Applications of quantum annealing in combinatorial optimization

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Summary

In this thesis we explore the use of quantum annealing in solving real-world combinatorial optimization in a variety of settings. With the continuous development of quantum hardware and software, the potential of quantum algorithms is slowly being unlocked. However, some of the peculiarities of quantum hardware have no exact parallel in classical computing, and so the work in this thesis develops new methods to address quantum annealing for combinatorial optimization. We start by describing the quantum annealing (QA) paradigm in Chapter 2. The underlying theory of the algorithm as a metaheuristic quantum optimization algorithm is discussed. We outlined the basics concerning the quantum processing units (QPUs) produced by D-Wave Systems which use the transverse-field Ising Hamiltonian to implement QA. The mathematical description of the objective functions and models admissible to QA QPUs is presented, which sets the context for general combinatorial optimization using these QPUs.

In Chapter 3 we take a deeper dive into solving problems directly using qubits in QA by studying the maximum independent set (MIS) problem, a well-known NP-hard problem. We find how minor-embedding arbitrarily-structured graphs to fixed QPU topologies affects performance, and what parameters are involved in doing so. We then further attempt to mitigate the performance issues encountered by embedding procedures by tuning a control parameter of the QPU, the *annealing offsets*. We use the (1+1)-CMA-ES algorithm to tune these offsets as a pre-processing step. We find that the tuning procedure improves the QPU performance on average, and the CMA-ES routine is more efficient in doing so compared to other tuning routines. The trade-off between using the tuning routine and not is explored. We then transition to solving real-world problems, as opposed to simple known models, in Chapter 4. We introduce two methods of solving combinatorial optimization problems: (i) deriving a new QUBO/Ising model from real-world data,

and (ii) processing real-world data and optimizing it using a well-known QUBO formulation. The former strategy is used to solve a real-world logistics combinatorial optimization problem, the *shipment rerouting problem*, an intermediate task of the more general less-than-truckload problem. The introduction of a mix of hard constraints (truck capacity) and simple optimization (reducing truck distances) into a combined QUBO results in a overhead in transformation which is not mitigated by the QUBO solvers tested. In the latter case of (ii), we use known discretization techniques to transform continuous time series data to a string of characters, which are then used to perform data reconstruction (from a known database). This is accomplished by formulating the reconstruction task as a set cover problem (another well-known NP-hard problem) which has a simple known QUBO formulation. We show how this set cover QUBO correctly reconstructs the original strings from fragments within the database, thus solving the original time series problem. We further show how to extend this to perform classification, which results in a semi-supervised classification algorithm.

In order to use quantum annealing in practice, we must overcome the QUBO/Ising modeling difficulties as well as the hardware limitations. To this end, we explore the use of hybrid quantum-classical algorithms in Chapter 5. We start by motivating a real-world traffic optimization problem: navigation a fleet of buses between known distances. A fully-automated hybrid optimization service is built and launched as part of a pilot project with a fleet of nine buses for the Web Summit 2019 conference. The connections between live traffic data used to build the traffic flow QUBO and accessing hybrid algorithms are constructed and deployed in practice. We find that while the service built results in stable navigation, the system is only able to handle small fleets. Therefore, in the second half of Chapter 5, we motivate another use-case for real-world optimization, the *paint shop problem*: painting a sequence of car bodies in a factory to minimize the number of color switches. We show how to construct an Ising model that directly represents the paint shop data with minimal overhead. By doing so, we include multiple techniques presented in the previous chapters of this thesis, and solve real-world problems directly on two different generations of D-Wave QPUs, as well as classical and hybrid algorithms. We are able to explore these Ising models at relatively large scales, where the problem sizes approach those of industrial relevance. While the performance of the hybrid algorithm tested showed the most utility, at the largest problem sizes a simple greedy algorithm was nearly as effective. We conclude on the work of this thesis in Chapter 6. The efficacy of the various methods used in this thesis are

discussed, and the future development work necessary to apply quantum annealing (and similar) algorithms in practice is explored.