Anyonic, cosmic, and chaotic: three faces of Majorana fermions

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Chapter 7

Can we use heavy nuclei to detect relic neutrinos?

7.1 Introduction

The ambitious goal of detection [83] and the measurement of the mass [222] of the relic neutrino relies on the precise experimental knowledge of the β-spectrum of radioactive elements [84, 85]. Relic neutrinos, which fill the totality of space in the form of an almost ideal gas of temperature $T_\nu \approx 1.95\,\mathrm{K}$, are expected to manifest themselves in rare neutrino capture events. Such events involving cosmic neutrinos of mass m_ν and a sample of radioactive atoms characterized by the β-decay energy Q would produce an extremely faint peak at the energy $Q + m_\nu c^2$ in the β-spectrum of the sample. We recall that for all radioactive elements the overwhelming bulk of the β-spectrum arises from spontaneous β-decay and forms a continuum with the upper cutoff energy $Q - m_\nu^0 c^2$ where m_ν^0 is the mass of the lightest neutrino. For this reason one expects the neutrino capture peak to be separated from the end of the spontaneous β-spectrum by an energy gap of at least one neutrino mass and for that reason to be discernible at least in principle.

Despite the simplicity of its theoretical premise, a neutrino capture experiment establishing the existence of relic neutrinos has not yet materialized. The reason for this is the weakness of the neutrino-matter interaction, which makes it difficult to achieve the sufficient number of capture events in a reasonably sized radioactive sample. The requirement of a large neutrino capture cross-section combined with other important considerations such as the manageable half-life time and the stability of the daughter isotope turn out to be so restrictive that only a handful of atoms can be viewed as viable candidates for the CνB detection experiment. From this perspective, Tritium has long been regarded as the best candidate β-emitter [85–87, 223–227], even though it was found that the workable sample of gaseous molecular Tritium falls short of the required activity levels by six orders of magnitude. Currently, the only viable alternative to the gas phase experiment is a solid state based architecture where the atomic tritium is adsorbed on a substrate [85].

The low event rate is not the only hindrance in the way of relic neutrino detection. The upper bounds on the neutrino mass [228] show that the energy gap between the signal from neutrino capture and the background is extremely small $m_\nu/Q \ll 1$ therefore the detection of the CνB requires extraordinary energy resolution. It has been demonstrated that the electromagnetic guidance system and the calorimetry module of the detection apparatus can be built to such stringent specifications [85], however, as it was found recently [91], deposition of β-emitters on a solid-state substrate produces a new fundamental limitation on the experimental resolution originating in the zero-point motion of the emitter's centre of mass. For Tritium on solid surfaces, the best theoretical resolution is $\Delta E \sim 0.5\,\mathrm{eV}$ which is an order of magnitude worse than what is required in order to see the relic neutrino peak. Furthermore, it was shown [91] that the main factor that determines it is the the ratio of the β-decay energy Q to the mass of the emitter nucleus m_{nucl}, namely $\gamma = \sqrt{Q^2 m_{\mathrm{e}}/m_{\mathrm{nucl}}^3}$. This finding opens a new avenue to search for a possible alternative for Tritium that would have both a sufficient event rate and low enough energy uncertainty. In the same work [91], it was found that the two promising candidates that have low enough γ-values are Thulium ($^{171}\mathrm{Tm}$) and Samarium ($^{151}\mathrm{Sm}$) with $\gamma_{^3\mathrm{H}}/\gamma_{^{171}\mathrm{Tm}} = 0.11$ and $\gamma_{^3\mathrm{H}}/\gamma_{^{151}\mathrm{Sm}} = 0.1$ respectively. This means that the intrinsic energy uncertainty for these isotopes is an order of magnitude smaller than that of Tritium. This value approaches the upper bound for the neutrino mass and therefore could, in principle, provide sufficient energy resolution for its detection.

