Playing dice with the universe: Bayesian statistical analyses of cosmological models and new observables

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Chapter 3

Constraints on α-attractor models

This chapter is based on:
Current and future constraints on single-field α-attractor models
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3.1 Introduction

In this chapter, we forecast the possible constraints that a future CMB StageIV (CMB-S4 hereafter) experiment may impose on inflationary observables in the optimistic scenario of a detection of non-vanishing tensor anisotropies in the Cosmic Microwave Background (CMB) polarization and temperature data. In general, the approach followed within the community (see e.g. (Planck Collaboration et al., 2014b, 2016; Chiang et al., 2010; Ade et al., 2016; Planck Collaboration, Aghanim, Akrami, Ashdown, et al., 2020b)) is to sample the inflationary parameters without assuming any specific inflationary model a priori. While this approach has the advantage of exploring the inflationary sector model-independently, it does not allow for a complete sampling study of the parameter space in a specific model. Moreover, the assumption that the inflationary observables are independent of one another is in contrast with the prediction of any theory of inflation, which, for instance, assumes the validity of the slow-roll conditions (see e.g. (Renzi, Shokri, & Melchiorri, 2020; Shokri, Renzi, & Melchiorri, 2019; Giarè, Di Valentino, & Melchiorri, 2019)). In this work, conversely to the current literature on the subject, we follow a model-dependent approach imposing a specific model *a priori* and calculate the inflationary observables directly imposing the slow-roll conditions on the inflationary potential.

As mentioned in Chapter 1, the ΛCDM model is based on the simplest inflationary paradigm: canonical slow-roll single-field inflation. Within this approach, the power spectra of scalar and tensor comoving curvature perturbations are parametrised as power laws:

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_\star^S}\right)^{n_s - 1 + \frac{\alpha_S}{2}\log k/k_\star^s} \tag{3.1}$$

$$P_T(k) = rA_s \left(\frac{k}{k_\star^t}\right)^{n_t} \tag{3.2}$$

where $k_\star^s = 0.05$ Mpc^{-1} and $k_\star^t = 0.002$ Mpc^{-1} and the subscripts stand for scalar and tensor perturbations respectively. The powers of the parametrizations are the scalar and tensor indices (n_s and n_t) and the scalar power spectrum have been further expanded in terms of the running of the spectral index α_S. Statistical analysis of recent cosmological observations (Planck observations of the Cosmic Microwave Background (CMB) (Planck Collaboration, Akrami, et al., 2020; Planck Collaboration, Aghanim, Akrami, Ashdown, et al., 2020c, 2020b) and Large Scale Structure (LSS) surveys (To et al., 2021; Heymans et al., 2020)) support this parametrisation for the scalar fluctuations with $10^9 A_S \approx 2.1$ and $n_s \approx 0.965$ (Planck Collaboration, Akrami, et al., 2020; Planck Collaboration, Aghanim, Akrami, Ashdown, et al., 2020c, 2020b).

In the last decade, the bound on the amplitude of PGWs (parametrized typically with the tensor-to-scalar ratio, r) has not yet seen significant improvement, where only an upper limit $r_{0.002} < 0.056$ at 95% C.L. has been provided in the last data release of the Planck Collaboration (Planck Collaboration, Akrami, et al., 2020) combining Planck and BICEP2/Keck array (BK15) data (Ade et al., 2018). Detecting those PGWs would give a direct measurement of the energy scale during inflation, as well as a clear distinguishable signature of the quantum origin of primordial fluctuations. In the upcoming decade, a new generation of CMB experiments (e.g. BICEP3 (Grayson et al., 2016), CLASS (Essinger-Hileman et al., 2014) , SPT-3G (Benson et al., 2014), Advanced ACTPol (Henderson et al., 2016), LBIRD (Suzuki et al., 2018) and CMB-S4 (K. N. Abazajian et al., 2016)) are expected to strongly improve the sensitivity on the B-modes polarization in the Cosmic Microwave Background (CMB), possibly revealing first evidences for inflationary tensor modes with amplitudes $r \sim 0.01 - 0.001$. That range is precisely expected in many well-motivated models, such as the Starobinsky inflation, which is considered the benchmark of future CMB experiments. However, while a measure of a non-vanishing r would be of key importance for inflationary theories, it will not allow understanding the inflationary mechanism in detail but only its energy scale. It is therefore timely to investigate, given future CMB experiments, what would be the freedom in a generic inflationary framework that is left in case of the optimistic scenario of a non-vanishing tensor-to-scalar ratio measure.

