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Applications of quantum annealing in combinatorial optimization

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Applications of quantum annealing in combinatorial optimization

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Glossary

\mathcal{T}	Universal Turing machine.
\mathcal{Q}	Universal quantum Turing machine.
\mathbb{R}	The field of real numbers.
\mathbf{B}	The domain of Boolean values, $\{0, 1\}$.
ψ	Wave function of a single qubit.
$ 0\rangle, 1\rangle$	Qubit computational basis states in bra-ket notation.
$\sigma_x, \sigma_y, \sigma_z$	The 2×2 Pauli-x, -y, and -z spin operators.
\mathcal{H}	Generic Hamiltonian operator.
\mathcal{H}_i	Initial Hamiltonian operator for Adiabatic Quantum Computing and quantum annealing.
\mathcal{H}_f	Final Hamiltonian operator for Adiabatic Quantum Computing and quantum annealing.
τ	Timescale of evolution for a time-dependent Hamiltonian.
x	Vector of N binary variables, $x_i \in \{0, 1\}$.
\mathbf{Q}	$N \times N$ QUBO matrix for binary variables.
s	Spin variable $s_i \in \{-1, 1\}$.
h_i	Linear weight for spin variable s_i (also known as bias).
J_{ij}	Quadratic interaction term for spin variables s_i and s_j (coupling strength).
$\mathcal{U}(a, b)$	The uniform random distribution between given points a and b .

CONTENTS

$\mathcal{N}(\mathbf{0}, \mathbf{I})$	The standard multivariate normal distribution.
\log	Natural logarithm.
\log_2	Natural logarithm (base 2).
${}_n C_k$	Binomial coefficient, “ n choose k ”.