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Intermittency and number expansions for random interval maps

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Intermittency and Number Expansions for Random Interval Maps

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