The γ-value introduced in the previous paragraph is defined in terms of the simple intrinsic characteristics of a nucleus such as its mass and Q-value and therefore is straightforward to calculate. In contrast, the neutrino capture cross-section has not been calculated for every isotope, in particular it is not known for either of the isotopes of interest, $^{171}\mathrm{Tm}$ and $^{151}\mathrm{Sm}$. The reason for this is twofold. Firstly, the theory of β-decay of certain nuclei (the ones that undergo the so-called *non-unique forbidden transitions*) is complicated [229–231] and does not provide a direct link between the observed half-life time and the predicted neutrino capture rate. Secondly, experimental β-spectra are not normally known with the energy resolution sufficient for a direct inference of the capture cross section. The goal of the present paper is to show how the neutrino capture cross section of a given radioactive isotope decaying through non-unique forbidden transitions can be estimated from the experimentally accessible β-spectrum of that isotope.

7.2 Quantum mechanics of β-interaction and crude estimate of neutrino capture

Neutrino capture and β decay are the same process driven by the weak interaction; they differ only in whether the (anti)neutrino is in the initial or final state. To establish the exact connection between their respective rates, we start from briefly reminding the main concepts of β decay theory. We consider the sibling processes of β-decay and neutrino capture by a generic nucleus

$$\begin{aligned} (A,Z) &\to (A,Z+1) + e^- + \bar{\nu}_e \\ \nu_e + (A,Z) &\to (A,Z+1) + e^-. \end{aligned} \tag{7.1}$$

which are driven by the same weak β-decay Hamiltonian

$$\mathcal{H}^\beta = \frac{G_\beta}{\sqrt{2}} \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_\nu \; \bar{p}\gamma_\mu (g_V + g_A\gamma_5) n + \text{h.c.}, \tag{7.2}$$

where $G_\beta = G_F \cos\theta_C$ and θ_C is Cabbibo angle, ψ_e, ψ_ν are electron and neutrino fields and p, n being the proton and neutron fields respectively. The vector g_V and axial g_A coupling constants are renormalized by strong interactions with $|g_A/g_V| \approx 1.27$ [232, 233].

The differential β-decay rate $d\Gamma_\beta$ and the capture cross-section for spin-averaged neutrino are given by the Fermi Golden Rule and can be written as[1]:

$$\begin{aligned} d\Gamma_\beta &= \frac{1}{2\pi^3} \times p_\nu E_\nu p_e E_e dE_e \times W_\beta(p_e, p_\nu) \\ (\sigma v)_\nu &= \lim_{p_\nu \to 0} \frac{1}{\pi} \times p_e E_e \times W_\nu(p_e, p_\nu), \end{aligned} \tag{7.3}$$

[1] Here we use the fact that absorption of antineutrino with momentum p_ν is equivalent to emission of neutrino with momentum $-p_\nu$

where $p_{e(\nu)}$ and $E_{e(\nu)}$ are the momenta and energies of the leptons, $W_\beta(p_e, p_\nu)$ is the average transition rate for the decay of an atom into two lepton plane waves with momenta p_e, p_ν, and $W_\nu(p_e, p_\nu)$ is the average transition rate for the capture of a neutrino having the momentum p_ν and the emission of an electron with momentum p_e.

The average transition rates are expressed in terms of transition amplitudes by

$$W_{\beta,\nu}(p_e, p_\nu) = \int \frac{d\Omega_e}{4\pi} \int \frac{d\Omega_\nu}{4\pi} \sum |\mathcal{M}_{\rm if}^{\beta,\nu}(\mathbf{p}_e, \mathbf{p}_\nu)|^2. \tag{7.4}$$

