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## **This is life: some thoughts on self-organized structure formation in active liquids and biological systems**

Hoffmann, L.A.

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# This Is Life

## Some Thoughts on Self-organized Structure Formation in Active Liquids and Biological Systems

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**Promotores:**

Dr. L. Giomi  
Prof.dr. K. E. Schalm

**Promotiecommissie:**

Prof.dr. D. Bartolo (ENS Lyon)  
Dr. M. Bauer (TU Delft)  
Prof.dr. J. Aarts  
Prof.dr. E. Eliel  
Dr. A. Morin  
Prof.dr. W. van Saarloos

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The front of the cover shows the shell of a turban snail, viewed from the top, in the foreground. A line is drawn on top of the snail to highlight the spiral that is visible on the shell. This unites two major themes covered in this thesis, topological defects and chirality. The spiral is reminiscent of the geometry of a  $+1$  topological defect. The fact that the shell of the snail contains a spiral makes it an example of a structure that is not left-right symmetric, i.e., the shell and its mirror image are distinct and the object is called chiral. On the other hand, the background is a picture of a rock which is mirrored at the spine of the thesis. The background is therefore mirror symmetric. The title is inspired by the title of the essay “This Is Water: Some Thoughts, Delivered on a Significant Occasion, about Living a Compassionate Life” by D. F. Wallace and the book “What Is Life?” by E. Schrödinger.

Dedicated to everyone who inspired me



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