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On topological properties of massless fermions in a magnetic field

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On topological properties of massless fermions in a magnetic field

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To Betti, to my friends and to my family.

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