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Scalability and uncertainty of Gaussian processes

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Scalability and uncertainty of Gaussian processes

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