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## Anyonic, cosmic, and chaotic: three faces of Majorana fermions

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# **Anyonic, cosmic, and chaotic: three faces of Majorana fermions**

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The cover shows artistic representation of the portraits of three scientists that made significant contribution to the fields studied in this thesis. Ettore Majorana (upper right) proposed a solution to the Dirac equation that corresponds to a fermion that can be its own anti-particle. Such fermion was later called Majorana. Alexei Kitaev (left) contributed a lot to the study of the SYK model. Wolfgang Pauli (lower right) invented neutrinos to explain  $\beta$ -decay spectrum of radioactive atoms. The green stripe symbolizes unification of the corresponding topics in this thesis. The plant on the back is Marjoram which in Ukrainian is spelled in the same way as Majorana. The cover is a digital painting made by the author of the thesis.

*To my first scientific mentor, V. Yu Reshetnyak*  
*Моему первому научному наставнику, В. Ю. Решетняку*



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Preface . . . . .	1
1.2	Majorana fermions as fundamental particles . . . . .	3
1.2.1	Dirac equation . . . . .	4
1.2.2	Majorana solution . . . . .	6
1.3	Majorana fermions in condensed matter physics . . . . .	6
1.3.1	The need in superconductivity . . . . .	7
1.3.2	Topologically protected zero modes . . . . .	8
1.3.3	Andreev levels . . . . .	10
1.4	Anyonic Majorana fermions . . . . .	12
1.5	Chaotic Majorana fermions . . . . .	18
1.5.1	Mean field solution for cSYK . . . . .	20
1.5.2	SYK in the lab . . . . .	24
1.6	Cosmic Majorana fermions . . . . .	26
1.6.1	$\beta$ decay on the surface . . . . .	29
1.7	This thesis . . . . .	34
1.7.1	Chapter 2 . . . . .	35
1.7.2	Chapter 3 . . . . .	35
1.7.3	Chapter 4 . . . . .	35
1.7.4	Chapter 5 . . . . .	35
1.7.5	Chapter 6 . . . . .	36
1.7.6	Chapter 7 . . . . .	36
1.7.7	Chapter 8 . . . . .	36
<b>2</b>	<b>Pfaffian formula for fermion parity fluctuations in a superconductor and application to Majorana fusion detection</b>	<b>39</b>
2.1	Introduction . . . . .	39
2.2	Pfaffian fermion-parity formula . . . . .	41

2.2.1	Kitaev's formula for an isolated system . . . . .	41
2.2.2	Pfaffian formula for a subsystem . . . . .	42
2.3	Connection with the Majorana fusion rule . . . . .	43
2.4	Random-matrix theory . . . . .	45
2.4.1	Skew Circular Real Ensemble . . . . .	45
2.4.2	Distribution of the local fermion parity in the skew-CRE	47
2.4.3	RMT model of weakly coupled quantum dots . . . . .	49
2.5	Effect of an isolated Majorana zero-mode . . . . .	52
2.6	Conclusion . . . . .	52
2.7	Appendix: Derivation of the Pfaffian formula from Klich's counting statistics theory . . . . .	53
2.8	Appendix: Moments of determinants of antisymmetric random matrices . . . . .	56
2.8.1	Principal minor of antisymmetric orthogonal matrix . .	56
2.8.2	Antisymmetric Hermitian matrix . . . . .	57
<b>3</b>	<b>Dynamical signatures of ground-state degeneracy to discriminate against Andreev Levels in a Majorana fusion experiment</b>	<b>59</b>
3.1	Introduction . . . . .	59
3.2	Adiabatic evolution to test for ground-state degeneracy . . . . .	60
3.3	Topologically degenerate ground state . . . . .	62
3.4	Accidentally degenerate Andreev levels . . . . .	63
3.5	Conclusion . . . . .	67
<b>4</b>	<b>Reentrant superconductivity in a quantum dot coupled to a Sachdev-Ye-Kitaev metal</b>	<b>69</b>
4.1	Introduction . . . . .	69
4.2	Main part . . . . .	70
4.3	Conclusion . . . . .	77
4.4	Appendix:Derivation of the gap equation . . . . .	78
4.5	Appendix: Saddle-point numerical analysis . . . . .	80
4.5.1	The algorithm . . . . .	80
4.5.2	Precision and grid . . . . .	82
<b>5</b>	<b>Quantum tunneling dynamics in a complex-valued Sachdev-Ye-Kitaev model quench-coupled to a cool bath</b>	<b>85</b>
5.1	Introduction . . . . .	85
5.2	The model . . . . .	86
5.3	Relaxation after the quench . . . . .	88

5.4	Tunneling current . . . . .	93
5.5	Conclusion . . . . .	97
5.6	Appendix: Derivation of the Kadanoff-Baym equations from the SYK saddle-point . . . . .	98
5.6.1	Saddle-point equations . . . . .	98
5.6.2	Reservoir as an external potential . . . . .	100
5.6.3	Dynamics of the SYK subsystem . . . . .	101
6	<b>Navigating the pitfalls of relic neutrino detection</b>	103
6.1	Introduction . . . . .	103
6.2	Defining the problem . . . . .	105
6.3	Estimate . . . . .	108
6.4	Discussion . . . . .	110
6.5	Appendix: Quantum derivation of the energy uncertainty . . . . .	112
7	<b>Can we use heavy nuclei to detect relic neutrinos?</b>	117
7.1	Introduction . . . . .	117
7.2	Quantum mechanics of $\beta$ -interaction and crude estimate of neu- trino capture . . . . .	119
7.2.1	Crude estimate of neutrino capture . . . . .	121
7.3	Experimental determination of the neutrino capture rate from the end of the $\beta$ decay spectrum . . . . .	123
7.4	Conclusion and discussion . . . . .	125
8	<b>Screening effects in the graphene-based relic neutrino detection exper-         iment</b>	129
8.1	Introduction . . . . .	129
8.2	Defining the problem . . . . .	130
8.3	Charge screening effects . . . . .	132
8.4	Electron-hole pair creation . . . . .	135
8.5	Conclusions . . . . .	136
8.6	Appendix: Average work performed by the electrons in graphene on the emitted $\beta$ -electron . . . . .	137
8.7	Appendix: Cross section of the process of the electron-hole cre- ation in graphene . . . . .	139
	<b>Bibliography</b>	143
	<b>Summary</b>	163

<b>Samenvatting</b>	<b>165</b>
<b>Curriculum Vitæ</b>	<b>167</b>
<b>List of Publications</b>	<b>169</b>