In particular, there is a general class of models called α-attractors, that has gained lots of popularity because of their agreement with observational constraints and the universality of their predictions for the inflationary observables (Iarygina, Sfakianakis, Wang, & Achucarro, 2019; Iarygina, Sfakianakis, Wang, & Achúcarro, 2020; Aresté Saló, Benisty, Guendelman, & Haro, 2021b, 2021a; Rodrigues, Santos da Costa, & Alcaniz, 2021). Recently, α-attractors have also been of the interest to study a possible connection between early and late physics in the context of Dark Energy

(Akrami, Kallosh, Linde, & Vardanyan, 2018; Akrami, Casas, Deng, & Vardanyan, 2021; Miranda, Fabris, & Piattella, 2017; Pozdeeva, 2020; Pozdeeva & Vernov, 2021). This set of models have also been embedded in a more general multi-field inflationary scenario and in $\mathcal{N} = 1$ supergravity. In the context of supergravity, the α-attractor can be represented by a potential of the form:

$$\frac{V(\varphi)}{V_0} = (\tanh(\beta\varphi/2))^{2n} \tag{3.3}$$

where $\beta^2 = 2/3\alpha$ and n is an arbitrary value. It is important to note that the "attractor behaviour" of this potential sits on the fact that the observable predictions are the same up to leading order regardless of the value of n while they differ only in sub-leading corrections. Assuming slow-roll inflation and the α-attractor form of the inflationary potential the observational predictions for the inflationary observables can be written as:

$$r = \frac{12\alpha}{N^2} \tag{3.4a}$$

$$n_s = 1 - \frac{2}{N} = 1 - \sqrt{\frac{r}{3\alpha}}, \tag{3.4b}$$

$$\alpha_S = -\frac{2}{N^2} = -\frac{r}{6\alpha}. \tag{3.4c}$$

where N is the number of e-folds to inflation to last. These definitions in terms of parameter α encompass several inflationary models and clearly reduce to the well-known Starobinsky inflation for $\alpha = 1$ (Kallosh, Linde, & Roest, 2013; Kallosh, Linde, Roest, & Yamada, 2017; Carrasco, Kallosh, Linde, & Roest, 2015). Moreover, for a broad class of potentials V, as long as $\alpha \ll O(1)$, the scalar spectral index n_s, its running α_S and the tensor-to-scalar ratio r converge to the functional form of equation (3.4) regardless of the kinectic terms of the theory. It has also been showed that this statement holds true in some multi-field inflation regimes (Achúcarro, Kallosh, Linde, Wang, & Welling, 2018), where the conditions that guarantee the universality of the observational predictions for the inflationary parameters are derived by imposing constraints on the potential. The universality of the observational constraints is one of the most important features of single-field α-attractor models.

3.2 Constraints from current CMB and LSS data

Current CMB (Planck Collaboration et al., 2014b, 2016; Chiang et al., 2010; Ade et al., 2016; Planck Collaboration, Aghanim, Akrami, Ashdown, et al., 2020b) and LSS (To et al., 2021) data are unable to constrain the tensor-to-scalar ratio if r is sampled independently from the scalar index n_s. However, by imposing the α-attractor model a priori, we force a specific functional relation between n_s and r that allows us to translate the sub-percentage constraints on n_s from current data into a constraint on r in the context of α-attractors.

