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Dormancy in stochastic interacting systems

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Dormancy in Stochastic Interacting Systems

Shubhamoy Nandan

Cover pages, part, and chapter openers: images obtained by taking screenshots of a simulation (available at <https://chainserver.pythonanywhere.com/hiv-dormancy>) made by the author of this thesis. The 3D model used in the simulation is of an HIV-cell which is known to exhibit dormancy (or *latency*) lasting up to 10 or 15 years.

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Dormancy in Stochastic Interacting Systems

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