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Anyonic, cosmic, and chaotic: three faces of Majorana fermions

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Citation

Cheipesh, Y. I. (2022, November 17). *Anyonic, cosmic, and chaotic: three faces of Majorana fermions. Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/3487143>

Version: Publisher's Version

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Anyonic, cosmic, and chaotic: three faces of Majorana fermions

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof. dr. ir. H. Bijl,
volgens besluit van het college voor promoties
te verdedigen op donderdag 17 november 2022
klokke 13.45 uur

door

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geboren te Uzhgorod, Oekraïne
in 1994

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Casimir PhD Series Delft-Leiden 2022-32

ISBN 978-90-8593-542-1

An electronic version of this thesis can be found at <https://openaccess.leidenuniv.nl>

The cover shows artistic representation of the portraits of three scientists that made significant contribution to the fields studied in this thesis. Ettore Majorana (upper right) proposed a solution to the Dirac equation that corresponds to a fermion that can be its own anti-particle. Such fermion was later called Majorana. Alexei Kitaev (left) contributed a lot to the study of the SYK model. Wolfgang Pauli (lower right) invented neutrinos to explain β -decay spectrum of radioactive atoms. The green stripe symbolizes unification of the corresponding topics in this thesis. The plant on the back is Marjoram which in Ukrainian is spelled in the same way as Majorana. The cover is a digital painting made by the author of the thesis.

To my first scientific mentor, V.Yu Reshetnyak
Моєму першому науковому наставнику, В. Ю. Решетняку

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