise that Kant uses to argue in Transcendental Aesthetic that space and time are intuitions. Kant argues that in the representation of time and space the parts cannot precede the whole; therefore, they are intuitive and not conceptual representations; since conceptual representations are precisely those in which the whole

⁴³¹ "Nun muss man zwar einen jeden Begriff als eine Vorstellung denken, die in einer unendlichen Menge von verschiedenen möglichen Vorstellungen (als ihr gemeinschaftliches Merkmal) enthalten ist, mithin diese unter sich enthält; aber kein Begriff, als ein solcher, kann so gedacht werden, als ob er eine unendliche Menge von Vorstellungen in sich enthielte" (*KrV*, B39).

is formed from the parts. Kant uses the definition of the concept as a representation by common marks to argue that space is not a conceptual representation but an intuitive one. The argument is introduced as a disjunctive syllogism. Our representations are either intuitive or conceptual. In intuitive representations, the whole precedes the part, in conceptual representations the part precedes the whole. In our representations of space and time, the whole precedes the part. Therefore, the representations of space and time are intuitive and not conceptual. Kant's argument is based on a disjunctive syllogism that assumes as valid the definition of his intuitive and conceptual representation.

It can be seen, the enormous importance of the definition of intuitive and conceptual representation, because the argument that Kant employs in the Transcendental Aesthetic, assumes that we accept this distinction. Conceptual representations are those that are obtained in a mediate way, by reference to intuition. The representation is obtained from common marks of the objects given to the intuition. These definitions that Kant introduces at the beginning of the *Critique of Pure Reason* are explained in the Dialectic. Kant explains that the gender is representation. Perception is a sensation with consciousness. The concept is the form of perception that is obtained by means of common marks of the objects. Kant presents the classification this way:

Bewußtsein (*perceptio*). Eine Perception, die sich lediglich auf das Subject als die Modification seines Zustandes bezieht, ist Empfindung (*sensatio*), eine objective Perception ist Erkenntniß (*cognitio*). Diese ist entweder Anschauung oder Begriff (*intuitus vel conceptus*). Jene bezieht sich unmittelbar auf den Gegenstand und ist einzeln, dieser mittelbar, vermittelst eines Merkmals, was mehreren Dingen gemein sein kann. (A320/ B 377)

The concept is a representation by common marks. This definition of the concept as representation by common marks follows that definition that Kant used in his Lectures on Logic. In the *Jäsche Logik*, we find a definition very similar to the one that Kant provides in the Introduction to Aesthetics and Dialectics:

Alle Erkenntnisse, das heißt: Alle mit Bewusstsein auf ein Objekt bezogene Vorstellungen sind entweder Anschauungen oder Begriffe- die Anschauung ist die einzellne Vorstellung (repraesentatio singularis), der Begriff eine allgemeine (repraesentatio discursiva) oder reflektierte Vorstellung" (*Logik*, AA: XXIV p.98)

Discursive or conceptual representation is a representation by common marks. As it is a representation by common marks, this representation is universal. The universal representation contains multiple parts, whose multiplicity is prior to the unity that contains it. For this reason, the concept contains a multiplicity under it. The multiple elements contained in the concept are different from each other. Thus, for example, the predicate red can correspond to multiple objects that differ from each other. The concept contains a multiplicity of possible representations, insofar as those possible multiple representations that the concept includes differ from each other ⁴³². Thus, in the conceptual representation, the parts precede the whole. The whole is the unity formed from the parts. That totality is always an indeterminate universal that can always contain multiple representations under it. The concept is a universal representation because it is a representation that is generated from what is common to all the objects that fall under it. Kant holds:

Da nur einzelne Dinge oder Individuen durchgängig bestimmt sind, so kann es auch nur durchgängig bestimmte Erkenntnisse als Anschauungen, nicht aber als Begriffe, geben; in Ansehung der Letzteren kann die logische Bestimmung nie als vollendet angesehen werden. 433

The conceptual representation is obtained by abstracting what is common in many objects. If a certain representation is not a common representation, it is not a concept. Therefore, the complete knowledge of the object can only be given by the singular object, because "only singular things or individuals are completely determined". Therefore, the possibility of complete determination is only possible as an intuitive representation; that is, "there can only be fully determined knowledge as intuitions (not as concepts)." Thus,

. .

⁴³² "Ein jeder Begriff enthält ein Mannigfaltiges unter sich, insofern es übereinstimmt, aber auch, insofern es verschieden ist. - Die Bestimmung eines Begriffs in Ansehung alles Möglichen, was unter ihm enthalten ist, sofern es einander entgegensetzt, d.i.. von einander unterschieden ist, heißt die logische Einteilung des Begriffs." *Logik*, AA: XXIV p.98.

⁴³³ *Logik*, AA: XXIV p.108.

with regard to intuitions, the logical determination can be complete, but "regarding concepts, the logical determination can never be considered as achieved." 434

Thus, Kant defines the concept as a form of representation by common marks that are abstracted from other the given representations. However, this definition is insufficient. Kant defines a priori concepts as rules, functions. The definition of the concept that Kant uses is insufficient to characterize the function of the concepts of the understanding. Kant uses this concept definition to characterize concepts in general.

Kant maintains that understanding is a source of concepts. As is well known, this is the result of Transcendental Logic. Understanding is the source of concepts, and those concepts are rules of unification of the multiple of intuition. The concept is the function of the understanding to provide unity to the multiplicity. The concept is this gathering form of the multiplicity of intuition. This form of reunion is produced by the understanding itself. The concepts "spring, pure and unmixed, out of the understanding which is an absolute unity, and therefore must be connected with each other according to a concept or idea." (A67-B 92). The concept is a function of the unity of the representations. The concept is a function of pure thinking to give unity to the various representations. In this sense, the concept is a rule. This rule is the function that determines the specific way in which multiplicity is unified. Since the understanding is

_

⁴³⁴ *Logik*, AA: XXIV, p. 108. These expressions of Jäsche Logic are also found in other Logic lessons. Thus, we find:

In the Logik Phillipi (early 1770s):

[&]quot;Ein Begriff ist eine allgemeine Vorstellung; Vorstellungen die nicht allgemein sind, sind keine Begriffe." AA: XXIV, p. 451

Logik Wiener (around 1780):

[&]quot;Conceptus communis kann ich nicht sagen, weil es eine tautologie seyn würde (...) Denn wenn eine Vorstellung nicht repraesentatio communis ist: so ist sie gar kein Begriff" AA: XXIV, p. 908.

[&]quot;Kein Begriff wird also ohne Vergleichung, ohne Wahrnehmung einer Einstimmung und ohne abstraction. Könnte ich nicht abstrahiren: so würde ich keinen Begriff haben,…" AA: XXIV, p. 909.

[&]quot;Ein Begriff ist also eine Vorstellung die vielen Dingen gemein ist." AA: XXIV, p. 905. Logik Dohna (early 1790s):

[&]quot;conceptus, enthält das, was mehrern Gegenständen gemein ist, nota communis." AA: XXIV, p. 752. Also: "Zum Gebrauche eines Begriffs wird Absonderung erfordert, aber dadurch wird der Begriff noch nicht gemacht. Letzteres geschieht 1. dadurch, daß etwas als Teilvorstellung betrachtet wird, die mehrern gemein sein kann, z.B. die rote Farbe. 2. wenn ich die Teilvorstellung als nota, als Erkenntnisgrund einer Sache betrachte, z.B. durch rot Blut, Rose usw. erkenne. Die 3te Handlung ist die Abstraktion, diese Teilvorstellung als Erkenntnisgrund, insofern ich von allen übrigen Teilvorstellungen absehe. Der Begriff ist also eine Teilvorstellung, sofern ich von allen übrigen dabei abstrahiere." Logik Pölitz:

[&]quot;repraesentatio ist das erste und allgemeinste und kann nicht erklärt werden," (...) "Erkenntniß ist entweder intuitus oder conceptus; intuitus, wenn ich nur einzelne Vorstellungen habe, conceptus wenn ich Vorstellungen hab, die vielen gemein sind, oder repraesentatio communis. Conceptus est repraesentatio communis weil der Begriff aufs Merkmal des Gegenstandes geht und also den Gegenstand mediate durchs Merkmal vorstellt und dies Merkmal kann vielen Dingen gemein seyn." AA: XXIV, p. 565.

the faculty of concepts, it is therefore also "the law of the synthetic unity of all phenomena" (A 128). Understanding is a source of concepts. For this reason, Kant affirmed at the beginning of Transcendental Aesthetic that concepts arise from the understanding, while intuitions rest on affections. While intuitions are grounded on affections, concepts are grounded in functions⁴³⁵. The concept must be understood as a rule, as a function. However, the Kantian definition of the concept as an abstraction of common marks does not seem to be a plausible expression of this function of the concept. Kant claimed that the concept is a representation by common marks that are abstracted from the representation. This corresponds to the way in which empirical concepts are formed but it is not a plausible definition to explain the operation of pure concepts of the understanding. The pure concept is productive, but the abstraction does not produce anything. 436 Therefore, abstraction is a negative concept 437. Kant seems to use the empirical concept formation model to explain concept formation theory in general. The problem is that, as Kant himself marks "the use of the pure concepts of the understanding would be completely altered, if one tried to treat them only as empirical products." (A 92)⁴³⁸. Then, a notion of concept is required that explains the function of the concept as a function.

Natorp claims that it was Leibniz who first understood the theory of concepts as functions. In his 1881 conference, "Leibniz and Materialism", published by Helmut

 $^{^{435}}$ A 68 - B 93.

⁴³⁶ "Durch Abstrahieren wird nicht nur nichts hervorgebracht, sondern vielmehr weggelassen" AA: XXIV, p. 754.

p. 754.

437 "[Abstrahieren ist im philosophischen Sinne ein negativer Begriff – nicht attendieren (in der Chemie positio)." AA: XXIV, p. 754. As Luciana Martinez explains: "El carácter general de las representaciones conceptuales se obtiene por medio de la abstracción. La abstracción se encuentra en el origen de la forma general de los conceptos, y no en el origen de su contenido. Ella no genera representaciones". Martínez, L., 2019, p.690.

⁴³⁸ In this line, Kemp Smith affirms that in the Transcendental Aesthetics Kant does not show that space and time are not concepts but that they are not empirical concepts. The only conclusion that can be drawn from this argument is that space and time are not generic class concepts. It is not shown that space and time belong to receptivity and not to spontaneity. Space and time have not been proven to be different from the categories. He holds: "Conception is always the representation of a class or genus." (...) Owing, however, to the narrowness of the field assigned to conception, the realm occupied by intuition is proportionately wide, and the conclusion is not as definite and as important as might at first sight appear. By itself, it amounts merely to the statement, which no one needs to challenge, that space is not a generic class concept., Kemp Smith, N, 1918, p.107. As Longuenesse explains, pure concepts (and also mathematical ones) follow the model of the generic concept. Longuenesse states: empirical concepts and a priori concepts (categories and mathematical concepts) "All equally are, however, *made as to their form.* Now, the only operations of the understanding to which Kant refers when he explains how the form of concepts is "made" are the three considered earlier: comparison, reflection, and abstraction:" Longuenesse, B., 1993, p.120. George Schrader considers that "there is nothing in common between α *priori* concepts and empirical concepts save the name." Schrader, G., 1958, p. 264.

