



Universiteit
Leiden
The Netherlands

Intermittency and number expansions for random interval maps

Zeegers, B.P.

Citation

Zeegers, B. P. (2023, February 14). *Intermittency and number expansions for random interval maps*. Retrieved from <https://hdl.handle.net/1887/3563041>

Version: Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/3563041>

Note: To cite this publication please use the final published version (if applicable).

Bibliography

- [A97] J. Aaronson. *An Introduction to Infinite Ergodic Theory*, volume 50 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 1997.
- [AGH18] N. Abbasi, M. Gharaei, and A. J. Homburg. Iterated function systems of logistic maps: synchronization and intermittency. *Nonlinearity*, 31(8):3880–3913, 2018.
- [AR66] L. M. Abramov and V. A. Rokhlin. The entropy of a skew product of measure-preserving transformations. *Amer. Math. Soc. Transl. Ser.*, 2(48):255–265, 1966.
- [ANV15] R. Aimino, M. Nicol, and S. Vaienti. Annealed and quenched limit theorems for random expanding dynamical systems. *Probab. Theory Related Fields*, 162(1-2):233–274, 2015.
- [AD00] K. B. Athreya and J. Dai. Random logistic maps. I. *J. Theoret. Probab.*, 13(2):595–608, 2000.
- [AS03] K. B. Athreya and H.-J. Schuh. Random logistic maps. II. The critical case. *J. Theoret. Probab.*, 16(4):813–830 (2004), 2003.
- [AFGTV21] J. Atnip, G. Froyland, C. González-Tokman, and S. Vaienti. Thermodynamic formalism for random weighted covering systems. *Comm. Math. Phys.*, 386(2):819–902, 2021.
- [BB16] W. Bahsoun and C. Bose. Mixing rates and limit theorems for random intermittent maps. *Nonlinearity*, 29(4):1417–1433, 2016.
- [BBD14] W. Bahsoun, C. Bose, and Y. Duan. Decay of correlation for random intermittent maps. *Nonlinearity*, 27(7):1543–1554, 2014.
- [BBR19] W. Bahsoun, C. Bose, and M. Ruziboev. Quenched decay of correlations for slowly mixing systems. *Trans. Amer. Math. Soc.*, 372(9):6547–6587, 2019.
- [BRS20] W. Bahsoun, M. Ruziboev, and B. Saussol. Linear response for random dynamical systems. *Adv. Math.*, 364:107011, 44, 2020.
- [BD17] S. Baker and K. Dajani. Induced random β -transformation. *Acta Arith.*, 178(1):1–14, 2017.