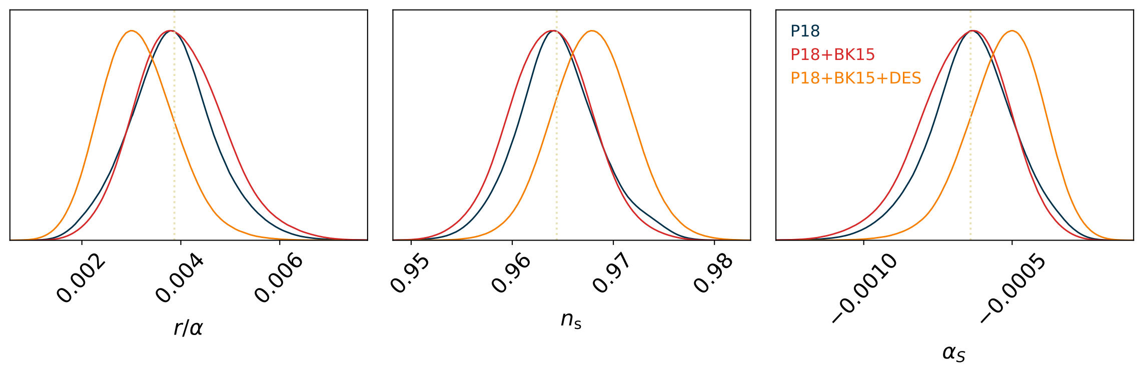

Figure 3.1: Posterior distribution for Planck 2018 data alone and combined with the Biceps/Keck 2015 B-mode data and with Large Scale Structure DES data. The dotted lines denote the expected values for Staronbinsky inflation ($\alpha = 1$ and $N \approx 60$).

In fact, by imposing an inflationary model we are selecting a subset of the parameter space allowed by the data when r and n_s are considered independently in cosmological parameter estimations. This is particularly evident if one considers the relation between r and n_s in the α-attractor model given by equation (3.4b).

Current data cannot break the $r-\alpha$ degeneracy, and therefore, sampling it in our analysis would give no insights on the α-attractor models. For this reason, instead of sampling r and α independently, we use the ratio r/α as a parameter for our MCMC analysis.

Along with the ratio r/α, we consider as independent parameters the other five standard ΛCDM ones: the baryon $\omega_b = \Omega_b h^2$ and the CDM $\omega_c = \Omega_c h^2$ densities, the Hubble constant H_0, the optical depth τ and the amplitude of scalar perturbations A_s. We also let free the running of the spectral index α_S. As α-attractors satisfy the usual inflationary consistency relation, we fix the index of tensor modes to $n_T = -r/8$. The uniform prior distribution imposed on these parameters are reported in Table 3.1.

Parameter	Prior range
$\Omega_b h^2$	$[0.005, 0.1]$
$\Omega_c h^2$	$[0.001, 0.99]$
H_0	$[20, 100]$
τ	$[0.01, 0.8]$
$r_{0.002}/\alpha \cdot 10^3$	$[0.5, 8]$
$\log(10^{10} A_s)$	$[1.61, 3.91]$
$\sum m_\nu$	$[0, 1]$
N_{eff}	$[2, 5]$

Table 3.1: Range of uniform priors distributions imposed on the sampled parameters during the analysis.

The predictions of the theoretical observational probes are calculated using the latest version of the cosmological Boltzmann integrator code `CAMB` (Lewis et al., 2000; Howlett et al., 2012). To compare our theoretical predictions with data, we use the full 2018 Planck temperature and polarization datasets which also includes multipoles $\ell < 30$ (Planck Collaboration, Aghanim, Akrami, Ashdown, et al., 2020a). We combine the Planck likelihood with the Biceps/Keck 2015 B-mode data (Ade et al., 2018) and the combination of galaxy clustering and weak lensing data from the first year of the Dark Energy Survey (DES Y1) (T. M. C. Abbott et al., 2018). The posterior distributions of the cosmological parameters have been explored using the publicly available version of the Bayesian analysis tool `Cobaya` (Torrado & Lewis, 2021). In particular, the posteriors have been sampled using the MCMC algorithm developed for CosmoMC (Lewis & Bridle, 2002; Lewis, 2013) and tailored for parameter spaces with a speed hierarchy.

