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[122] The chiral symmetry at $\mu = 0$ is broken by the mass term $M(k)$ in the Hamiltonian (6.14). This residual chiral symmetry breaking is visible in Fig. 6.3 as a very small splitting of the $\mu = 0$ Landau levels (green flat bands).

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