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## Hysterons and pathways in mechanical metamaterials

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# **Hysterons and Pathways in Mechanical Metamaterials**

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geboren te Luoyang, China  
in 1991

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The cover shows the transition pathways of a biholey mechanical metamaterial with three hysteron.

To my family.

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