Here $\mathcal{M}_{\rm if}$ is the quantum transition amplitude between the initial and the final state induced by the reduced weak interaction Hamiltonian [230, 231, 234]

$$\mathcal{M}_{\rm if} = \frac{G_\beta}{\sqrt{2}} \int \bar{\psi}_e(\mathbf{r})\gamma^\mu(1-\gamma_5)\psi_\nu(\mathbf{r}) \times J^\mu_{\rm nuclear}(\boldsymbol{r})\, d\boldsymbol{r}, \tag{7.5}$$

which encapsulates all information about the changes in the internal nuclear structure in a function $J^\mu_{\rm nuclear}(\mathbf{r})$. This function cannot be calculated from first principles, however its transformation properties under the symmetry group of space are known for each transition. The summation symbol in Eq. (7.4) is a shorthand for the sum over the spin quantum numbers of the out-states as well as averaging over the spins of the in-states. The averaging over the directions of $\mathbf{p}_e$ and $\mathbf{p}_\nu$ is shown explicitly. Two important remarks are in order

1 For an overwhelming part of the β-spectrum one can consider the neutrino as a massless (Weyl) particle in both the energy conservation law and the wave functions entering the transition amplitudes. There exists a tiny energy window on the order of m_ν near the high-energy end of the β-spectrum where the neutrino mass plays a role, however the resolution required for the observation of the β-spectrum inside that window is by far beyond the reach of the existing experimental technique. Since the existing β-decay experiment cannot distinguish between the massive and massless cases, *we shall throughout this note discuss the function $W_\beta(p_e, p_\nu)$ assuming the $m_\nu \to 0$ limit.*

2 Our main focus is on neutrino capture processes involving the cosmic neutrino background. For such neutrinos $p_\nu \ll m_\nu$, which is the opposite of the ultra-relativistic limit discussed in item 1. It is straightfoward to see that for a left-handed particle with a Majorana mass term,

$$W_\nu(p_e, 0) = \frac{1}{2} \lim_{p_\nu \to 0} W_\beta(p_e, p_\nu) \tag{7.6}$$

Indeed, in the $p_\nu \to 0$ limit the incoming massive neutrino is a superposition of a left-handed Weyl particle and a right-handed Weyl antiparticle $|\text{Majorana}\rangle = (|\nu\rangle + |\bar{\nu}\rangle)/\sqrt{2}$. In a process where an electron is created, the operator (7.5) only picks one term of the two, hence the corresponding transition rate is one half of the transition rate W_β of a Weyl neutrino.

7.2.1 Crude estimate of neutrino capture

In this subsection, we want to provide a simple order-of-magnitude estimate for neutrino capture cross-section. To this end, we assume that the matrix element has no dependence on the lepton energy and reduces to a constant encoding the information about the initial and final nuclear states

$$\sum |\mathcal{M}^{\beta}_{\text{if}}(p_e, p_\nu)|^2 = \text{const.} \tag{7.7}$$

Such an approximation neglects the Coulomb interaction between the emitted electron and the nucleus. It also assumes that the selection rules admit for the existence of the emission channel with the total angular momentum of leptons $J = 0$.

Assuming Eq. (7.7) to be true, all the structural information about the nuclei gets absorbed into a constant numerical factor, therefore the ratio of the β decay and the neutrino capture rates, Eqns. (7.3), is completely determined by the phase volume factors $p_\nu^2 p_e E_e$ and $p_e E_e$ accordingly. Using Eq. (7.6), this gives rise to the following relationship between the capture cross-section $(\sigma v)_\nu$, the total lifetime $\tau = (\int d\Gamma_\beta)^{-1}$ of a β-decaying isotope, and the total kinetic energy Q released in the reaction:

$$(\sigma v)_\nu = \tau^{-1} \frac{(2\pi)^{-1} p_e E_e}{(2\pi^3)^{-1} \int_{m_e}^{m_e+Q} E'_e p'_e (Q - T'_e)^2 \, dE'_e}, \tag{7.8}$$

with $T_e = E_e - m_e$ being the kinetic energy of the electron, and neutrino momentum in β decay is $p_\nu = Q - T_e$. In the particular case of nonrelativistic electron $Q \ll m_e$, this relation gives the following simple scaling:

$$(\sigma v)_{\text{est.}} = 5.3 \cdot 10^{-46} \, \text{cm}^2 \times \frac{1 \, \text{year}}{\tau} \times \left(\frac{100 \, \text{keV}}{Q} \right)^3 . \tag{7.9}$$

