Dormancy in stochastic interacting systems

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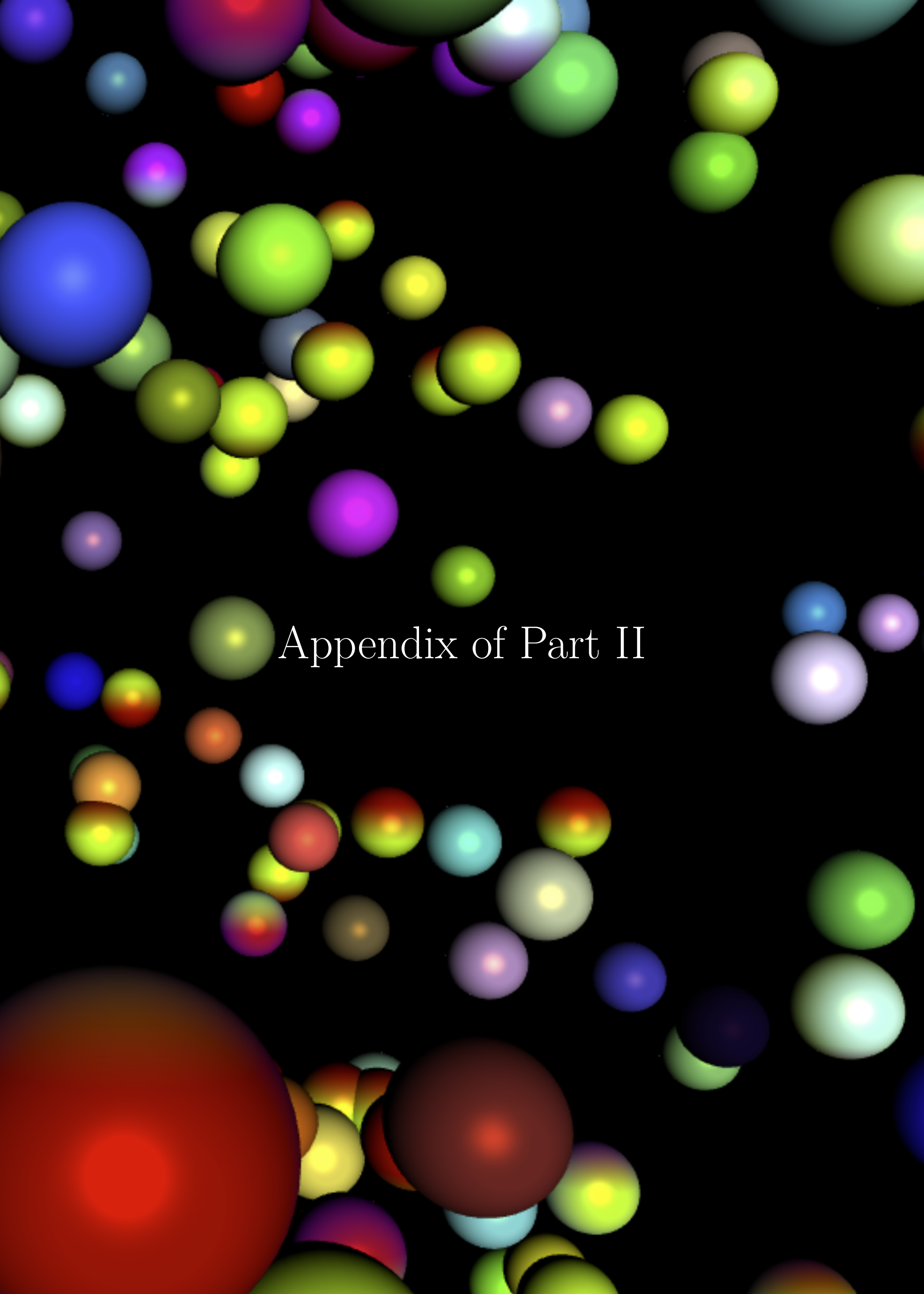
Appendix of Part II