The 1D posterior of r/α, n_s and α_S resulting from our Bayesian statistical analysis employing Planck 2018 data in combination with DES and BK15 data are reported in Figure 3.1. Given the sub-percentage constraints on the scalar index, we found a sub-percentage constraint on the tensor amplitude *i.e.* $r/\alpha = 0.00387^{+0.00078}_{-0.00094}$ for Planck 2018 data alone. Due to this correlation, we see also that there is virtually no difference between the results using the Planck 2018 data and combining them with the Biceps/Keck 2015 data ($r/\alpha = 0.00400^{+0.00076}_{-0.00095}$) as the constraint on the scalar index is unchanged (if not for a statistically insignificant shift in the posterior mean between the two runs). When LSS data (i.e: DES) are included in the analysis, a shift in the spectral index n_s with respect to CMB data is found. DES data prefer a slightly higher value for n_s, shifting accordingly the running of the spectral index α_S and the tensor-to-scalar ratio r. However, the results remain consistent with that from PK18 and PK18+BK15 within 1σ (see also Figure 3.1).

Incidentally, we also obtain a constraint on the running of the scalar index α_S (related to r/α by equation (3.4c)) away from zero at 4 standard deviation *i.e.* $\alpha_S = -6.4^{+1.6}_{-1.3} \cdot 10^{-4}$. It is worth noting that (as for r) this is due to the specific correlation which arises in α-attractor inflation between the parameters of the scalar and tensor spectrum. This result, however, points out that future measurements of $r_{0.002}$ and $n_{\rm nrun}$ could potentially rule out the α-attractor model: they are key parameters in studying the viability of an inflationary model and should be considered in the future analysis of CMB and LSS data.

We conclude this section with the following two considerations:

- Given $n_s \approx 0.965$ and $r \to 0$, the best-fit model for CMB and LSS data is the case of $\alpha = 1$, which corresponds to Starobinsky inflation, and this cannot be distinguished from a generic model with $\alpha \neq 1$ unless future experiments provides a measure of either polarization B-modes or α_S.

- Current data are consistent with $r \to 0$. Nevertheless, not all values of α are allowed. Instead, they set an upper limit on the value of the tensor-to-scalar ratio and this knowledge can be used to obtain an upper limit for α:

$$\alpha \lesssim \frac{r_{\rm lim}}{r_0} \equiv \alpha_{\rm lim}, \tag{3.5}$$

where $r_{\rm lim}$ is the experimental threshold for a given experimental configuration and r_0 is the mean of the r/α posterior. Being $r_{\rm lim} \approx 0.1$ for P18 data and $r_{\rm lim} \approx 0.06$ for P18+BK15 data combined (Planck Collaboration, Aghanim, Akrami, Ashdown, et al., 2020b), we correspondingly find $\alpha_{\rm lim} \approx 25$ and $\alpha_{\rm lim} \approx 15$ for P18 and P18+BK15 respectively. These upper limits would be the same that one would obtain running an MCMC analysis with r and α considered independently (see Appendix 3.5) .