Holzhey in 1985, Natorp argues that it is necessary to redefine the notion of concept⁴³⁹. He introduces this idea within the, by then traditional, debate between materialism and idealism. Natorp's central thesis is that a mechanistic position is not necessarily materialistic. Mechanicism does not lead to materialism. At the mechanical conception of nature was one of the great achievements of early modernity. In the seventeenth century, it emerged as a widely accepted idea that nature as a whole behaves mechanically. That is to say, that "the totality of nature in all its phenomena and connections does not represent more than a perfect mechanism." Mechanical laws control everything. It is accepted that "the simplest forms and laws of events, as taught by mechanics, control and unite the totality of the inexhaustible variety of natural things." However, along with this idea, there was the belief that the mechanical conception of nature was associated with the materialistic conception. The early modern conception held that the endorsement of mechanism led directly to an acceptance of materialism. According to Natorp, before Leibniz, materialism and mechanism were considered to have a close and unavoidable connection⁴⁴³.

For Natorp, one of the achievements of Leibniz's proposal is to show that there is no direct relationship between mechanicism and materialism. On the contrary, the mechanistic conception leads to idealism, and consequently, to the need to introduce an idealistic conception of the notion of concept. Leibniz rejects materialism but accepts mechanism. And with this, he objects that materialism is an inevitable consequence of

⁴³⁹ Edgar Scott shows the importance that Cohen's approach to Leibniz had. He maintains that "…in Leibniz's arguments against Descartes' view that matter's essence is extension, Cohen would have found a problem with his own account of knowledge, a problem that was potentially devastating by his own lights. Leibniz's arguments revealed to Cohen that reality must be conceived by appeal to non-extensive magnitudes, in addition to extensive magnitudes. But then, the Anticipations would appear as the chapter of the first *Critique* that provides an account of just those non-extensive magnitudes." Edgard also explains the influence that Natorp has on Cohen's reading. He also exhibits the differences of their approaches to the issue. Edgar, S., 2021, p. 203.

⁴⁴⁰ Cf. Holzhey, H., 2011, esp. p.7

^{441 &}quot;die gesammte Natur in allen ihren Erscheinungen und Zusammenhängen nichts als einen vollkommenen Mechanismus / darstelle. So waren in wenigen Jahrzehnten die Anschauungen des ganzen Mittelalters gestürzt und der Sieg der modernen Weltauffassung entschieden." LM, p.5

⁴⁴² "es wird begreiflicher, wie gewisse einfachste Formen und Gesetze des Geschehens, wie sie die Mechanik lehrt, die ganze für uns unerschöpfliche Mannigfaltigkeit der Naturdinge beherrschen und zusammenknüpfen." Natorp. P., LM, p. 6.

⁴⁴³ "Es ist bekannt, dass der Materialismus keine Erscheinung der neusten Zeit, vielmehr fast so alt ist wie die wissenschaftliche Erforschung der Natur. Wann und wo immer man versucht hat, die Zusammenhänge der Erscheinungen auf mechanischem Wege zu erklären, ergab sich der Materialismus als scheinbare Consequenz. So zu Leibniz' Zeit." Natorp. P., LM, p. 5.

mechanicism. From his early writings, Leibniz rejects materialism but he endorses a mechanistic conception of nature⁴⁴⁴.

Natorp introduces two arguments: 1) based on the problem of the relationship between sensation and thought (materialism would be incapable of explaining this relation) 2) The need to think about unity in matter. Materialism cannot give a mechanical account of the unity of phaenomena. For our purposes, we will focus on the second issue.

Natorp's argument begins by showing the insufficiency of materialism to mechanically explain nature. The mechanistic conception, Natorp will argue, leads to idealism. Natorp defines mechanism as that conception according to which everything in nature is connected by mechanical laws. There is no spontaneous causation. He defines materialism as the conception that holds that the entire universe is composed of material entities. For materialism, matter is the ultimate substance of the real. What is real is matter and, consequently, the first object of senses: *phaenomena*.

For Natorp, the first problem of materialism is the need to introduce a principle of unity in phenomena. Leibniz shows that matter by itself cannot be a source of unity that phenomena themselves require. Mere matter cannot be the principle of determination. It is necessary to introduce an active principle to think of nature. Natural events can be explained by the legality that governs them, but it does not happen the other way round. The legality of the events cannot be explained by the materiality that constitutes them. Natorp points out:

Zugegeben, dass aus Grosse, Figur und Bewegung der Körper alle besondern Erscheinungen der Natur erklärbar seien, so lässt sich doch schon irgendwelche bestimmte Grosse und Figur aus der Materie als blosser Ausdehnung nicht ableiten: die Ausdehnung in sich betrachtet entbehrt jeglicher Determination; und ferner folgt aus ihrem Begriff zwar Beweglichkeit, aber nicht wirkliche Bewegung; es muss daher schon ein actives, immaterielles Princip eingeführt werden, damit selbst ein bloss mechanisches Geschehen nur irgend verständlich werde. 445

^{444 &}quot;Der "Mechanismus" behielt den Sieg; und Leibniz hat sich sein ganzes Leben hindurch mit ganzer Entschiedenheit und selbst mit Begeisterung zu ihm bekannt, während er die materialistischen Folgerungen nicht minder entschieden, und ebenfalls schon früh, zurückwies. [...] Seine Anerkennung der modernen, mechanistischen Naturauffassung ist in der That aufrichtig und rückhaltlos;" Natorp. P., LM, p.5.

⁴⁴⁵ Natorp, P., LM, p. 9.

The mere extension has no determination, and its determination cannot be the matter, since matter, as a mere extension, lacks determination in itself. Matter has no determination by itself. It cannot have an internal principle of order. For the mechanical explanation to take place it is necessary to introduce a non-material active principle. Mechanism leads beyond materialism by exhibiting this necessity. The mechanical explanation of nature demands an ideal principle of unity. To explain nature, it is necessary to introduce an active principle, mechanic but not material. Leibniz shows the need to introduce an immaterial principle for the understanding of mechanical phenomena. In this way, for Natorp, it would be clear that the mechanical conception is anti-materialist because it can be seen that matter is not the ultimate substance of the universe since matter does not have unity by itself but, at the same time, it cannot be a source of unity. Materialism relies on the notion of *pheanomena* but the *pheanomena* themselves demand a unity that matter cannot provide. The matter has no unity by itself but, at the same time, it cannot be a source of unity.

For Natorp, Leibniz shows that matter by itself cannot be a source of unity. The introduction of a unity -and the peculiarity of the type of unity introduced- requires a non-material principle. This non-material principle of matter, which determines what matter is, is force. The concept of force shows the connection between the mechanistic conception and idealism, since it is this metaphysical concept that allows us to explain the behavior of matter itself. For this reason, "the concept of force makes the transition from mechanical to metaphysical conception" That is, the matter cannot be a principle of action. It does not behold any principle to act. Then, there cannot be an identical subject of action unless one admits a formal principle. This formal principle is the force, which generates both: the movement and the unity of what is moving. The phenomena require forces, but forces lead to the concept of law. This was Leibniz's innovation. Leibniz saw that concepts are modes of establishing relationships and not representations abstracted from things.

⁴⁴⁶ As Scott Edgar explains, according to Natorp, "Leibniz's arguments assume a connection between the concept of reality and the concept of a genuine unity or genuine individual. On this assumption, conceiving of the real requires conceiving of unities or individuals. This is just the point Natorp draws attention to when he recalls the Parmenidean and Platonic antecedents to Leibniz's arguments: a thing must have the right kind of unity to be a being properly so called." Edgar, S. 2021, p.219.

^{447 &}quot;es muss daher schon ein actives, immaterielles Princip eingeführt werden, damit selbst ein bloss mechanisches Geschehen nur irgend verständlich werde. Daher ist es der Begriff der Kraft, welcher für Leibniz den Uebergang bildet von der streng mechanischen Auffassung der immanenten Zusammenhänge der Natur zu einer Metaphysik, welche ihre Principien höher hernimmt..." Natorp. P., L, p. 9.

Matter has no unity of its own. There is nothing in matter that can provide that unity. The unity and the peculiarity of the type of unity that requires a non-material principle. Therefore, an active, immaterial principle must be introduced, so that even a purely mechanical event can be understood in some way. The need to seek this principle of unity is what in history has led to the concept of substance. The notion of substance has come to satisfy this requirement. The mistake has been to seek that principle of unity in matter. As an immaterial principle, the concept of force shows how mechanism does not lead to materialism but to idealism. Precisely, by the concept of substance, materialism is overcome. Leibniz showed that only by introducing this formal principle, we can differentiate appearance from phaenomena. To be a phenomenon, in contrast to mere appearance, is to be a case of a law. As we explained in chapter 3, the unity of the legality is consciousness itself. For Natorp, the act of thinking itself can be defined as the search for unity. Thinking is comprehending the multiple in a unity. To think and, ergo, to conceptualize, is to give multiplicity a specific form of unity. The concepts, precisely, are modes of uniting. The ways of giving unity to the multiple are the concepts. Natorp maintains:

Was diese verlangte Einheit sei, lässt sich durch nichts Sinnliches deutlich machen, hingegen versteht es sich sofort durch die Reflexion auf die Grundbeschaffenheit unsres Denkens, welches, wiewohl eine Vielheit von Objecten umfassend, doch diese stets in einer Einheit darstellt, in einer Concentration gleichsam, welche eben das ausmacht, was wir Denken oder Bewusstsein nennen. Leibniz sah ein, dass auf solcher formalen, ideellen, begrifflichen Einheit das beruht, was die Wahrheit der Phänomene, die Substanz oder das Wesen der Dinge im Unterschied von der blossen Erscheinung ausmacht.⁴⁴⁸

Natorp claims that thinking is giving unity to the multiple and that concepts are precisely these modes of reunion. Unity is that provisional point of view. The articulating unity is the law that regulates the ways in which the multiplicity is reunited in a unity. This

⁴⁴⁸ Natorp, P., LM, p. 9.

provisional point of view is the law. The law is the expression of the unity of the point of view. Leibniz, Natorp claims, was the first to see this required unity in the unity of the law. The law is this ideal unity that shapes multiplicity. The law allows a 'representation' of the substance of the thing because it allows to articulate a specific point of view, "no longer material and sensible, but formal and ideal." Natorp states:

Nur unter Einer Gestalt kann die "Substanz", welche vor unsern Sinnen in den Formen des Raumes und der Zeit bloss erscheint, von uns gedacht werden: unter der Gestalt des Gesetzes. Im Begriff des Gesetzes, als der eigentlichen Darstellung der Substanz der Dinge, hat denn Leibniz ein ganz und gar / nicht mehr materiales und sinnliches, sondern formales und ideelles Princip erreicht; die Metaphysik des Materialismus war damit erst gründlich überwunden, während zugleich alles Berechtigte desselben, nämlich die Forderung der strengen Durchführung des Naturbegriffs, der ja auf nichts beruht als auf dem Begriff des Gesetzes, ungeschmälert erhalten blieb. 449

For Natorp, the phenomenon can only be constituted thanks to this unity of the law. Shapeless multiplicity, as a mere appearance, can only acquire the form of a phenomenon thanks to this ideal unity. The unity of the phenomenon constituted by virtue of the law contrasts with mere appearance. To be a phenomenon is to be a multiplicity united under the point of view of the law. This was Leibniz's discovery. For Natorp, Leibniz recognized that on the basis of such a conceptual unity that the essence of things (the substance) contrasts with mere appearance. Therefore, the substance must be understood as a legal determination of the phenomenon. The substance of the phenomenon is this legality which, by giving it an objective determination, concomitantly gives it reality as opposed to mere appearance. To be real is to be a case of the law. To be constituted by it is the objective, or real. To think of an object as real is to think of it as being a case of the law. Natorp takes the examples which have already been introduced by Cohen: the algebraic series and the generation of a curve⁴⁵⁰. As matter itself does not contain any

⁴⁴⁹ Natorp. P. LM, p. 9.