APPENDIX C

Appendix: Chapter 5

Inverse of the boundary-layer matrix

The inverse of the matrix M_ϵ defined in (5.115) is given by (α_1 and α_2 are as in (5.116))

$$M_\epsilon^{-1} := \frac{1}{Z}\begin{bmatrix} -m_{13} & -m_{14} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31}(\alpha_2) & m_{32}(\alpha_2) & m_{33}(\alpha_2) & m_{34}(\alpha_2) \\ -m_{31}(\alpha_1) & -m_{32}(\alpha_1) & -m_{33}(\alpha_1) & -m_{34}(\alpha_1) \end{bmatrix}, \tag{C.1}$$

where

$$\begin{aligned} Z &:= \alpha_1^{N+1}[\alpha_2(1-\epsilon)(\alpha_2^{N-1}+1)+2\epsilon(\alpha_2^{N+1}+1)] \\ &\qquad\qquad \times [\alpha_2(1+N)(1-\epsilon)(\alpha_2^{N-1}-1)+2\epsilon(N+\epsilon)(\alpha_2^{1+N}-1)], \\ m_{13} &:= \alpha_1^{N+1}[\alpha_2(1-\epsilon)(\alpha_2^{N-1}+1)+2\epsilon(\alpha_2^{N+1}+1)] \\ &\qquad\qquad \times [\alpha_2(1-\epsilon)(\alpha_2^{N-1}-1)+\epsilon(\alpha_2^{N+1}-1)], \\ m_{14} &:= \epsilon\,\alpha_1^{N+1}[\alpha_2(1-\epsilon)(\alpha_2^{N-1}+1)+2\epsilon(\alpha_2^{N+1}+1)]\,(\alpha_2^{N+1}-1), \\ m_{21} &:= (1+N)(1-\epsilon)^2(\alpha_2^{N-1}-\alpha_1^{N-1})-\epsilon(1-\epsilon)^2(\alpha_2-\alpha_1) \\ &\qquad + \epsilon^2(1+2N+\epsilon)(\alpha_2^{N+1}-\alpha_1^{N+1})+\epsilon(1-\epsilon)(2+3N+\epsilon)(\alpha_2^{N}-\alpha_1^{N}), \\ m_{22} &:= \epsilon\,[(1-\epsilon)(1+N)(\alpha_2^{N}-\alpha_1^{N})+\epsilon(1+2N+\epsilon)(\alpha_2^{N+1}-\alpha_1^{N+1})], \\ m_{23} &:= \epsilon\,(1-\epsilon)[(N+\epsilon)(\alpha_2-\alpha_1)-(1-\epsilon)(\alpha_2^{N}-\alpha_1^{N})-\epsilon(\alpha_2^{N+1}-\alpha_1^{N+1})], \\ m_{24} &:= -\epsilon(1-\epsilon)[(1+N)(\alpha_2-\alpha_1)+\epsilon\,(\alpha_2^{N+1}-\alpha_1^{N+1})], \end{aligned} \tag{C.2}$$

and the polynomials $m_{31}(z), m_{32}(z), m_{33}(z), m_{34}(z)$ are defined as

$$\begin{aligned} m_{31}(z) &:= -(1-\epsilon)^2\,z-\epsilon\,(1-\epsilon)+(1-\epsilon)(N+\epsilon)\,z^N-\epsilon(1-2N-3\epsilon)\,z^{N+1}, \\ m_{32}(z) &:= -(1-\epsilon)(1+N)z^N-\epsilon\,(1-\epsilon)-\epsilon(1+2N+\epsilon)\,z^{N+1}, \\ m_{33}(z) &:= (1-\epsilon)^2\,z^N+\epsilon\,(1-\epsilon)\,z^{N+1}-(1-\epsilon)(N+\epsilon)\,z+\epsilon(1-2N-3\epsilon), \\ m_{34}(z) &:= (1+N)(1-\epsilon)\,z+\epsilon\,(1-\epsilon)\,z^{N+1}+\epsilon(1+2N+\epsilon). \end{aligned} \tag{C.3}$$

We remark that most of the terms appearing in the inverse simplify because of (5.117). We define the four vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3, \vec{c}_4$ as the respective rows of M_ϵ^{-1}, i.e.,

$$\begin{aligned} \vec{c}_1 &:= (M_\epsilon^{-1})^T\vec{e}_1, \quad \vec{c}_2 := (M_\epsilon^{-1})^T\vec{e}_2, \\ \vec{c}_3 &:= (M_\epsilon^{-1})^T\vec{e}_3, \quad \vec{c}_4 := (M_\epsilon^{-1})^T\vec{e}_4, \end{aligned} \tag{C.4}$$

where

$$\vec{e}_1 := \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T, \quad \vec{e}_2 := \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T,$$
$$\vec{e}_3 := \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T, \quad \vec{e}_4 := \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T.$$

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