- [B00] V. Baladi. *Positive Transfer Operators and Decay of Correlations*, volume 16. World scientific, 2000.
- [BI08] L. Barreira and G. Iommi. Partial quotients of continued fractions and β -expansions. *Nonlinearity*, 21(10):2211–2219, 2008.
- [BBDK96] J. Barrionuevo, R. M. Burton, K. Dajani, and C. Kraaikamp. Ergodic properties of generalized Lüroth series. *Acta Arith.*, 74(4):311–327, 1996.
- [BPV86] P. Bergé, Y. Pomeau, and C. Vidal. *Order within Chaos*. J. Wiley, New York, 1986.
- [B92] T. Bogenschütz. Entropy, pressure, and a variational principle for random dynamical systems. *Random Comput. Dynam.*, 1(1):99–116, 1992.
- [B93] T. Bogenschütz. *Equilibrium States for Random Dynamical Systems*. PhD thesis, Bremen University, 1993.
- [BC92] T. Bogenschütz and H. Crauel. The Abramov-Rokhlin formula. In *Ergodic Theory and Related Topics, III (Güstrow, 1990)*, volume 1514 of *Lecture Notes in Math.*, pages 32–35. Springer, Berlin, 1992.
- [BQT21] C. Bose, A. Quas, and M. Tanzi. Random composition of L-S-V maps sampled over large parameter ranges. *Nonlinearity*, 34(6):3641–3675, 2021.
- [BDK99] W. Bosma, K. Dajani, and C. Kraaikamp. Entropy and counting correct digits. Technical Report 9925, University of Nijmegen, 1999.
- [BG97] A. Boyarsky and P. Góra. *Laws of Chaos: Invariant Measures and Dynamical Systems in One Dimension*. Springer Science & Business Media, 1997.
- [B82] M. G. Branton. *The Dynamics of Random Maps*. PhD thesis, University of North Carolina at Chapel Hill, 1982.
- [BLvS03] H. Bruin, S. Luzzatto, and S. van Strien. Decay of correlations in one-dimensional dynamics. *Ann. Sci. École Norm. Sup. (4)*, 36(4):621–646, 2003.
- [BSvS03] H. Bruin, W. Shen, and S. van Strien. Invariant measures exist without a growth condition. *Comm. Math. Phys.*, 241(2-3):287–306, 2003.
- [B99] J. Buzzi. Exponential decay of correlations for random Lasota-Yorke maps. *Comm. Math. Phys.*, 208(1):25–54, 1999.
- [C02] N. Carlsson. A contractivity condition for iterated function systems. *J. Theoret. Probab.*, 15(3):613–630, 2002.
- [CCD21] E. Charlier, C. Cisternino, and K. Dajani. Dynamical behavior of alternate base expansions. *Ergodic Theory Dynam. Systems*, pages 1–34, 2021.

-
- [CE83] P. Collet and J.-P. Eckmann. Positive Liapunov exponents and absolute continuity for maps of the interval. *Ergodic Theory Dynam. Systems*, 3(1):13–46, 1983.
- [DF01] K. Dajani and A. Fieldsteel. Equipartition of interval partitions and an application to number theory. *Proc. Amer. Math. Soc.*, 129(12):3453–3460, 2001.
- [DJ17] K. Dajani and K. Jiang. Shrinking random β -transformation. *Indag. Math. (N.S.)*, 28(1):74–83, 2017.
- [DK07] K. Dajani and C. Kalle. Random β -expansions with deleted digits. *Discrete Contin. Dyn. Syst.*, 18(1):199–217, 2007.
- [DK13] K. Dajani and C. Kalle. Local dimensions for the random β -transformation. *New York J. Math.*, 19:285–303, 2013.
- [DK20] K. Dajani and C. Kalle. Invariant measures, matching and the frequency of 0 for signed binary expansions. *Publ. Res. Inst. Math. Sci.*, 56(4):701–742, 2020.
- [DK21] K. Dajani and C. Kalle. *A First Course in Ergodic Theory*. CRC Press, 2021.
- [DKM21] K. Dajani, C. Kalle, and M. Maggioni. Matching for random systems with an application to minimal weight expansions. *Nonlinearity*, 34(6):3676–3708, 2021.
- [DK03] K. Dajani and C. Kraaikamp. Random β -expansions. *Ergodic Theory Dynam. Systems*, 23(2):461–479, 2003.
- [DO18] K. Dajani and M. Oomen. Random N -continued fraction expansions. *J. Approx. Theory*, 227:1–26, 2018.
- [DdV05] K. Dajani and M. de Vries. Measures of maximal entropy for random β -expansions. *J. Eur. Math. Soc. (JEMS)*, 7(1):51–68, 2005.
- [DdV07] K. Dajani and M. de Vries. Invariant densities for random β -expansions. *J. Eur. Math. Soc. (JEMS)*, 9(1):157–176, 2007.
- [DDGV02] I. Daubechies, R. A. DeVore, C. S. Güntürk, and V. A. Vaishampayan. Beta expansions: a new approach to digitally corrected A/D conversion. *Proc. IEEE Int. Symp. Circ. Syst.*, 2:784–787, 2002.
- [DDGV06] I. Daubechies, R. A. DeVore, C. S. Güntürk, and V. A. Vaishampayan. A/D conversion with imperfect quantizers. *IEEE Trans. Inform. Theory*, 52(3):874–885, 2006.
- [DGWY10] I. Daubechies, S. Güntürk, Y. Wang, and Ö. Yılmaz. The golden ratio encoder. *IEEE Trans. Inform. Theory*, 56(10):5097–5110, 2010.