⁴⁵⁰ As Hernán Pringe explains, for Cohen, the introduction of the infinitesimal calculus came to solve these issues.:"...according to Cohen, the history of infinitesimal calculus shows that there are three fundamental problems that the notion of infinitesimal enables us to solve. Firstly, the geometrical problem of tangents; secondly, the algebraic problem of series and, finally, the dynamical problem of velocity and acceleration."

principle to act, to establish an identical subject of movement, it is necessary to introduce a principle to guarantee this identity. Leibniz showed that "an identical subject of any movement cannot exist without a formal principle of force", and "the immutable unity of the changing states of the same subject, in which its entire sequence is expressed as an algebraic series"⁴⁵¹.

As we explained in Chapter 3, the relation is the fundamental concept of thinking⁴⁵². The act of thinking consists in establishing relationships, and the number is the first expression of this procedure⁴⁵³. In the algebraic series, every number can be conceived as a member. Every member can be defined by the position that it occupies. Indeed, the series is compounded by the members but, the members do not precede the relation they have among themselves. The law of the series determines the nature of the members and the relations among each other. Every member is defined by the position it assumes. The parts cannot precede the whole, as Kant explained the concept formation. Nor can the members be obtained by abstraction from any previous given data. In this case, parts and whole arise simultaneously⁴⁵⁴. Besides, every member can be considered

Pringe, H., 2020b, p.142. Also, Giovanelli, M, 2011 pp. 213ss..According to Marco Giovanelli, Natorp's account of "infinitesimal method" is completely different from Cohen's conception. He considers that "Natorp puts forward a conception of the "infinitesimal method" that is actually very far from Cohen's." Giovanelli, 2011, p. 215. I consider that while in Cohen, the core of the argument is the problem of the infinitesimals (Cf. Pringe, H., 2020b, esp., p.276), in Natorp, the basis of his proposal is the broader problem of the theory of the concept formation. This aspect of the Neo-Kantian approach to Leibniz has not been sufficiently highlighted. In general, scholars focus on the problem of the infinitesimal calculus. Cf. Holzhey, Helmut. 1986, Giovanelli, Marco 2011, Scott, Edgard, 2021. Indeed, this was the core of the Cohenian reading. However, I think that Natorp's reading of Leibniz makes emphasis in the theory of concept formation and not in the problem of infinitesimals.

⁴⁵¹ "denn diese bedeutet nichts weiter als diejenige unveränderliche Einheit der wechselnden Zustände desselben Subjects, worin deren ganze Folge ausgedrückt ist wie eine algebraische Reihe in ihrer Formel oder wie alle Punkte einer Curve in der Gleichung, welche die Natur oder / das Gesetz der Curve ausdrückt. So verhält es sich ja thatsächlich schon bei jeder auch bloss derivativen Kraft; so sagen wir, es folge aus der Natur eines in einer gegebenen Graden mit gegebener Geschwindigkeit bewegten Körpers, dass er, von Störungen abgesehn, in gewisser Zeit einen gewissen Punkt der Graden erreicht…" Natorp. P., LM, p. 9.

⁴⁵² "Aber der wahrhaft letzte Grundbegriff des mathematischen und alles strengen Denkens überhaupt ist vielmehr die Relation. Es ist Täuschung, dass man die Termini voraus haben könnte, um erst aus ihrem Zusammentritt die Relation hervorgehen zu lassen. Mit Recht fragte bereits Plato: Waren die zwei etwa nicht zwei, bevor man sie zusammenthat? Mathematik hat Überhaupt nichts zu thun, sie hat nur zu betrachten, und zwar zuletzt nichts anderes als Relationen. Die Relata sind erst gesetzt durch die Relation als deren Termini." Natorp. P., EGM, p. 3

⁴⁵³ Numbering is the first expression of thinking. This point was developed in chapter 3.

⁴⁵⁴ "Darin liegt nun aber der Hinweis auf ein logisches Moment; das in der Zahl von Anfang an schlummerte und doch bis dahin tief versteckt blieb; das in seiner fundamentalen Bedeutung für die Denkschöpfung der Zahl überhaupt von den Arithmetikem erst verhältnismäßig spät beachtet worden ist; nämlich jenes logische Moment, dem Kant den Name der "Relation" beilegt, welches in Wahrheit aber vielmehr eine eigene Relation von Relationen darstellt. Sein genauer Ausdruck in der Sprache der Arithmetik ist die Funktion. Die Große als Veränderliche enthüllt ihre eigentliche Bedeutung erst, sofern dabei mitgedacht wird an eine gesetzliche Beziehung, gemäß welcher eine Wertreihe einer anderen von Glied zu Glied korrespondiert Nicht die Größe ist veränderandlich; die Größe als das Wiegroß muß vielmehr fest bleiben, und die Größe

a whole by themselves, and contain a multiplicity in it. The law of formation guarantees that the relation among the terms is always the same. In all the different variations of the relations among the terms, the qualitative unity is conserved. The qualitative unity of the law subsists. For this reason, there is no preeminence of the qualitative relation over the quantitative, as the law expresses the "unlimited possibility of composition and division" of the terms. The continuity required is on the ground of the possibility of thinking of positing relations⁴⁵⁵. This is the way in which the concepts are made. A priori concepts, such as mathematical concepts are generated as operations of thinking. They are functional concepts and not thing-concepts⁴⁵⁶. This is a clarification of the notion of concept as rule that the Kantian system required.

This is a point in common between Leibniz and Natorp. Leibniz and Natorp agree on the impossibility of empty concepts. In an article of 2005, "Gedanken ohne Inhalt sind leer "457, Mario Caimi shows that one of the innovations of the Kantian proposals was the introduction of the possibility of empty concepts. For Leibnizian rationalism, concepts have a content per se. On the contrary, according to Kant, pure concepts are empty. Mario Caimi exhibits that this is a novelty of the Kantian system: the possibility of empty concepts. I would like to suggest that this is another agreement between Leibniz and Neo-Kantians against Kant. As we explained in Chapter 1, for Leibniz, the difference between intuitive cognition and intellectual cognition is a question of degree. The representations of the sensibility and understanding have the same root. More precisely, concepts and intuitions do not come from different origins, but they have the same source. The difference between these two types of perception is the degree they achieve in the determination of the object. Actually, they are different degrees of the same function. For Neo-Kantianism, concepts are functions, modes of relations. The relation is introduced with the relata. There is no unity without a content and no content can be conceived unless it is thought under a unity. Thus, both for Neo-Kantians and Leibniz, there are no empty concepts. The problem of giving content to the concepts arises as a result of this novelty of the Kantian system: the possibility of empty concepts⁴⁵⁸. For Kant, the relation between

als Kontinuum bedeutet nur die Allheit der Werte je unter einem gegebenen Gattungsbegriff; sie ist die Bedingung der Veränderlichkeit, aber ist selbst nicht veränderlich." Natorp, P., LGEW, p.202.

⁴⁵⁵ "Kontinuität ist ein so ursprüngliches, unverbrüchliches Gesetz des Denkens, dass überhaupt irgendwelche Diskretion sich nur als Diskretion eines Kontinuums will denken lassen. Also gibt is für das reine Denken das Kontinuum der Beziehungssine oder Richtungen ebenso wie das Kontinuum der Werte." Natorp, P., LGEW, p.237

 ^{456 &}quot;die mathematischen Begriffe Funktionsbegriffe, nicht Dingbegriffe sind." Natorp, P., LGEW, p. 144.
 457 Caimi, M., 2005

⁴⁵⁸ Cf. Caimi, M., 2005, esp.142ss.

the representation and what is real is no longer grounded on the possibility of a complete analysis but on the possibility of giving content to concepts, which are empty merely by themselves. The introduction of the possibility of empty concepts comes along with the requirement of an external element to give content to the conceptual representations: intuition. According to Leibniz and Neo-Kantians, every concept has content. For Leibniz, as the concept is always composed of simple elements, it is never empty. An empty concept is not truly a concept but a mere notion, a chimera. There are not empty concepts but those that contain a contradiction. For the Leibnizian conception, all non-contradictory concepts have content and then, all knowledge can arise from them. According to Neo-Kantians, the function introduces the relation and the relata at the same time. As it was for Leibniz, for them too, concepts can never be empty and, therefore, there is no necessary reference of concepts to intuition to have content. Leibniz and Neo-Kantians agree on the impossibility of empty concepts.

Conclusion

In the first chapter, we studied the Kantian distinction between intuition and concepts. We exhibited that Kant inherited the definition of these notions. Concepts were defined as abstractive representations. Then, the question arose: how can essentially universal representations be related to singular objects? Kant concluded that there can only be fully determined knowledge as intuitions. More specifically, Kant argued that space and time are forms of intuition. The problem of the incongruent counterparts led to this result. However, now we see that the problem arose at first because of the generic definition of concepts. Indeed, space and time are not generic concepts, they are not internal properties that can be abstracted from things. However, neither are the pure concepts of the understanding nor the mathematical concepts. Then, when Kant concluded that space and time are not concepts, he should have concluded that they are not formed as empirical concepts. Kant defined concepts under the model of empirical concepts. As we showed in Chapter 1, this was the model of the Aristotelian-scholastic definition. We showed the insufficiency of this model to understand the notion of concepts as functions. The new theory of the concept formation is much more consistent with the Kantian proposal and explains the operative of non-empirical concepts, such as

mathematical concepts or even pure concepts of the understanding. In Chapter 2, we exhibited that neither the subjectivist perspective nor logicism could give a satisfactory answer to the Kantian question. Kant held that one of the central problems of knowledge was resumed in the following question: "... on what foundation rests the relationship of what we call representation in us with the object?" Kant answered this question by arguing that human beings need concepts and intuition. Kant believes our knowledge may relate to objects only by means of intuition. We exhibited that for Natorp, the question of how cognition may refer to the object requires rethinking the method of philosophy. The reformulation of the notions of intuition and concept is necessary for the accomplishment of a consistent idealism, for which intuition is a task of thinking and the concepts are functions and not mere processes of abstraction. We exhibited that the answer to the Kantian question relies on the exhibition of the functional character of concepts.

This investigation has made two main advances in relation to the existing studies on the Neo-Kantian interpretation of the distinction between intuitions and concepts. First, we have exhibited the positive role that the notion of intuition plays within Paul Natorp's system. Second, we have clarified the relation between his conception of the distinction between intuitions and concepts and the problem of method.