In order to quantify the error introduced by the simplifying assumptions leading up to Eq. (7.7), we introduce a correction factor δ such that the actual cross-section is given by

$$(\sigma v)_\nu = \delta \times (\sigma v)_{\text{est.}} \tag{7.10}$$

The values of δ for a number of elements where the exact results for the neutrino capture cross-section are known [86] are given in Fig. 7.1. One can see that in all those cases δ is is reasonably close to unity.

Isotope	Q, keV	τ, year	$(\sigma v)_\nu$, $10^{-46}\,\mathrm{cm}^2$	δ
^{3}H	18.591	17.8	39.2	0.86
^{63}Ni	66.945	145	$6.9 \cdot 10^{-2}$	0.57
^{93}Zr	60.63	$2.27 \cdot 10^6$	$1.20 \cdot 10^{-5}$	1.15
^{106}Ru	39.4	1.48	29.4	0.51
^{107}Pd	33	$9.38 \cdot 10^6$	$1.29 \cdot 10^{-5}$	0.83
^{187}Re	2.646	$6.28 \cdot 10^{10}$	$2.16 \cdot 10^{-6}$	0.48

Table 7.1. Neutrino capture cross-sections for different isotopes from [86]. Note that $(\sigma v)_\nu$ differ from those of [86] by two due to neutrino spin averaging, as pointed out in [227]. One can see that the parameter δ defined by Eq. (7.10) varies only by a factor of two from the identity that signals that Eq. (7.9) gives a good approximation for the capture rates of the given isotopes.

We are interested in neutrino capture by possible candidates for solid-state based CνB detection experiments — ^{171}Tm and ^{151}Sm. For these isotopes, the parameterization (7.10) reads

$$
\begin{aligned}
(\sigma v)_{^{171}\mathrm{Tm}} &= 2.1 \cdot 10^{-46}\,\mathrm{cm}^2 \times \delta_{^{171}\mathrm{Tm}} \approx 0.054\ (\sigma v)_{^3\mathrm{H}} \times \delta_{^{171}\mathrm{Tm}} \\
(\sigma v)_{^{151}\mathrm{Sm}} &= 9.1 \cdot 10^{-48}\,\mathrm{cm}^2 \times \delta_{^{151}\mathrm{Sm}} \approx 0.0023\ (\sigma v)_{^3\mathrm{H}} \times \delta_{^{151}\mathrm{Sm}}.
\end{aligned} \tag{7.11}
$$

However, unlike the isotopes listed in Table 7.1, the theoretical values of the δ factors for ^{171}Tm and ^{151}Sm are not known. This is because both isotopes have a rather peculiar structure of the matrix element (7.5), as explained in the following paragraph.

For purely illustrative purposes we neglect the effect of the Coulomb attraction between the β-electron and the daughter nucleus, bearing in mind that in practice such an approximation may result in significant inaccuracy. We recall that the function $J^\mu_{\mathrm{nuclear}}(\boldsymbol{r})$ is mainly localized inside the nucleus $r < R$, and decays rapidly with increasing r for $r > R$. Here $R = A^{1/3} \times 1.2 \times 10^{-13}$ cm is the radius of the nucleus. Since the typical lepton momentum is on the order $1\,\mathrm{MeV} \ll R^{-1}$, one can expand the matrix elements and the sum $\sum |\mathcal{M}_{\mathrm{if}}|^2$ as a series in small parameters $p_{e/\nu} R \ll 1$[2]

[2]If Coulomb attraction is taken into account, the constants in this expansion get multiplied by correction factors $F_i(p_e)$, which do not depend on unknown nuclear physics and can be computed explicitly.