- [DY06] I. Daubechies and Ö. Yilmaz. Robust and practical analog-to-digital conversion with exponential precision. *IEEE Trans. Inform. Theory*, 52(8):3533–3545, 2006.
- [EW11] M. Einsiedler and T. Ward. Ergodic theory with a view towards number theory, volume 259 of Graduate Texts in Mathematics. Springer-Verlag London, Ltd., London, 2011
- [EJK90] P. Erdős, I. Joó, and V. Komornik. Characterization of the unique expansions $1 = \sum_{i=1}^{\infty} q^{-n_i}$ and related problems. *Bull. Soc. Math. France*, 118(3):377–390, 1990.
- [F97] C. Faivre. On decimal and continued fraction expansions of a real number. *Acta Arith.*, 82(2):119–128, 1997.
- [F98] C. Faivre. A central limit theorem related to decimal and continued fraction expansion. *Arch. Math. (Basel)*, 70(6):455–463, 1998.
- [F01] C. Faivre. On calculating a continued fraction expansion from a decimal expansion. *Acta Sci. Math. (Szeged)*, 67(3-4):505–519, 2001.
- [FWL16] L. Fang, M. Wu, and B. Li. Limit theorems related to beta-expansion and continued fraction expansion. *J. Number Theory*, 163:385–405, 2016.
- [FWL19] L. Fang, M. Wu, and B. Li. Beta-expansion and continued fraction expansion of real numbers. *Acta Arith.*, 187(3):233–253, 2019.
- [F99] G. Froyland. Ulam’s method for random interval maps. *Nonlinearity*, 12(4):1029–1052, 1999.
- [GH17] M. Gharaei and A. J. Homburg. Random interval diffeomorphisms. *Discrete Cont. Dynam. Syst. Ser. S*, 10(2):241–272, 2017.
- [G04] S. Gouëzel. Sharp polynomial estimates for the decay of correlations. *Israel J. Math.*, 139:29–65, 2004.
- [G12] C.S. Güntürk. Mathematics of analog-to-digital conversion. *Comm. Pure Appl. Math.*, 65(12):1671–1696, 2012.
- [HKST15] J. Heinonen, P. Koskela, N. Shanmugalingam, and J. T. Tyson. *Sobolev Spaces on Metric Measure Spaces. An approach based on upper gradients*, volume 27 of New Mathematical Monographs. Cambridge University Press, Cambridge, 2015.
- [H09] A. Herczegh. Central limit theorems in ergodic theory. Master’s thesis, Eötvös Loránd University, 2009.
- [HKR⁺22] A. J. Homburg, C. Kalle, M. Ruziboev, E. Verbitskiy, and B. Zeegers. Critical intermittency in random interval maps. *Comm. Math. Phys.*, 394(1):1–37, 2022.