As we pointed out at the beginning of the investigation, the commentators agree that the pillar of the Neo-Kantianism proposal lies in overcoming the distinction between intuitions and concepts. Many researchers have recognized this aspect of the Neo-Kantian proposal. Éric Dufour, Marco Giovanelli, Reiner Munk, Rudolf Malter, Helmut Holzhey, Thomas Mormann, Christian Krijnen, Nicolas Warren, Hernán Pringe, Massimo Ferrari, among others, highlighted that the Neo-Kantian program is grounded on the conception that thinking can create both the singular and the universal representation 1459. Neo-Kantian scholars agree on the fact that "intuition is ultimately to be reduced to thinking" 1460. Certainly, Paul Natorp, one of the main representatives of the school, considers that accomplishing a genuine idealism requires clarifying the role of intuition in the process of knowing, by exhibiting how thinking produces nature as a whole. Indeed, one of the pillars of the Marburg Neo-Kantian "return to Kant" relies on a new approach to the dualism between intuitions and concepts. As commentators exhibited, for Natorp

 ⁴⁵⁹ Ferrari, M., 1997, p. 118.Dufour, É.; 2003, p.90. Giovanelli, M., 2005, p.116. Munk, R. 2005, p. 8.
 Holzhey, H., 2010, p.25. Giovanelli, M., 2011, p. 217. Mormann, T.; 2013, p. 241. Malter, R., 1981, p. 539.
 Krijnen, C., 2013, p. 168. Warren, N., 2015, p.90. Pringe, H., 2020, pp.137 ss.
 ⁴⁶⁰ Kim, A., 2015, p. 48.

intuitions and concepts do not have their origin in different faculties of the human mindas Kant thought-, but they are modes of thinking. However, the studies have failed to exhibit the positive role that the intuitive moment has in the Neo-Kantian theory of knowledge. This investigation has shown that even when for Natorp the Kantian distinction between intuition and concepts needs to be revised, there still remains a positive role of intuition: exhibiting the limits of human thinking. As it was clarified in chapter 5, for Natorp, the intuitive factor is a demand for knowledge. The Kantian mistake is to turn the demand into an accomplishment of the demand. The introduction of the intuition in the system is a reminder that the complete determination of objectivity can never be achieved. Natorp accepts the introduction of intuition as a demand. Therefore, our investigation has shown the very positive role of intuition. This role has not been sufficiently recognized by scholars. This investigation makes an improvement in this direction by exhibiting both moments in Natorp's approach: a) The sovereignty of thinking in the creation of singularity and, b) the positive role of intuition.

Second, we exhibited that the reformulation of the distinction between intuitions and concepts comes along with the introduction of the task of accomplishing a consistent idealism. The problem of the distinction between intuitions and concepts is introduced with the problem of the possibility of the prosecution of a genuine idealism. Rethinking the method of philosophy ended up in a new way to understand the distinction between intuitions and concepts. We proved our thesis with two main arguments. First, we showed that the new distinction between intuitions and concepts is based on the criticism of psychologism and logicism. We explained how Natorp reformulates the Kantian distinction between intuitions and concepts arguing against these tendencies. Psychologism and logicism misunderstood the problem of the relation of concepts and intuitions due to methodological errors. Second, we exhibited that the clue to understanding how Natorp conceives the problem of the relation between intuition and concepts rests on his conception of the method of philosophy. In this way, we challenged the most canonical reading of Neo-Kantianism. Existing scholarship considers that the Neo-Kantian method consists in departing from the fact of science. Helmut Holzhey, Jünger Stoltenberg, Frederick Beiser, Alan Kim, Eric Dufour⁴⁶¹, among others, assume that the Neo-Kantian transcendental method takes the science of nature as a point of departure in the investigation. According to this conception, the Neo-Kantian method

⁴⁶¹ Dufour, E., 2003, Kim, A., 2015, p. 48, Holzhey, H., 2010, p. 34. Stolzenber, 2010, p. 133. Beiser, F., 2014, p.466., p. 23.

"...begins with 'the fact of science', that is, the acceptance of mathematical physics as a datum; it then explains how that fact is possible, specifying the conditions for a mathematical knowledge of nature" Holzhey, one of the most important scholars within the Neo-Kantian studies, considers the concept of "category" to have only a historiographical function that only makes sense when Natorp refers to the Kantian system. According to him: "In his book *Die logischen Grundlagen der ecxakten Wissenschaften* of 1910, Paul Natorp employed the concept of 'category' only in a historical sense when referring to Kant." It is interesting to note that assuming that Natorp departed from the fact of sciences, scholars have neglected to explain the role of Natorp's deduction of categories. This is clearly seen in the reviews of Natorp's *Logishen Grundlagen der exaten Wissenshaften*. Morris Cohen holds in his review:

In the second chapter, we have a modernized deduction of the categories. The dry bones of the Kantian framework receive a great deal of flesh and blood. In the end, however, they turn out to be our old friends the twelve, marching in four groups of three each. If it were not for the fact that students at our colleges do not read German, this chapter could profitably be recommended to those who are reading Kant for the first time and who generally cannot grasp what these categories are about.⁴⁶⁴

In our thesis, we proposed a new approach to the problem. The point of departure to overcome the heterogeneity between intuitions and concepts relies on the deduction of categories. In the deduction of categories, Natorp showed that the object is constructed *in* and *by* thinking. The Kantian question of how our representation can legitimacy relate to the objects should be reformulated in terms of how thinking is able to produce objectivity. Thinking creates objectivity by the fundamental producers: the categories. Our investigation makes an advance in this direction. We explain not only the relation of the problem of method to the reformulation of the distinction between intuition and concepts, but we also showed that overcoming the heterogeneity between these two modes of representation demands such a method. We exhibited that only by this method, it is possible to overcome the heterogeneity between intuitions and concepts and, therefore, to

⁴⁶² Beider, F., 2014, p. 498.

⁴⁶³ Holzhey, H., 2005, p. 70.

⁴⁶⁴ Cohen, M., 1911, p. 694.

achieve a genuine idealism. The way in which Natorp carries out this task has not been developed either by his contemporary readers or by contemporary scholars. We consider that our investigation makes an interesting contribution in this direction.

Samenvatting

In het eerste hoofdstuk hebben we het Kantiaanse onderscheid tussen aanschouwingen en begrippen bestudeerd. We lieten zien dat Kant de definitie van deze begrippen overnam van eerdere denkers. Concepten werden gedefinieerd als abstraherende voorstellingen. Vervolgens rees de vraag: hoe kunnen in wezen universele voorstellingen in verband worden gebracht met singuliere objecten? Kant concludeerde dat er alleen sprake kan zijn van volledig bepaalde kennis als aanschouwingen. Meer in het bijzonder stelde Kant dat ruimte en tijd vormen van aanschouwing zijn. Het probleem van de incongruente tegendelen leidde tot dit resultaat. Nu zien we echter dat het probleem aanvankelijk ontstond door de generieke definitie van begrippen. Ruimte en tijd zijn inderdaad geen generieke begrippen, het zijn geen interne eigenschappen die geabstraheerd kunnen worden van de dingen. Het zijn echter ook niet de zuivere begrippen van het verstand of de wiskundige begrippen. Toen Kant dus concludeerde dat ruimte en tijd geen concepten zijn, had hij moeten concluderen dat ze niet als empirische concepten worden gevormd. Kant definieerde concepten volgens het model van de empirische concepten. Zoals we in hoofdstuk 1 hebben laten zien, was dit het model van de Aristotelisch-scholastieke definitie. Wij hebben aangetoond dat dit model ontoereikend is om de notie van begrippen als functies te begrijpen. De nieuwe theorie van de begripsvorming is veel meer in overeenstemming met het Kantiaanse voorstel en verklaart de werking van nietempirische concepten, zoals wiskundige concepten of zelfs zuivere concepten van het verstand.

In hoofdstuk 2 hebben we laten zien dat noch het subjectivistische perspectief noch het logicisme een bevredigend antwoord konden geven op de Kantiaanse vraag. Kant stelde dat een van de centrale problemen van kennis opgenomen werd in de volgende vraag: "... op welk fundament berust de relatie van wat wij representatie in ons noemen met het object?" Kant beantwoordde deze vraag door te stellen dat de mens concepten en aanschouwing nodig heeft. Volgens Kant kan onze kennis zich alleen door middel van aanschouwing verhouden tot objecten. Wij stellen vast dat voor Natorp de vraag hoe cognitie kan verwijzen naar het object een opnieuw doordenken van de filosofische methode vereist. De herformulering van de begrippen intuïtie en concept is noodzakelijk voor de verwezenlijking van een consistent idealisme, waarvoor de intuïtie een taak van het denken is en de concepten functies zijn en niet louter processen van abstractie. Wij stelden vast dat het antwoord op de Kantiaanse vraag berust op het tonen van het functionele karakter van begrippen.

Dit onderzoek heeft twee belangrijke vorderingen gemaakt ten opzichte van de bestaande studies over de Neo-Kantiaanse interpretatie van het onderscheid tussen aanschouwingen en begrippen. Ten eerste hebben wij de positieve rol aangetoond die het begrip aanschouwing speelt binnen het systeem van Paul Natorp. Ten tweede hebben wij de relatie verduidelijkt tussen zijn opvatting van het onderscheid tussen aanschouwingen en concepten en het probleem van de methode.

Zoals wij aan het begin van het onderzoek aangaven, zijn de commentatoren het erover eens dat de pijler van het Neo-Kantiaanse voorstel rust op het overwinnen van het onderscheid tussen aanschouwingen en concepten. Veel onderzoekers hebben dit aspect van het Neo-Kantiaanse voorstel herkend. Onder meer Éric Dufour, Marco Giovanelli, Reiner Munk, Rudolf Malter, Helmut Holzhey, Thomas Mormann, Christian Krijnen, Nicolas Warren, Hernán Pringe en Massimo Ferrari benadrukten dat het Neo-Kantiaanse programma gebaseerd is op de opvatting dat het denken zowel de singuliere als de universele voorstelling kan scheppen. Neo-Kantiaanse geleerden zijn het erover eens dat "aanschouwing uiteindelijk tot denken moet worden gereduceerd". Zeker Paul Natorp, een van de belangrijkste vertegenwoordigers van de school, is van mening dat voor een werkelijk idealisme de rol van de aanschouwing in het kenproces moet worden verduidelijkt, door aan te tonen hoe het denken de natuur als geheel voortbrengt. Een van de pijlers van de Marburgse Neo-Kantiaanse "terugkeer naar Kant" berust inderdaad op een nieuwe benadering van het dualisme tussen aanschouwingen en concepten. Zoals commentatoren aantoonden, hebben voor Natorp aanschouwingen en concepten niet hun oorsprong in verschillende vermogens van de menselijke geest - zoals Kant dacht - maar zijn het denkwijzen. De studies hebben echter niet de positieve rol aangetoond die het aanschouwingsmoment in de Neo-Kantiaanse kennistheorie speelt. Dit onderzoek heeft aangetoond dat zelfs wanneer voor Natorp het Kantiaanse onderscheid tussen aanschouwing en concepten moet worden herzien, er nog steeds een positieve rol van de aanschouwing overblijft: het tonen van de grenzen van het menselijk denken. Zoals in hoofdstuk 5 is verduidelijkt, is de aanschouwelijke factor voor Natorp een vraag naar kennis. De Kantiaanse vergissing is de vraag om te zetten in een vervulling van de vraag. De introductie van de aanschouwing in het systeem herinnert eraan dat de volledige bepaling van de objectiviteit nooit bereikt kan worden. Natorp aanvaardt de invoering van de aanschouwing als een eis. Ons onderzoek heeft dus de zeer positieve rol van de aanschouwing aangetoond. Deze rol is nog onvoldoende erkend door geleerden. Dit onderzoek brengt een verbetering in deze richting door beide momenten in Natorp's

benadering zichtbaar te maken: a) de soevereiniteit van het denken in het creëren van singulariteit en, b) de positieve rol van aanschouwing.