3.3 Forecast for future CMB-S4 observations

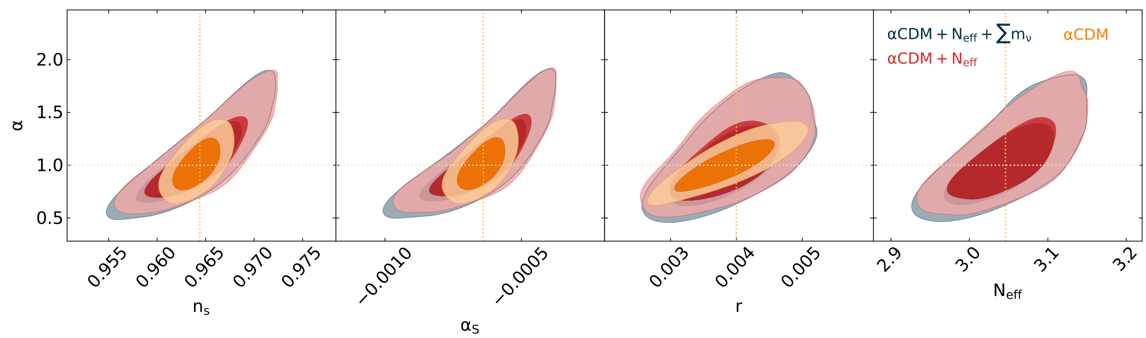

Figure 3.2: Forecasted 2D contours at 68% and 95% C.L. for α attractor inflationary parameters for a CMB-S4 experiments with and without allowing the neutrino sector to vary (αCDM, αCDM+$N_{\rm eff}$ and αCDM+$N_{\rm eff} + \sum m_\nu$ respectively).

While current data are unable to constraint the value of α given the current experimental sensitivity, future generation CMB experiments are expected to strongly improve the sensitivity on the B-mode polarization signal of the CMB, possibly discovering evidence of a primordial tensor mode with amplitude in the range of $r \sim 0.01 - 0.001$ (Ade et al., 2019; Lee et al., 2019; K. Abazajian et al., 2022; Hazumi et al., 2020). In particular, this is the range of predictions for many well-motivated inflationary models, such as Starobinsky inflation, considered the benchmark for future CMB observations (Ade et al., 2019; Lee et al., 2019; K. Abazajian et al., 2022; Hazumi et al., 2020). In this section, we study the optimistic scenario of a future detection in the CMB anisotropies of non-vanishing tensor amplitude and we forecast the constraints achievable with a CMBS4-like experiment on the parameters of the α-attractor model.

We consider as a baseline model a minimal extended ΛCDM cosmology with the inclusion of non-vanishing tensor-to-scalar ratio, r and α. This extended model constitutes our simulated data sets. The value of r is chosen correspondingly to the best-fit value obtained with a Starobinsky model using only Planck 2018 data *i.e.* $r = 0.00387$, while we fix $\alpha = 1$. The value of the scalar index and its running are also fixed to $n_s = 0.964$ and $\alpha_S = 0.0006$. The remaining ΛCDM parameters values are: $\omega_b = 0.0221$, $\omega_c = 0.12$, $H_0 = 67.3$, $\tau = 0.06$ and $\ln(10^{10}A_s) = 3.05$. As the new generation of CMB experiment also expects to set some light in the neutrino sector, we also explore the number of effective degrees of freedom of relativistic species N_{eff} and the sum of the neutrino masses $\sum m_\nu$ in the forecast.

Both simulated data and theoretical models are computed with the latest version of the Boltzmann code `CAMB` (Lewis et al., 2000; Howlett et al., 2012). To extract constraints on cosmological parameters, we make use of the Monte Carlo Markov Chain (MCMC) code CosmoMC (Lewis & Bridle, 2002; Lewis, 2013) which compares theory with a simulated dataset using a given likelihood.

As in (Di Valentino, Holz, Melchiorri, & Renzi, 2018; Renzi, Hogg, Martinelli, & Nesseris, 2021; Renzi, Cabass, Di Valentino, Melchiorri, & Pagano, 2018; Cabass et al., 2016), we built our forecasts for future CMB experiments following a well-established and common method. Using the set of fiducial parameters described above, we compute the angular power spectra of temperature C_ℓ^{TT}, E and B polarization $C_\ell^{EE,BB}$ and cross temperature-polarization C_ℓ^{TE} anisotropies. We produce synthetic realization of future data adding to the theoretical power spectra, an exponential noise of the form (Perotto, Lesgourgues, Hannestad, Tu, & Y Y Wong, 2006):

$$N_\ell = w^{-1} \exp(\ell(\ell+1)\theta^2/8\ln 2) \tag{3.6}$$

where θ is the FWHM angular resolution and w^{-1} is the experimental sensitivity expressed in μK arcmin. The polarization noise is derived equivalently assuming $w_p^{-1} = 2w^{-1}$ since one detector measures two polarization states. The simulated spectra, realized accordingly to the previous discussion, are compared with theoretical ones using a "CMB-like" likelihood as in (Perotto et al., 2006; Audren, Lesgourgues, Bird, Haehnelt, & Viel, 2013)