-
- [HPR21] A. J. Homburg, H. Peters, and V. Rabodonandrianandraina. Critical intermittency in rational maps. Preprint, available at arXiv:1909.05559v2, 2021.
- [H04] H. Hu. Decay of correlations for piecewise smooth maps with indifferent fixed points. *Ergodic Theory and Dynamical Systems*, 24(2):495–524, 2004.
- [I12] T. Inoue. Invariant measures for position dependent random maps with continuous random parameters. *Studia Math.*, 208(1):11–29, 2012.
- [J81] M. V. Jakobson. Absolutely continuous invariant measures for one-parameter families of one-dimensional maps. *Comm. Math. Phys.*, 81(1):39–88, 1981.
- [JW09] D. Jiménez and Y. Wang. The $\beta\alpha$ -encoders for robust A/D conversion. *Acta Appl. Math.*, 107(1-3):313–323, 2009.
- [JM16] Y. Jitsumatsu and K. Matsumura. A β -ary to binary conversion for random number generation using a β encoder. *Nonlinear Theory and Its Applications, IEICE*, 38–55, 2016.
- [JMKA13] Y. Jitsumatsu, K. Matsumura, T. Kohda, and K. Aihara. Pseudo-random number generator using beta-encoder cmos circuit. *The 3rd Int. Symp. Innovative Mathematical Modelling*, page 107, 2013.
- [JSS11] T. Jordan, P. Shmerkin, and B. Solomyak. Multifractal structure of Bernoulli convolutions. *Math. Proc. Cambridge Philos. Soc.*, 151(3):521–539, 2011.
- [KKV17] C. Kalle, T. Kempton, and E. Verbitskiy. The random continued fraction transformation. *Nonlinearity*, 30(3):1182–1203, 2017.
- [KM22a] C. Kalle and M. Maggioni. Invariant densities for random systems of the interval. *Ergodic Theory Dynam. Systems*, 42(1):141–179, 2022.
- [KM22b] C. Kalle and M. Maggioni. On approximation by random Lüroth expansions. *Int. J. Number Theory*, 18(5):1013–1046, 2022.
- [KMTV22] C. Kalle, V. Matache, M. Tsujii, and E. Verbitskiy. Invariant densities for random continued fractions. *J. Math. Anal. Appl.*, 512(2):Paper No. 126163, 28, 2022.
- [KVZ22] C. Kalle, E. Verbitskiy, and B. Zeegers. Random Lochs’ Theorem. *Studia Math.*, 267(2):201–239, 2022.
- [KZ22] C. Kalle and B. Zeegers. Decay of correlations for critically intermittent systems. Preprint, available at arXiv:2206.07601, 2022.

- [KN92] G. Keller and T. Nowicki. Spectral theory, zeta functions and the distribution of periodic points for Collet-Eckmann maps. *Comm. Math. Phys.*, 149(1):31–69, 1992.
- [K14] T. Kempton. On the invariant density of the random β -transformation. *Acta Math. Hungar.*, 142(2):403–419, 2014.
- [K86] Y. Kifer. *Ergodic Theory of Random Transformations*, volume 10 of *Progress in Probability and Statistics*. Birkhäuser Boston, Inc., Boston, MA, 1986.
- [KHA12] T. Kohda, Y. Horio, and K. Aihara. Beta-expansion attractors observed in A/D converters. *Chaos*, 22(4):1–18, 047512, 2012.
- [KHTA12] T. Kohda, Y. Horio, Y. Takahashi, and K. Aihara. Beta encoders: symbolic dynamics and electronic implementation. *Internat. J. Bifur. Chaos Appl. Sci. Engrg.*, 22(9):1–55, 1230031, 2012.
- [KJ16] I. Koji and Y. Jitsumatsu. Random number generation using outputs from multiple beta encoders. In *Proceedings of Nonlinear Theory and Its Applications 2016*, pages 249–252, 2016.
- [K58] A. N. Kolmogorov. A new metric invariant of transient dynamical systems and automorphisms in Lebesgue spaces. In *Doklady of Russian Academy of Sciences*, volume 119, pages 861–864, 1958.
- [LMS04] A. Lasota, J. Myjak, and T. Szarek. Markov operators and semifractals. In *Fractal geometry and stochastics III*, volume 57 of *Progr. Probab.*, pages 3–22. Birkhäuser, Basel, 2004.
- [LY73] A. Lasota and J. A. Yorke. On the existence of invariant measures for piecewise monotonic transformations. *Trans. Amer. Math. Soc.*, 186:481–488 (1974), 1973.
- [LY88] F. Ledrappier and L.-S. Young. Entropy formula for random transformations. *Probab. Theory Related Fields*, 80(2):217–240, 1988.
- [L96] C. Liverani. Central limit theorem for deterministic systems. In *International Conference on Dynamical Systems (Montevideo, 1995)*, volume 362 of *Pitman Res. Notes Math. Ser.*, pages 56–75. Longman, Harlow, 1996.
- [LSV99] C. Liverani, B. Saussol, and S. Vaienti. A probabilistic approach to intermittency. *Ergodic Theory Dynam. Systems*, 19(3):671–685, 1999.
- [LW08] B. Li and J. Wu. Beta-expansion and continued fraction expansion. *J. Math. Anal. Appl.*, 339(2):1322–1331, 2008.
- [L64] G. Lochs. Vergleich der Genauigkeit von Dezimalbruch und Kettenbruch. *Abh. Math. Sem. Univ. Hamburg*, 27:142–144, 1964.