Ten tweede hebben wij aangetoond dat de herformulering van het onderscheid tussen aanschouwingen en concepten gepaard gaat met de invoering van de taak om een consistent idealisme te verwezenlijken. Het probleem van het onderscheid tussen aanschouwingen en concepten wordt geïntroduceerd met het probleem van de mogelijkheid van een echt idealisme. Een heroverweging van de filosofische methode leidde tot een nieuw begrip van het onderscheid tussen aanschouwingen en concepten. Wij hebben onze stelling met twee hoofdargumenten bewezen. Ten eerste toonden wij aan dat het nieuwe onderscheid tussen aanschouwingen en begrippen gebaseerd is op de kritiek op het psychologisme en het logicisme. Wij hebben uitgelegd hoe Natorp het Kantiaanse onderscheid tussen aanschouwingen en concepten herformuleert en daarbij deze tendensen bestrijdt. Psychologisme en logicisme hebben het probleem van de relatie tussen concepten en aanschouwingen door methodologische fouten verkeerd begrepen. Ten tweede hebben wij aangetoond dat de sleutel tot het begrijpen van hoe Natorp het probleem van de relatie tussen aanschouwing en begrippen opvat, berust op zijn opvatting van de methode van de filosofie. Op die manier hebben wij de meest canonieke lezing van het Neo-Kantianisme in twijfel getrokken. Volgens de bestaande wetenschap bestaat de Neo-Kantiaanse methode erin te beginnen met het gegeven van de wetenschap. Onder anderen Helmut Holzhey, Jünger Stoltenberg, Frederick Beiser, Alan Kim en Ëric Dufour gaan ervan uit dat de Neo-Kantiaanse transcendentale methode de natuurwetenschap als uitgangspunt neemt in het onderzoek. Volgens deze opvatting begint de Neo-Kantiaanse methode "...met 'het feit van de wetenschap', dat wil zeggen de aanvaarding van de mathematische fysica als een gegeven; vervolgens wordt uitgelegd hoe dat feit mogelijk is, waarbij de voorwaarden voor een mathematische kennis van de natuur worden gespecificeerd". Holzhey, een van de belangrijkste geleerden binnen de Neo-Kantiaanse studies, vindt dat het begrip "categorie" slechts een historiografische functie heeft die alleen zin heeft als Natorp naar het Kantiaanse systeem verwijst. Hij stelt: "In zijn boek Die logischen Grundlagen der exakten Wissenschaften uit 1910 heeft Paul Natorp het begrip 'categorie' alleen in historische zin gebruikt wanneer hij naar Kant verwijst." Het is interessant op te merken dat in de veronderstelling dat Natorp begon met het feit van de wetenschappen, geleerden verzuimd hebben de rol van Natorp's deductie van categorieën te verklaren. Dit blijkt duidelijk uit de besprekingen van Natorp's Logischen Grundlagen der exakten Wissenschaften. Morris Cohen stelt in zijn recensie:

In het tweede hoofdstuk zien we een gemoderniseerde deductie van de categorieën. De kale beenderen van het Kantiaanse kader krijgen veel vlees en bloed. Uiteindelijk blijken het echter onze oude vrienden de twaalf te zijn, marcherend in vier groepen van elk drie. Ware het niet dat de studenten aan onze hogescholen geen Duits lezen, dan zou dit hoofdstuk op nuttige wijze kunnen worden aanbevolen aan degenen die Kant voor het eerst lezen en die doorgaans niet kunnen bevatten waar deze categorieën over gaan.

In ons proefschrift hebben wij een nieuwe benadering van het probleem voorgesteld. Het uitgangspunt om de heterogeniteit tussen aanschouwingen en concepten te overwinnen berust op de deductie van categorieën. In de deductie van categorieën liet Natorp zien dat het object wordt geconstrueerd in en door het denken. De Kantiaanse vraag hoe onze voorstelling zich legitiem kan verhouden tot de objecten moet worden geherformuleerd in termen van hoe het denken in staat is objectiviteit te produceren. Het denken creëert objectiviteit door de fundamentele producenten: de categorieën. Ons onderzoek gaat in die richting. Wij verklaren niet alleen de relatie van het probleem van de methode tot de herformulering van het onderscheid tussen aanschouwing en begrippen, maar wij hebben ook aangetoond dat het overwinnen van de heterogeniteit tussen deze twee wijzen van voorstelling een dergelijke methode vereist. Wij toonden aan dat het alleen door deze methode mogelijk is de heterogeniteit tussen aanschouwingen en concepten te overwinnen en dus tot een echt idealisme te komen. De manier waarop Natorp deze taak uitvoert is noch door zijn hedendaagse lezers, noch door hedendaagse geleerden ontwikkeld. Wij zijn van mening dat ons onderzoek een interessante bijdrage in deze richting levert.

Bibliography

- Aguilar Martínez, Ernesto; "El Platón de Natorp", Bajo Palabra. Revista de Filosofía II Época, Nº 9 (2014):223-232.
- Allison, Henry; Kant's Transcendental Idealism. An Interpretation and Defense; Revised and Enlarged Edition, Yale University Press, New Haven and London, 2004.
- Anderson, Lanier; "Neo-Kantianism and the Roots of Anti-Psychologism",
 British Journal for the History of Philosophy, 13: 2, 2010, pp. 287-323.
- Beck, Jakob Sigismund; Erläuternder Auszug aus den kritischen Schriften des Prof. I. Kant, auf Anrathen desselben, Erster Band, Riga 1793.
- Beck, Lewis; "Two ways of Reading Kant's Letter to Herz: Comments on Carl",
 in: Förster, Eckart (ed.), Kant's Transcendental Deductions: The Three Critiques
 and the Opus Postumum, Stanford University Press, Stanford, 1989, pp. 21 26.
- Beck, Lewis; *Early German Philosophy: Kant and His Predecessors*, Harvard University Press Cambridge, 1996.
- Beiser, Frederick, *After Hegel: German philosophy, 1840-1900*, Princeton University Press, Princeton, 2014.
- Beiser; Frederick C., The Genesis of Neo-Kantianism, 1796-1880, Oxford University Press, Oxford, 2014.
- Beneke, Firedrich Eduard; Kant und die philosophische Aufgabe unserer Zeit. eine Jubeldenkschrift auf die Kritik der reinen Vernunft. Berlin, Posen Bronberg, Mittler, 1832.
- Beneke, Friedrich Eduard, *Die Philosophie in ihrem Verhältnisse zur Erfahrung,* zur Spekulation und zum Leben, Berlin, Posen Bronberg, Mittler, 1833.
- Beneke, Friedrich Eduard; *System der metaphysik und religionsphilosophie*, F. Dümmler, Berlin, 1840.
- Beneke, Friedrich Eduard; *System der Logik als Kunstlehre des Denkens. Erster Teil*, Ferdinand Dümmler, Berlin, 1842.
- Bennett, Jonathan, "The Difference Between Right and Left", *American Philosophical Quarterly*, Vol. 7/3, 1970, pp. 175-191.
- Beneke, F., Dressler, J. G., Raue, G; The Elements of Psychology: On The Principles Of Beneke, Stated And Illustrated In A Simple And Popular Manner, James Parker and Co, Oxford and London, 1871.

- Bérubé, Camille; *La connaissance de l'individuel au moyen âge*, Presses Universitaires de France, Paris, 1964.
- Brandt, Francis Burke; *Friedrich Eduard Beneke, the man and his philosophy:* an introductory study, Columbia College, New York, 1895.
- Boler, John F.; "Intuitive and abstractive cognition", in: *The Cambridge History of Later Medieval Philosophy*; N. Kretzmann et al (eds.), Cambridge University Press, Cambridge, 1982, pp.460-478.
- Brandt, Reinhard "Transzendentale Ästhetik, §§1-3 (A19/B33-A30/B45)", Klassiker Auslegen Interpreting Philosophical Classics, Otfried Höffe (Ed.), Berlín, 1998.
- Burke, Brandt; Friedrich Eduard Beneke, the Man and His Philosophy: An Introductory Study, Macmillan, New York, 1895.
- Buroker, Jill Vance; *Space and Incongruence: The Origin of Kant's Idealism,* Springer, Dordrecht, 1981.
- Caffarena, José Gómez; "Estudio preliminar", in: Kant Immanuel, Principios formales del mundo sensible e inteligible (Disertación de 1770), trad. Caffarena José Gómez, Consejo superior de investigaciones científicas, Madrid, 1996.
- Caimi, Mario; "La sensación en la Crítica de la razón pura", en: Cuadernos de filosofía, Buenos Aires, 1983, pp. 109 – 119.
- Caimi, Mario, "Introducción y notas", to: Kant, I., *Prolegomena*, trad. Caimi,
 Mario; Istmo, Madrid, 1999, pp. 7-10.
- Caimi, Mario; "About the Argumentative Structure of the Transcendental Aesthetic", *Studi Kantiani* 9, Pisa, 1996, pp. 27-46.
- Caimi, Mario; "Gedanken ohne Inhalt sind leer", in: *Kant-Studien*, 96, Berlin, 2005, pp. 135-146
- Caimi, Mario; "Introducción", to: Descartes, R., Discurso del método, Trad.
 Mario Caimi, Colihue, Buenos Aires, 2006.
- Caimi, Mario; Leçons sur Kant. La déduction transcendantale dans la deuxième édition de la Critique de la raison pure, Paris, Publications de la Sorbonne, 2007.
- Caimi, Mario; "Introducción", a: Kant, I. Crítica de la razón pura, Colihue, Buenos Aires, 2009.
- Carl, Wolfgang; "Kant's First Drafts of the Deduction of the Deduction of the Categories", in: Förster, Eckart (ed.), *Kant's Transcendental Deductions: The*

- *Three Critiques and the Opus Postumum*, Stanford University Press, Stanford, 1989, pp. 3 20.
- Carvajal, Julián; "Finitud radical y moralidad a la luz del debate Heidegger-Cassirer sobre el kantismo", *Fragmentos* de *Filosofía*, 5, 2007, pp. 239-262.
- Cassirer, Ernst; The Problem of Knowledge. Philosophy, Science, and History since Hegel, William H. W., Charles W. Hendel (trans.), Yale University Press, New Haven, 1950.
- Cassirer, Ernst; Kants Leben und Lehre, Bruno Cassirer, Berlin, 1921.
- Cazeaux, Clive; "Metaphor and Heidegger's Kant", *The Review of Metaphysics*, Vol. 49, No. 2 (Dec., 1995), pp. 341-364.
- Chenet, François Xavier; *L'assise de l'ontologie critique: L'esthétique transcendantale*, Presses Universitaires de Lille, Paris, 1994.
- Cleve van, James, Frederick, Robert (eds.); The Philosophy of Right and Left. Incongruent Counterparts and the Nature of Space, Kluwer Academic Publishers, Dordrecht, 1991.
- Cleve van, James, Problems from Kant, Oxford University Press, New York/Oxford, 1999.
- Cohen, Hermann, *System der Philosophie: Logik der reinen Erkenntniss*, Bruno Cassirer, Berlin, 1902.
- Cohen, Hermann, Kommentar zu Immanuel Kants Kritik der reinen Vernunft, Verlag der Dürrschen Buchhandlung, Leipzig, 1907.
- Conant, James. "Why Kant Is Not a Kantian", Philosophical Topics, Vol. 44, n.1. Spring, 2016.
- Dale, Jacquette (Ed.); Philosophy, Psychology, and Psychologism Critical and Historical Readings on the Psychological Turn in Philosophy, Kluwer Academic Publishers, Dordrecht, 2003.
- Day, Sebastian; Intuitive Cognition A Key To The Significance Of The Later Scholastics, Franciscan Institute Publications, New York, 1947.
- Detlefsen Michael, "Philosophy of mathematics in the twentieth century" in: *Philosophy of Science, Logic and Mathematics in the Twentieth Century*.
 Routledge History of Philosophy Volume IX, Stuart G. Shanker (ed), London and New York, Routledge, 2004.