$$\sum |\mathcal{M}_{\rm if}|^2 = c_0 + c_1 \cdot p_e R + c_2 \cdot p_\nu R + \ldots \tag{7.12}$$

The constants c_i in this expression are in essence combinations of the spherical multipole moments of $J^{\mu}_{\rm nuclear}(\mathbf{r})$ containing structural information about the many-body wave functions of the parent and daughter nuclei. The simplifying approximation (7.7) amounts to keeping only the leading-order term c_0 in the expansion (7.12), which in many cases is well justified. For some isotopes, however, electroweak selection rules demand that $c_0 = 0$. Indeed, if the mother and daughter isotopes have different spin and parity then at least one of the leptons is required to carry a non-vanishing orbital angular momentum. Since a lepton's wave function corresponding to the orbital angular momentum l has the asymptotic form $(pr)^l$ at small r, the matrix element of such a transition, Eq. (7.5), will necessarily contain terms proportional to $(p_e R)^l (p_\nu R)^{l'}$ with $l + l' > 0$. The worst case scenario, known as a *forbidden non-unique transition*, is when the selection rules admit for the presence of several commensurate leading-order terms on the fight hand side of the asymptotic expansion Eq. (7.12). For such a transition one has to assume that the matrix element (7.12) contains several unknown constants c_i each multiplying its own unique function of energy. If that happens, the cancellation of the unknown constants, such as the one seen in Eq. (7.8), does not occur and the neutrino capture cross-section cannot be inferred from the isotope's life time. This is precisely what happens for ^{171}Tm and ^{151}Sm. We conclude, that for the isotopes of our interest, ^{171}Tm and ^{151}Sm, the values of the δ factors, Eq. (7.11), are beyond the reach of pure theory, which naturally brings us to the next section.

7.3 Experimental determination of the neutrino capture rate from the end of the β decay spectrum

We have established that for isotopes such as ^{171}Tm and ^{151}Sm the knowledge of the lifetime and the Q-value is insufficient in order to predict the neutrino capture cross-section. Here, we discuss how the required cross-section can be inferred directly from the experimentally measured β-spectrum. Our approach is based on two key observations. Firstly, both the emission and capture processes are governed by the same unknown structure function $W_\beta(p_e, p_\nu)$, albeit taken at different values of arguments. Specifically, a capture process corresponds to the limit $p_\nu \to 0$ and $p_e = \sqrt{(Q + m_e)^2 - m_e^2}$, whilst in a spontaneous β-decay process $p_e = \sqrt{(Q + m_e - p_\nu)^2 - m_e^2}$, where p_ν can take any value between 0 and Q, resulting in a broad β-spectrum. Secondly, the function $W_\beta(p_e, p_\nu)$ is an analytic function of both arguments near the end point $p_\nu = 0$ of the β-spectrum [93]. We recall that in our discussion $W_\beta(p_e, p_\nu)$ is the rate involving transitions with massless neutrino states (see discussion at the end of section 7.2).

Using the analyticity of $W_\beta(p_e, p_\nu)$ and making use of equations (7.3) and (7.6) we write the following expansion[3] for the observable β-spectrum near the edge $p_\nu = 0$

$$\frac{\pi^2}{p_\nu^2}\frac{d\Gamma_\beta}{dE_e} = (\sigma v)_\nu \times \left[1 + \alpha_1 p_\nu/Q + O(p_\nu^2/Q^2)\right] \tag{7.13}$$

where α_1 is a constant. The characteristic energy scale where the linear approximation is applicable can be estimated from the microscopic theory of β decay [93]. For the purposes of the present work, we notice that the physics of β decay of heavy nuclei involves three important of energy scales, that is Q, m_e, and $1/R_0$ where R_0 is the radius of the nucleus. The smallest of the three defines the energy range where the expansion (7.13) works well. For ^{171}Tm and ^{151}Sm the smallest energy scale is Q.