- [L06] S. Luzzatto. Stochastic-like behaviour in nonuniformly expanding maps. In *Handbook of Dynamical Systems. Vol. 1B*, pages 265–326. Elsevier B. V., Amsterdam, 2006.
- [LM13] S. Luzzatto and I. Melbourne. Statistical properties and decay of correlations for interval maps with critical points and singularities. *Comm. Math. Phys.*, 320(1):21–35, 2013.
- [L02] M. Lyubich. Almost every real quadratic map is either regular or stochastic. *Ann. of Math. (2)*, 156(1):1–78, 2002.
- [M57] G. W. Mackey. Borel structure in groups and their duals. *Trans. Amer. Math. Soc.*, 85:134–165, 1957.
- [MIS⁺15] T. Makino, Y. Iwata, K. Shinohara, Y. Jitsumatsu, M. Hotta, H. San, and K. Aihara. Rigorous estimates of quantization error for A/D converters based on beta-map. *Nonlinear Theory and Its Applications*, 6(1):99–111, 2015.
- [MP80] P. Manneville and Y. Pomeau. Different ways to turbulence in dissipative dynamical systems. *Phys. D*, 1(2):219–226, 1980.
- [dMvS93] W. de Melo and S. van Strien. *One-Dimensional Dynamics*, Springer-Verlag, New York, 1993.
- [M81] M. Misiurewicz. Absolutely continuous measures for certain maps of an interval. *Inst. Hautes Études Sci. Publ. Math.*, (53):17–51, 1981.
- [M85a] T. Morita. Asymptotic behavior of one-dimensional random dynamical systems. *J. Math. Soc. Japan*, 37(4):651–663, 1985.
- [M85b] T. Morita. Random iteration of one-dimensional transformations. *Osaka Journal of Mathematics*, 22(3):489–518, 1985.
- [M86] T. Morita. Entropy of random dynamical systems. *Proc. Japan Acad. Ser. A Math. Sci.*, 62(4):121–124, 1986.
- [M05] R. Murray. Approximation of invariant measures for a class of maps with indifferent fixed points. *Mathematics Research Report Series II No. 106*. University of Waikato, 2005.
- [NPPT21] M. Nicol, F. Perez Pereira, and A. Török. Large deviations and central limit theorems for sequential and random systems of intermittent maps. *Ergodic Theory Dynam. Systems*, 41(9):2805–2832, 2021.
- [NTV18] M. Nicol, A. Török, and S. Vaienti. Central limit theorems for sequential and random intermittent dynamical systems. *Ergodic Theory Dynam. Systems*, 38:1127–1153, 2018.
- [NvS91] T. Nowicki and S. van Strien. Invariant measures exist under a summability condition for unimodal maps. *Invent. Math.*, 105(1):123–136, 1991.