- Dewalque, Arnaud; "La critique néokantienne de Kant et l'instauration d'une théorie conceptualiste de la perception", Dialogue, Vol. 49/03, 2010, pp 413 – 433.
- Dewalque, Arnaud; «Peut-on fondé sur la logique pure ? la controversée entre Natorp et Rickert », *Dialogue*, 46/01, 2014, pp.43-68.
- Dietmar H. Heidemann, "Anschauung überhaupt", In: Natur und Freiheit. Akten des XII. Internationalen Kant-Kongresses, V. L. Waibel, M. Ruffing and D. Wagner (Eds), De Gruyter, Berlin, 2019, pp. 743-760.
- Dressler, Johann Gottlieb; *Elements of Psychology on the Principles of Beneke*, James Parker, Oxford and London, 1871.
- Duarte Amaral, Lucas Alessandro, "Remarks on the theoretical context of Cassirer's philosophical project", *Argumentos*, ano 7, n. 13 - Fortaleza, jan./jun. 2015, pp. 247-255.
- Dufour, É. "Le statut du singulier: Kant et le néokantisme de l'École de Marbourg" Kant-Studien 93, 2002, pp 324–350.
- Dufour, Éric, Les Néokantiens. Valeur er vérité, Vrin, Paris, 2003.
- Dufour, Éric; "Négation et altérité chez Hermann Cohen et Paul Natorp", in: Fiorato, Pierfrancesco; Negation, Unendlichkeit und Andersheit im Neukantianismus, Königshausen & Neumann, 2009, pp. 29-46.
- Dufour, Éric, Paul Natorp. De la psychologie générale à la systématique philosophique, Vrin, Paris, 2010.
- Earman, John; "Kant, Incongruous Counterparts, and the Nature of Space and Space-Time" in: Cleve van, James, Frederick, Robert (eds.); *The Philosophy of Right and Left. Incongruent Counterparts and the Nature of Space*, Kluwer Academic Publishers, Dordrecht, 1991, pp. 131-149.
- Earman, John; "On the Other Hand...: A reconsideration of Kant Incongruent Counterparts, and Absolute Space" in: Cleve van, James, Frederick, Robert (eds.); *The Philosophy of Right and Left. Incongruent Counterparts and the Nature of Space*, Kluwer Academic Publishers, Dordrecht, 1991, pp. 235-255.
- Edgar, Scott. "Paul Natorp and the emergence of anti-psychologism in the nineteenth century", *Studies in History and Philosophy of Science*, Volume 39, Issue 1, March 2008, Pages 54-65.
- Edgar, Scott. "Leibniz's influence on Hermann Cohen's Interpretation of Kant", Kant e-Prints, Campinas, série 2, v. 16, n. 2, maio-ago. 2021, pp. 200-230.

- Falkenburg, Brigitte, "Intuition and Cosmology: The Puzzle of Incongruent Counterparts", in: Carson E., Huber R. (eds) *Intuition and the Axiomatic Method*. The Western Ontario Series in Philosophy of Science, vol 70. Springer, Dordrecht, 2006, pp. 157-180.
- Falkenstein, Lorne; "Kant's Account of Intuition", *Canadian Journal of Philosophy*, Vol. 21, n.2, June 1991, pp.165-193.
- Falkenstein, Lorne; Kant's intuitionism. A Commentary on the Transcendental Aesthetic, University of Toronto Press Incorporated, Toronto Buffalo London, 1995.
- Ferrari, Massimo; Introduzione a il Neocriticismo, Laterza, Roma, 1997.
- Förster, Eckart (ed.), *Kant's Transcendental Deductions: The Three Critiques and the Opus Postumum*, Stanford University Press, Stanford, 1989.
- Friedman, Michael, *Kant and the Exact Sciences. Cambridge*, London: Harvard University Press, 1995.
- Frege, Gottlob; Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens, Halle an der Saale: Verlag von Louis Nebert, 1879.
- Frege, Gottlob; Die Grundlagen der Arithmetik: Eine logisch-mathematische Untersuchung über den Begriff der Zahl. Breslau: W. Koebner, 1884. [Translation: J. L. Austin, 1974. The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number, 2nd ed. Blackwell.]
- Frege, Gottlob; *Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet*, Jena: Verlag Hermann Pohle, Band I, 1893.
- Frege, Gottlob; *Posthumous Writings*, trans. Peter Long and Roger White, H. Hermes, F. Kambartel and F. Kaulbach, Basil Blackwell, Oxford, 1979.
- Fronterotta, Francesco; «L'interprétation néo-kantienne de la théorie platonicienne des idées et son «héritage» philosophique ». In: Revue Philosophique de Louvain. Quatrième série, Tome 98, N°2, 2000. pp. 318-340.
- Gatto, Alfredo. "La maschera cartesiana: René Descartes nella Critica della ragion pura di Kant". Con-Textos Kantianos. International Journal of Philosophy, [S.l.], n. 5, p. 138-149, jun. 2017.
- Edel, Geert, "Cohen und die analytische Philosophie der Gegenwart," in R. Brandt and F. Orlik (eds.), *Philosophisches Denken Politisches Wirken: Hermann-Cohen-Kolloquium Marburg 1992*, Hildesheim: Georg Olms, 1993.

- Edel, Geert, "Der Zusammenhang der Systemteile in ihrer methodischen Verknüpfung." in: *Hermann Cohen und die Erkenntnistheorie*, Wolfgang Marx, Ernst Wolfgang Orth, Königshausen & Neumann, 2001.
- Gigliotti, Giana; "Beweis and Aufweis: Transcendental a priori and Metaphysical a priori in Cohen's Neo-Kantianism" in: Munk, Reiner (ed.), Hermanns Cohen Critical Idealism, Srpinger, Dordrecht, 2005, pp. 97- 131.
- Gilson, Etienne; Jean Duns Scot: introduction a ses positions fondamentales, Vrin, Paris, 1952.
- Giovanelli, Marco, "La distinzione kantiana tra 'concetto' e 'intuizione' e il problema della realta delle 'relazioni'", 10:113-146 (2005).
- Giovanelli, Marco, Reality and Negation Kant's Principle of Anticipations of Perception. An Investigation of its Impact on the Post-Kantian Debate, Springer, New York, 2011.
- Gloy, Karen; "Die Kantische Differenz von Begriff und Anschauung und ihre Begründung", *Kant-Studien*, 75, n. 1, 1984, pp. 1-37.
- González Porta, Mario Ariel, "Züruck zu Kant!", Dois pontos, Curitiba, São Carlos, vol. 2, n. 2, 2005 pp.35-59.
- González Porta, Mario Ariel, "Frege y Natorp: Platonismos, Antipsicologismos y Teorías de la Subjetividad", 2006.
- González Porta, Mario Ariel, "La teoría del número en Natorp y Cassirer", La ciencia de los filósofos, 1996, pp. 199-222.
- González Porta, Mario Ariel, Estudos Neokantianos, Edições Loyola, São Paulo,
 2011.
- González Porta, Mario Ariel "Prolegómenos a los "Prolegómenos" (Revisando la participación neokantiana en el Psychologismusstreit)", Kant e-Prints. Revista Internacional de filosofía, Publicações CLE, Campinas, serie 2, v.16, n.2, 2021, pp. 164-184.
- Hegel, G. W.F., *Glauben und Wissen*, Felix Meiner, Hamburg, 1986.
- Heidegger, Martin; Gesamtausgabe, Vittorio Klostermann, Frankfurt am Main, 1975-ss.
- Helmholtz; Hermann v., Die Tatsachen in der Wahrnehmung, Hirschwald, Berlin, 1879.
- Helmholtz; Hermann v., "Über das Verhältnis der Naturwissenschaften zur Gesamtheit der Wissenschaft", *Physikalische Blätter* 6/4, 1950, pp. 145-152.

- Hinske, Norbert; "Einleitung in die dritte Abteilung der Reihe Forschungen und Materialien zur deutschen Aufklärung (FMDA)" in Lambert-Index. Band 1: Stellenindex zu Johann Heinrich Lambert "Neues Organon I". In Zusammenarbeit mit Heinrich P. Delfosse (= FMDA III, 1) Stuttgart-Bad, Cannstatt, 1983, pp.V-XI.
- Hinske, Norbert; "Prefacio", en: Kant, Immanuel; Lógica; acompañada de una selección de "Reflexiones" del legado de Kant, María Jesús Vázquez Lobeiras (ed), Akal, D.L., Madrid, 2000.
- Hintikka, Jaakko; "Kantian Intuitions," *Inquiry* 15, 1972, pp. 341–5.
- Holzhey, Helmut, "Introduction" in: Hermann Cohen Critical Idealism, Munk Reinier, Springer, Dordrecht, 2005.
- Holzhey, Helmut, Die Leibniz-Rezeption im "Neu-Kantianismus" der Marburger Schule. Leibniz: Werk und Wirkung IV, Internationaler Leibniz-Kongress, 1986, pp. 289-300.
- Holzhey, Helmut, *Cohen und Natorp*, Schwabe, Basel/Sturrgart, 1986.
- Holzhey, Helmut; "Neo-Kantianism and Phenomenology: The Problem of Intuition", in: *Neo-Kantianism in Contemporary Philosophy*, Indiana University Press, Bloomington and Indianapolis, 2010, pp. 25-40.
- Jacquette, Dale (ed.); *Handbook of the Philosophy of Science. Philosophy of Logic*, Elsevier, North Holland, 2007.
- Jauernig, Anja; "Kant's Critique of the Leibnizian Philosophy: Contra the Leibnizians, but Pro Leibniz" in: *Kant and the Early Moderns*, D. Garber and B. Longuenesse (eds.), Princeton, Princeton University Press, 2008, pp. 41-63.
- Jourdain, Philip E. B., "Reviewed Work: *Die logischen Grundlagen der exakten Wissenschaften.* by Paul Natorp", *Mind*, Vol. 20, No. 80 (Oct., 1911), pp. 552-560.
- Kant, I Immanuel; Gesammelte Schriften, Hrsg.: Bd. 1-22. Prussishe Akademie der Wissenschaften Bd. 23. Deutsche Akademie der Wissenshaften zu Berlin, ab Bd. 24. Akademie der Wissenshaften zu Göttingen, 1900ss.
- Kant, Immanuel; Correspondence, Zweig, Arnulf (Trad.), Cambridge University Press, Cambridge, 1999.
- Kemp Smith, Norman; *A Commentary to Kant's Critique of Pure Reason*, Macmillan and co., London, 1918.
- Kemp Smith, Norman; "The Paradox of Incongruous Counterparts" in: Cleve van, James, Frederick, Robert (eds.); *The Philosophy of Right and Left*.