Now we are in position to discuss the experimental procedure. We assume a finite energy resolution ΔE of the experiment (say, 1 keV). We propose a way to deduce the neutrino capture rate of the ^{171}Tm and ^{151}Sm from the end of their experimentally measured β spectra:

1. Define some experimentally accessible energy resolution $\Delta E \ll Q$ and measure the number of β decay events N in several energy bins[4] $T_e \in [Q - (n+1)\Delta E, Q - n\Delta E]$ as a function of the electron energy residue $\varepsilon_n = \Delta E(n + 1/2)$

2. We assume that all the decay events are detected. In this case, one can check whether the experimental points $N(\varepsilon_n) \times (\varepsilon_n \text{ in keV})^{-2}$ fit the linear curve. If so, continue the obtained fit up till the value $\varepsilon_n = 0$.

[3]Such a linear behaviour can be seen in the spectra generated by the BetaShape software, which predicts $(\sigma v)_\nu = 1.2 \cdot 10^{-46}\,\text{cm}^2(^{171}\text{Tm})$, $4.8 \cdot 10^{-48}\,\text{cm}^2(^{151}\text{Sm})$ and $\alpha_1 = 0.25(^{171}\text{Tm})$, $0.21(^{151}\text{Sm})$. For further discussion see Sec. 7.4 and our accompanying paper [93].

[4]We note that the spectrum itself behaves as $d\Gamma/dE_e \sim p_\nu^2$ and, therefore, events within a single bin are not uniformly distributed. Most of the events occur near the left side of a bin, which may introduce an additional systematic uncertainty. A possible way to avoid this problem and is to measure the integral number of events $N(p_\nu) = \int_{Q-p_\nu}^{Q} \frac{dN}{dT_e} dT_e$ and consider the function $N(p_\nu) \cdot p_\nu^{-3}$. This can be also fitted by a linear function and therefore used to extract $(\sigma v)_\nu$. In addition, this method allows to collect more statistics compared to the one with bins for sufficiently large p_ν.

3. Assuming that the time of the measurement is $T_{\rm m} \ll \tau$ and there are $N_{\rm at}$ decaying atoms, the neutrino capture rate can be estimated as

$$(\sigma v)_\nu = \frac{7.0 \cdot 10^{-37}\,{\rm cm}^2}{(T_{\rm m}\ \text{in hours})(\Delta E\ \text{in keV})} \times \frac{1}{N_{\rm at}} \left(\frac{N(\varepsilon_n)}{(\varepsilon_n\ \text{in keV})^2} \right) \Bigg|_{\varepsilon_n = 0} \quad (7.14)$$

A remark should be made concerning the generality of (7.14). Until now we neglected possible contributions to the electron spectrum due to β-decay into excited states of daughter nuclear or/and electronic shell of the atom. Let us comment on these contributions:

1. Excited nuclear states have typical energies $E_{\rm ex} \sim 10\,{\rm keV}$, for instance, 66.7 keV for ^{171}Yb [235] (daughter isotope for ^{171}Tm) and 21.5 keV of ^{151}Eu [236] (daughter isotope of ^{151}Sm). They do not contribute to the spectrum near the endpoint for $T_e > Q - E_{\rm ex}$. Therefore, they are not relevant for the energy resolution of order 1 keV.

2. Atomic excitations are of order 1 eV. If the energy resolution is much above this scale, Eq. (7.14) overestimates the value of cross-section. For $Z \sim 60$, the probability to excite the electronic configuration is expected to be less than 30% [237], which translates into the same possible error in the value of the cross-section.

The corrections discussed above may only introduce a difference by a prefactor of order one are therefore are beyond our considerations.