- [P84] S. Pelikan. Invariant densities for random maps of the interval. *Transactions of the American Mathematical Society*, 281(2):813–825, 1984.
- [P89] K. Petersen. *Ergodic Theory*, volume 2 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1989.
- [P80] G. Pianigiani. First return map and invariant measures. *Israel J. Math.*, 35(1-2):32–48, 1980.
- [PM80] Y. Pomeau and P. Manneville. Intermittent transition to turbulence in dissipative dynamical systems. *Comm. Math. Phys.*, 74(2):189–197, 1980.
- [R57a] A. Rényi. On algorithms for the generation of real numbers. *Magyar Tud. Akad. Mat. Fiz. Oszt. Közl.*, 7:265–293, 1957.
- [R57b] A. Rényi. Representations for real numbers and their ergodic properties. *Acta Math. Acad. Sci. Hungar*, 8:477–493, 1957.
- [R52] V. A. Rokhlin. On the fundamental ideas of measure theory. *Amer. Math. Soc. Translation*, 1952(71):55, 1952.
- [SKM⁺13] H. San, T. Kato, T. Maruyama, K. Aihara, and M. Hotta. Non-binary pipeline analog-to-digital converter based on beta-expansion. *IEICE Trans. Fundamentals*, 96(2):415–421, 2013.
- [S09] O. M. Sarig. Lecture notes on thermodynamic formalism for topological markov shifts. *Penn State*, 2009.
- [S48] C. E. Shannon. A mathematical theory of communication. *The Bell system technical journal*, 27(3):379–423, 1948.
- [SJO15] Y. Shu, Y. Jitsumatsu, and K. Oda. Performance evaluation of a random number generation using a beta encoder. *NOLTA2015*, pages 511–514, 2015.
- [S03] N. Sidorov. Almost every number has a continuum of β -expansions. *Amer. Math. Monthly*, 110(9):838–842, 2003.
- [S59] Ya. G. Sinai. On the notion of entropy of a dynamical system. *Doklady of Russian Academy of Sciences*, 124:768–771, 1959.
- [S19] S. Suzuki. Invariant density functions of random β -transformations. *Ergodic Theory Dynam. Systems*, 39(4):1099–1120, 2019.
- [T80] M. Thaler. Estimates of the invariant densities of endomorphisms with indifferent fixed points. *Israel J. Math.*, 37(4):303–314, 1980.
- [T04] R. Toledano. A note on the Lebesgue differentiation theorem in spaces of homogeneous type. *Real Anal. Exchange*, 29(1):335–339, 2003/04.

-
- [VO16] M. Viana and K. Oliveira. *Foundations of Ergodic Theory*, volume 151 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2016.
- [W00] P. Walters. *An Introduction to Ergodic Theory*, volume 79. Springer Science & Business Media, 2000.
- [W08] R. Ward. On robustness properties of beta encoders and golden ratio encoders. *IEEE Trans. Inform. Theory*, 54(9):4324–4334, 2008.
- [W06] J. Wu. Continued fraction and decimal expansions of an irrational number. *Adv. Math.*, 206(2):684–694, 2006.
- [W08] J. Wu. An iterated logarithm law related to decimal and continued fraction expansions. *Monatsh. Math.*, 153(1):83–87, 2008.
- [Y92] L.-S. Young. Decay of correlations for certain quadratic maps. *Comm. Math. Phys.*, 146(1):123–138, 1992.
- [Y98] L.-S. Young. Statistical properties of dynamical systems with some hyperbolicity. *Ann. of Math.*, 147:585–650, 1998.
- [Y99] L.-S. Young. Recurrence times and rates of mixing. *Israel J. Math.*, 110:153–188, 1999.
- [Z] B. Zeegers. Intermittency generated by attracting and weakly repelling fixed points. To appear in *Indag. Math.* DOI: 10.1016/j.indag.2022.12.002.
- [Z18] B. Zeegers. On invariant densities and Lochs’ Theorem for random piecewise monotonic interval maps. Master’s thesis, Leiden University, 2018.
- [Z08] Y. Zhu. On local entropy of random transformations. *Stoch. Dyn.*, 8(2):197–207, 2008.