- *Incongruent Counterparts and the Nature of Space*, Kluwer Academic Publishers, Dordrecht, 1991, pp. 43 47.
- Kim, Alan; "Neo-Kantian Ideas of History" in: Warren, Nicolas; Staiti, Andrea;
 (comps.), New Approaches to Neo-Kantianism, Cambridge University Press,
 Cambridge, 2015.
- Kitcher, Patricia; Kant's Transcendental Psychology, Oxford University Press, Oxford, 1990.
- Kolb, Daniel C., "Thought and Intuition in Kant's Critical System" in: *Journal of the History of Philosophy*, Vol. 24/2, 1986, pp. 223-24.
- Krijnen, Christian; "Der Wahrheitsbegriff im Neukantianismus", in: Die Geschichte des philosophischen Begriffs der Wahrheit Hsg. Markus Enders und Jan Szaif, Walter de Gruyter, Berlin, 2006.
- Krijnen, Christian; "Neokantismus" in: Bonk, Thomas (Hrsg.) *Lexikon der Erkenntnistheorie*, WBG, Darmstad, 2013.
- Krijnen Christian; "Transzendentaler Idealismus und empirischer Realismus" in:
 Christian Krijnen / Kurt Walter Zeidler (Hrsg.), Wissenschaftsphilosophie im Neukantianismus. Ansätze Kontroversen Wirkungen, Königshausen & Neumann, Würzburg, 2014, pp. 11-52.
- Külpe, Oswald; *Immanuel Kant: Darstellung und Würdigung*, Teubner, Leipzig, 1907.
- Külpe, Oswald, Einleitung in die Philosophie, Hs. Messer, Leipzig, 1921.
- Külpe, Oswald, *Introducción a la filosofía*, trad. Carlos Jesinghaus, Ed. Poblete,
 Buenos Aires-Madrid, 1931.
- Kusch, Martin; Psychologism. A case Study in the Sociology of Philosophical Knowledge, London, 1995.
- Kusch, Martin; *Psychologism: The Sociology of Philosophical Knowledge*, Taylor & Francis e-Library, 2005.
- Laks, André; "Plato Between Cohen and Natorp: Aspects of the neo-Kantian Interpretation of the Platonic Ideas (Manuscript), P. Natorp, *Plato's Theory of Ideas* (edited with an introduction by V. Politis, trans. by V. Politis and J. Connolly), Academia-Verlag, Sankt Augustin, International Plato Studies, vol. 16, 2004, p. 453-483.

- Leibniz, Gottfried Wilhelm. *Die Leibniz-Handschriften der Königlichen Öffentlichen Bibliothek zu Hannover*. ed. Eduard Bodemann (Hildesheim: Olms, 1966). Reprint, Hannover, 1989.
- Leibniz, Gottfried Wilhelm. *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*. ed. Carl Immanuel Gerhardt (Berlin: Weidmann, 1875).
- Lema-Hincapié; Andrés; "La carta de Kant a Marcus Herz: 21 de febrero 1772"; *Praxis Filosófica;* Nueva serie, No. 18 (Ene.-Jun.), 2004, pp. 59-78.
- Lembeck, Karl-Heinz; "Begründungsphilosophische Perspektiven: Husserl und Natorp über Anschauung", *Phänomenologische Forschungen*, *änomenologische Forschungen*, 2003, pp. 97-108.
- Longuenesse, Béatrice, Kant and the Capacity to Judge: Sensibility and Discursivity in the Transcendental Analytic of the Critique of Pure Reason, Trad.
 Wolfe, C., Princeton University Press, Princeton and Oxford, 1998.
- Malter, Rudolf and Beck, Hamilton; "Main Currents in the German Interpretation
 of the Critique of Pure Reason Since the Beginnings of Neo-Kantianism", *Journal*of the History of Ideas, Vol. 42, No. 3 (Jul. Sep., 1981), pp. 531-551.
- Marion, Jean; "Cartesian metaphysics and the role of the simple natures". In: J.
 Cottingham (Ed.), *The Cambridge Companion to Descartes*, Cambridge University Press, Cambridge, 1992, pp. 115-139.
- Martínez, Luciana, "La doctrina kantiana de la definición en las lecciones de lógica (1770-1782)", Revista Anales del Seminario de Historia de la Filosofía 36 (3), 683-704, 2019.
- Maxwell, "J. J., Note on a Review of Prof. Natorp's Die Logischen Grundlagen", etc, *Mind* 21:302, 1912, 302-303.
- Mensch, Jennifer; "The Key to All Metaphysics: Kant's Letter to Herz, 1772",
 Kantian Review, Vol 12/02. July 2007, pp 109 127.
- Messer, August, Geschichte der Philosophie im 19. Jahrhundert bis zu Gegenwart, Quelle- Meyer, Leipzig, 1920.
- Moledo, Fernando; Los años silenciosos de Kant: Aspectos de la génesis de la Deducción Trascendental en la década de 1770. Seguido de la traducción del Legado de Duisburg (ca. 1775), Buenos Aires, Prometeo, 2014.
- Moran Dermot, Cohen Joseph; The Husserl Dictionary, Continuum Philosophy Dictionaries, Continuum, New York, 2012.

- Mormann, Thomas and Katz, Mikhail: "Infinitesimals as an Issue of Neo-Kantian Philosophy of Science" in: *Hopos: The Journal of the International Society for* the History of Philosophy of Science, Vol. 3, No. 2 (Fall 2013), pp. 236-280.
- Munk, Reiner (ed.), Hermanns Cohen Critical Idealism, Srpinger, Dordrecht, 2005.
- Nagel, Gordon. The Structure of Experience: Kant's System of Principles.
 Chicago: University of Chicago Press, 1983.
- Natorp, Paul. "Über objective und subjective Begründung der Erkentnnis I.",
 Philosophische Monatshefte, Bd XXIII, Heidelberg, 1887.
- "Thema und Disposition der aristotelischen *Metaphysik*", *Philosophische Monatshefte*, 24, 1888, p. 37-65; 540-574.
- "Über Aristoteles' *Metaphysik* K 1-8, 1065 a 26", *Archiv für Geschichte der Philosophie*, 1, 1888, p. 178-193.
- "Quantität und Qualität im Begriff, Urteil und gegenständlicher Erkentnnis",
 Philosophische Monatshefte, Bd XXVII, Heidelberg, 1891.
- Sozialpädagogik. Theorie der Willenserziehung auf der Grundlage der Gemeinschaft, Fr. Frommanns, Stuttgart, 1899.
- "Die erkenntisteoretischen Grundlagen der Mathematik" in: *Unterrichtsblätter Für Mathematik und Naturwissenschaften*, F. Pietzker, Verlag von Otto Sall, Berlin, 1902.
- Platos Ideenlehre: Eine Einführung in den Idealismus, Leipzig, Dürr. 1903.
- Philosophische Propädeutik, Marburg, 1904.
- «Nombre, temps, espace, dans leurs rapports avec les fonctions primitives de la pensé», Bibliothèque du Congrès International de Philosophie, vol. 1, Philosophie générale et Métaphysique., Colin, Paris, 1900, pp. 343-389. Allgemeine Psychologie in Leitsätzen zu akademischen Vorlesungen, Elwert, Marbug, 1904.
- Die logischen Grundlagen der ecxakten Wissenschaften, Berlin, 1910.
- Logik. Grundlegung und logischer Aufbau der Mathematik und mathematischen Naturwissenschaften in Leitsätzen zu akademischen Vorlesungen, Marburg: Elwert. 1910.
- Philosophie. Ihr Problem und ihre Probleme. Göttingen. 1911.
- "Kant und die Marburger Schule", *Kant-Studien* 17, 1912, pp. 193 221.

- Platos Ideenlehre. Zweite, durchsehene und um einen metakritischen Anhang vermehrte Ausgabe, Leipzig, Felix Meiner Verlag, 1921.
- Plato's Theory of ideas. An introduction to Idealism, Trad. V., Politis and J.
 Connolly, Sankt Augustin: Academia Verlag, 2004.
- "Descartes'-Erkenntnistheorie. Eine Studie zur Vorgeschichte des Kriticismus", Elwert, Marburg, 1882.
- "Leibniz und der Materialismus", (1881). Aus dem Nachlaß herausgegeben (Helmut Holzhey), *Studia Leibnitiana*, Bd. 17, H. 1, Franz Steiner Verlag, 1985, pp. 3-14.
- Platos Ideenlehre, Leipzig, Meiner, 1903
- "Zu den logischen Grundlagen der neueren Mathematik", *Archiv für systematische Philosophie*, Bd. 7, 1901, pp. 177-209/372 -438.
- "Zur Frage der logischen Methode. Mit Beziehung auf Edm. Husserls 'Prolegomena zur reinen Logik", Kant-Studien 6, 1901, 270–283.
- O'Connor, John K. "Precedents in Aristotle and Brentano for Husserl's Concern with 'Metabasis." *The Review of Metaphysics*, vol. 61, no. 4, Philosophy Education Society Inc., 2008, pp. 737–57.
- Parsons, Charles, "The Transcendental Aesthetic", The Cambridge Companion to Kant, Paul Guyer (ed.), Cambridge University Press, New York, 1992, pp. 62-100.
- Parsons, Charles; "Kant's Philosophy of Arithmetic." In: Posy C.J. (eds) Kant's Philosophy of Mathematics. Synthese Library (Studies in Epistemology, Logic, Methodology, and Philosophy of Science), vol 219. Springer, Dordrecht, 1992b, pp. 42-79.
- Parsons, Charles, "The Kantian Legacy in Twentieth Century Foundations of Mathematics", in: Parsons, Charles, *Philosophy of Mathematics in the Twentieth* Century. Selected Essays. Harvard University Press, Cambridge and London, 2014, pp. 11-39.
- Pasnau, Robert, "Cognition," in Williams T., The Cambridge Companion to Duns Scotus, New York, Cambridge University Press, 2002, pp.285-311.
- Paton, James; *Kant's* Metaphysic of *Experience*, Allen & Unwin, London, 1970.
- Philonenko, Alexis; *L'Oeuvre de Kant*, Vrin, Paris, 1969.
- Pippin, Robert B; *Kant's Theory of Form. An* Essay *on* the *Critique of Pure Reason*, Yale University Press, Haven and London, 1982.