7.4 Conclusion and discussion

The most promising route towards the relic neutrino detection is currently through the use of solid state based detectors where the β emitters are adsorbed on a substrate. Such a design has the potential to to achieve sufficient density of emitters in a controllable way (such that electron scattering remains suppressed), and hence get a sufficient number of capture events. However, any β decay experiment that uses bound emitters (either in molecular form or adsorbed on a substrate) suffers from an irreducible intrinsic energy uncertainty due to the emitter's zero-point motion. It was shown in [91] that such an uncertainty is proportional to the dimensionless parameter $\gamma = \sqrt{Q^2 m_e / m_{\rm nucl}^3}$, Q being the energy released in the β decay, $m_e, m_{\rm nucl}$ - masses of the electron and nucleus respectively. It was also shown that this parameter is too large for

^{3}H, therefore Tritium-based detectors are unable to achieve the required energy resolution. Instead, the most promising candidates are ^{171}Tm and ^{151}Sm as they have the intrinsic energy uncertainty that is an order of magnitude lower than that of ^{3}H.

However, contrary to the case of ^{3}H for which the neutrino cross section is known [86, 87], theoretical calculation of $(\sigma v)_\nu$ for ^{171}Tm and ^{151}Sm poses a challenge. The quantum numbers (spin and parity) of the parent and daughter nuclei for these isotopes differ, hence the leptons are required to have a non-zero total orbital momentum. The latter can be composed in a non-unique way, which results in several different unknown nuclear constants entering the matrix element (7.5) that do not factor out.

We propose a way to estimate the relic neutrino capture cross section. Our proposal relies on the experimental measurement of the spectrum of β-decay near the endpoint. We show, that the extraction of the relic neutrino cross section can be achieved using the experimental data (via Eq. (7.14)) even if the energy resolution ΔE of the experiment that is much larger than neutrino mass $\Delta E \gg m_\nu$.

Finally, to get a rough idea of the feasibility of the relic neutrino capture experiment based on ^{171}Tm ($Q = 96.5\,\text{keV}$, $\tau = 2.77\,\text{years}$) or ^{151}Sm ($Q = 76.6\,\text{keV}$, $\tau = 130\,\text{years}$), we estimate the corresponding cross sections using the β-decay spectra computed in BetaShape [238, 239]. For ^{171}Tm and ^{151}Sm, this code uses the so-called ξ-approximation, whose validity has to be established on the case to case basis.

BetaShape predicts the following neutrino capture rates $\Gamma_\nu = \eta_\nu (\sigma v)_\nu$ per single atom:

$$\frac{\Gamma_{\text{capture}}}{\text{y}^{-1}} = \frac{\eta_\nu}{\langle \eta_\nu \rangle} \begin{cases} 12.7\,(6.4) \times 10^{-27} & ^{171}\text{Tm} \\ 5.1\,(2.5) \times 10^{-28}, & ^{151}\text{Sm} \end{cases} \tag{7.15}$$

for Majorana (Dirac) neutrino, where η_ν is the local cosmic number density of one neutrino species which could be significantly larger than the average over the universe $\langle \eta_\nu \rangle \sim 56\,\text{cm}^{-3}$ due to gravitational clustering. The corresponding cross-sections are in agreement with the crude estimate ($\delta \approx 0.5$).

Since the emitters in the solid-state based experiments are attached to the substrate atom by atom, the single event exposure based on the estimate (7.4) corresponds to $2 \cdot 10^{27}\,\text{atoms} \cdot \text{year}$ of ^{151}Sm or $10^{26}\,\text{atoms} \cdot \text{year}$ of ^{171}Tm. For comparison, the same number of events can be achieved with $2 \cdot 10^{24}\,\text{atoms} \cdot \text{year}$ of ^{3}H. According to this, using ^{171}Tm as β emitter in a full size CνB experiment is promising since it can provide with *both* sufficient event rate *and* energy resolution for the relic neutrino detection.

We emphasize that the results based on BetaShape might be inaccurate and the measurement is still needed to confirm them. We discuss the approximation used in BetaShape in the follow-up paper [93], together with independent theoretical bounds on $(\sigma v)_\nu$.