- Pinto, Luis; La dénégation de l'origine [Hermann Cohen, de la sociologie à la philosophie transcendantale], Actes de la recherche en sciences sociales, Vol. 109, Nro. 1, 1995, pp. 41-59.
- Placencia, Luis; Espinosa, Ruth; "Introducción" a: Frege, G., *Escritos Filosóficos*, trans. Placencia, Luis Espinosa, Colihue, Buenos Aires, 2017.
- Politis, Vasilis; "Anti-Realist Interpretations of Plato: Paul Natorp", *International Journal of Philosophical Studies*, 9:1, 2001, 47-62.
- Poma, Andrea; The Critical Philosophy of Hermann Cohen, Trad. Denton John,
 State University of New York Press, Albany, 1997.
- Poma, Andrea; Yearning for Form and other Essays on Hermann Cohen's Thought, Springer, Dordrecht, 2006.
- Pringe, Hernán; "El sujeto en Cohen", El giro subjetivista de la Filosofía Moderna Perspectivas históricas y debates contemporáneos; Cátedra de Filosofía Moderna, Mar del Plata, 2010, pp. 446-454.
- Pringe, Hernán; Critique of the Quantum Power of Judgment, De Gruyter, Berlin-New York, 2011.
- Pringe, Hernán; "Legalidad empírica y realidad efectiva.", *Ideas y Valores* 64.158
 (2015): 21-39.
- Pringe, Hernán; "Cohen's Logik der reinen Erkenntnis and Cassirer's Substanzbegriff und Funktionsbegriff", *Kant Yearbook*, vol. 12, no. 1, 2020, pp. 137-168.
- Pringe, Hernán; "Conocimiento puro y cálculo infinitesimal en el neokantismo de Hermann Cohen", *Revista Latinoamericana de Filosofía*, Vol. 46 N°2 l, 2020b.
- Pringe, Hernán; "Infinitesimal Method and Judgment of Origin", Kant e-Print,
 Campinas, serie 2, Vol 16, n.2, 2021, pp. 185-199.
- Reale, Giovanni; "Introduzioni" in: Paul Natorp, Dottrina platonica delle Idee.
 Una introduzione all'Idealismo, Reale, Giovanni; Cicero, Vincenzo; Vita e
 Pensiero, Milano, 1999.
- Riehl, Alois *Hermann von Helmholtz in seinem Verhältnis zu Kant*. Reuther & Reichard, Berlin, 1904.
- Robson, Kent; "Kant's Concept of Intuition" in: *Akten des 4. Internationalen Kant-Kongresses*, Mainz. 6.-10. April 1974, Berlin-Nueva York, Hrsg. von Gerhard Funke, 1974, pp. 240-246.

- Rolf, George and Rusnock, Paul; "A Last Shot at Kant and Incongruent Counterparts", *Kant-Studien* 86 (3), Walter de Gruyter, 1995, pp. 257-277.
- Rolf, George and Rusnock, Paul; "Review Essays: Snails Rolled Up Contrary to All Sense." Reviewed Work: The Philosophy of Right and Left: Incongruent Counterparts and the Nature of Space by James Van Cleve, Robert E. Frederick", Philosophy and Phenomenological Research, Vol. 54, No. 2 (Jun., 1994), pp. 459-466.
- Rosales, Alberto; Sein und Subjektivität bei Kant: Zum subjektiven Ursprung der Kategorien, Kantstudien-Ergänzungshefte, 135, De Gruyter, Berlin – New York, 2000.
- Russell, Bertrand A. W., *An Essay on the Foundations of Geometry. With a new Foreword by Morris Kline*, Dover Publications, New York, 1956.
- Sánchez Rodríguez, M., La teoría de los tipos de representación en Leibniz y sus principales influencias en la estética y la lógica de la Ilustración alemana, *Cultura*. *Revista de História e Teoria das Ideias*, Vol. 32, 2013, pp. 271-295.
- Schneider, Hans, *Die Einheit als Grundprinzip der Philosophie P. Natorps*, Bottrop i W., Buch- und kunstdruckerei W. Postberg, 1936.
- Schrader, George, "Kant's Theory on concepts", *Kant-Studien* vol. 49, no. 1-4, 1958, pp. 264-278.
- Scotus, John Duns; God and Creatures. The Quadlibetal Questions, A. B. Wolter,
 F. Alluntis; Princeton University Press, Princeton and London, 1975.
- Scotus, John Duns; Questions on the Metaphysics of Aristotle Vol. I, trans. Girard
 J. Etzkorn and Allan Wolter, Franciscan Institute Publications, New York, 1997.
- Scott, Kermit; "Ockham on Evidence, Necessity and Intuition", *Journal of the History of Philosophy* 7, 1969, pp. 27-49.
- Seron, Denis; «L'idéalisme de Cohen et Natorp: Entre Kant et Platon», dans S. Delcomminette & A. Mazzú (éds.), L'Idée platonicienne dans la philosophie contemporaine: Jalons, Paris, Vrin, coll. «Tradition de la pensée classique», 2012, p. 97-116 (Pre-publication draft).
- Severo Passos, Rogério; "A puzzle about incongruent counterparts and the critique of pure reason", *Pacific Philosophical Quarterly* 88 (4), 2007, 507–521.
- Shabel, Lisa; "The Transcendental Aesthetic" in: *The Cambridge Companion to Kant's Critique of Pure Reason*, Paul Guyer, ed., Cambridge University Press, 2010, pp. 93-117.

- Smit, Houston; "Kant on Marks and the Immediacy of Intuition", *The Philosophical Review*, Vol. 109, No. 2, 2000, pp. 235-266.
- Smyth, Daniel; "Infinity and givenness: Kant on the intuitive origin of spatial representation", *Canadian Journal of Philosophy* 44 (5-6), 2014, pp. 551-579.
- Stadler, August; "§1 der transscendentalen Aesthetik. Erster Absatz. Aus einem Konversatorium für Anfänger" Kant-Studien 1 (1-3), 1897, pp. 100-107.
- Stati, Andrea, Warren Nicolás de. (eds), *New approaches to Neo-kantianism*, Cambridge University Press, Cambridge, 2015.
- Stephenson, Andrew, "Kant on object dependence and Halucination", *The Philosophical Quarterly*, Volume 65, Issue 260, July 2015, Pages 486–508.
- Stolzenberg, Jünger, "The Highest Principle and the Principle of Origin in Hermann Cohen's theoretical philosophy", in: Makreel, R. Luft, S (eds), *Neo-Kantianism in Contemporary Philosophy*, Indiana University Press, 2010, pp. 132-149.
- Strawson, Peter; *The Bounds of Sense. An Essay on Kant's Critique of Pure Reason*, Methuen & Co. Ltd., London, 1966.
- Stumpf, Carl, Über den psychologischen Ursprung der Raumvorstellung, S. Hirzel, Leipzig, 1873.
- Sullivan Peter M., "Frege's logic", in: Gabbay, Dov M., John Woods (eds),
 Handbook of the History of Logic. Vol. 3: The Rise of Modern Logic. From
 Leibniz to Frege, ELSEVIER, Amsterdam, 2004, pp.659-750.
- Thompson, Manley; "Singular Terms and Intuitions in Kant's Epistemology", *The Review of Metaphysics*, Vol. 26, No. 2 (Dec., 1972), pp. 314-343.
- Tonelli, Giorgio; "Wiederaufleben der deutsch-aristotelischen Terminologie bei Kant während der Entstehung der "Kritik der reinen Vernunft", Archiv für Begriffsgeschichte, Vol. 9, 1964, pp. 233-242.
- Torretti, Roberto; "La geometría en el pensamiento de Kant", *Logos. Anales del Seminario de Metafísica*, Vol. 9, Madrid, 1974, pp. 9-61.
- Torretti, Roberto; *Manuel Kant. Estudio sobre los fundamentos de la filosofía crítica*, Charcas, Buenos Aires, 1967.
- Vaihinger, Hans; *Commentar zu Kants Kritik der reinen Vernunft*, Zweiter Band, Union Deutsche Verlasgesselschaft, Stuttgard, Berlin, Leipzig, 1892.

- Van Clave, James; "Foundationalism, Epistemic Principles, and the Cartesian Circle", in: J. Cottingham (ed.), *Descartes*, Oxford University Press, New York, 1998.
- Van de Pitte, Frederick P; Intuition and Judgment in Descartes' Theory of Truth, Journal of the History of Philosophy, Vol 26/3, Johns Hopkins University Press, 1988, pp. 453-470.
- Vleeschauwer, Herman Jean; La evolución del pensamiento kantiano. Historia de una doctrina, trad. Guerra, Ricardo; Universidad Nacional Autónoma de México, México, 1962.
- Walford, David; "Towards an Interpretation of Kant's 1768 *Gegenden im Raume* Essay, Kant-Studien 92. Jahrg., Walter de Gruyter 2001, S. 407-439.
- Walsh, Sean; "Incongruent Counterparts and Causality", *Kant-Studien* 98. Jahrg., Walter de Gruyter, 2007, S. 418–430.
- Warren, Nicolas; Staiti, Andrea; (comps.), New Approaches to Neo-Kantianism,
 Cambridge University Press, Cambridge, 2015.
- Wiendelband, Wilhelm, Lehrbuch der Geschichte der Philosophie. Dritte Auflage, Mohr, Tübingen und Leipzig, 1903.
- Wilson, Kirk Dallas; "Kant on Intuition", *The Philosophical Quarterly*, Vol. 25, No. 100 (Jul. 1975), pp. 247-265.
- Willaschek, Marcus; "The Sensibility of Human Intuition. Kant's Causal Condition on Accounts of Representation" In Rainer Enskat (ed.), Kants Theorie der Erfahrung, De Gruyter. 2015, Berlin, pp. 129-150.
- Zerbudis Ezequiel, "Incongruent Counterparts and the Origin of Kant's Distinction Between Sensibility and Understanding", Archiv für Geschichte der Philosophie, vol. 94, Berlin, 2012, pp. 326 352.
- Zweig, Arnulf, "Introduction", in; Kant, I. Correspondence, Trad. and Ed. Zweig,
 A., Cambridge University Press, New York, 1999.

Acknowledgments

This thesis would have never been written without the patience and support of my supervisor Prof.dr. H. Pringe. It would be difficult to find so many virtues in a single person. Every interesting point of this project, I owe it to him. Mistakes are mine.

Words cannot express my gratitude to my dear friend and colleague Dr. Luciana Martinez. This thesis would not have been possible without her support, expert advice, and encouragement.

During my undergraduate studies and while writing this project, I had the honour of attending Prof. Mario Caimi seminars. When someone asks me, what philosophy is, I recommend reading any of his lectures.

I am grateful for the support of the University Diego Portal. Particularly, I would like to express my gratitude to Prof. Juan Manuel Garrido, who helped me at the beginning of the project.

I would also like to thank to all the members of the Grupo de Estudios Kantianos of Buenos Aires who generously discussed philosophy with me.

I would like to express my gratitude to Prof.dr. Frans A.J. de Haas and Dr. V.A. Gijsbers, and to all the members of Leiden University.

This thesis was funded by the doctorate scholarship of the CONICYT (Chilean National Committee for Scientific and Technological Research). Throughout the years of my research, I have also received funding from Diego Portales University.

Laura Pelegrín studied Philosophy at the University of Buenos Aires (UBA). She graduated in 2012. In 2010, she obtained a scholarship for undergraduate students in this institution with a project on contemporary readings of Immanuel Kant. In 2013, she obtained a PhD scholarship from the Universidad Diego Portales in Santiago de Chile. In 2014, she obtained a PhD scholarship from CONICYT in Chile. She developed her PhD in the UDP with a *Cotutelle* in Leiden University with a project on Neo-Kantianism. She has given seminars at The University Diego Portales and at the University of Buenos Aires. She has worked at Universidad Maimonides, Universidad del Museo Argentino, and in The University of Buenos Aires. She is a member of the Grupo de Estudios Kantianos of the Institute of Philosophy in the UBA, and of the Grupo Idealismo Crítico in the UDP. She is an associate researcher at the UDP. She has presented the results of her research at various scientific meetings and has published in national and international academic journals.