For this chapter, we have constructed synthetic realizations of CMB data for only one experimental configuration, namely CMB-S4 (see e.g. (K. N. Abazajian et al., 2016)). The CMB-S4 dataset is constructed using $\theta = 3'$ and $w = 1\,^{-}$K arcmin, and it operates over the range of multipoles $5 \leq \ell \leq 3000$, with a sky coverage of the 40%. Furthermore CMB-S4 is expected to reach a target sensitivity on the tensor-to-scalar ratio of $\Delta r \sim 0.0006$, whose goal is to provide a 95% upper limit of $r < 0.001$. Therefore the value chosen for our fiducial model is well within the scope of an experiment like CMB-S4. However, the corresponding sensitivity on the value of the running of the scalar index, α_S would be only $\Delta\alpha_S = 0.002$ which would clearly not be enough for a joint detection of r and α_S assuming Starobinsky inflation. Thus, it may not be possible to distinguish between a generic α-attractor model with $r \sim 0.004$ and Starobinsky inflation despite a future detection of a non-vanishing tensor amplitude.

In α-attractors, however, the uncertainties about the correct shape of the inflationary potential, defining the value of r, n_s and α_S, are parameterized with the α parameter. Therefore a measure of the value of α would also give us insights about the correct shape of the inflationary potential and correspondingly on the correct theory of inflation. A CMBS4-like experiment will be able to give such insights provided a detection of a non-vanishing tensor amplitude.

Current data only place a loose upper bound $0 \leq \alpha \lesssim 15$ (Kallosh & Linde, 2019) correspondingly to P18+BK15 upper limit on the tensor amplitude $r < 0.056$

at 95%C.L. (Planck Collaboration, Akrami, et al., 2020). To correctly explore the available parameter space for α we therefore employ a logarithmic prior on its value $-6 \leq \log_{10} \alpha \leq 1$ while we keep the priors on the other parameters as of Table 3.1. We refer to this model as αCDM. From our CMB-S4 forecasts we obtain a 15% bound on the parameter $\alpha = 1.01^{+0.14}_{-0.18}$, clearly showing the ability of future CMB experiments of bounding single-field slow-roll inflationary models. Models with $\alpha \geq 2$ and $\alpha \leq 0.5$ would be potentially excluded at more than 2 standard deviations in the optimistic scenario of a PGWs detection with amplitude in the range of the Starobinsky model. We eventually extend this baseline model including the number of relativistic neutrino species $N_{\rm eff}$, (αCDM$+N_{\rm eff}$). When $N_{\rm eff}$ is varied, we find a 5% reduction of the accuracy with which α is measured *i.e.* $\alpha = 1.07^{+0.18}_{-0.23}$ while the bound on the tensor-to-scalar ratio is basically the same in the two cases, *i.e.* $\sigma(r) = 0.00050$. Conversely we found an increase in the error budget of the scalar index and running, passing from $\sigma(n_s) = 0.0016$ and $\sigma(\alpha_S) = 0.00006$ (αCDM) to $\sigma(n_s) = 0.0035$ and $\sigma(\alpha_S) = 0.0001$ (αCDM $+ N_{\rm eff}$) an worsening of a factor around two in both cases. It is worth stressing that Primordial Gravitational waves may also contribute to the number of relativistic species being themselves relativistic degrees of freedom (Cabass et al., 2016; T. L. Smith, Pierpaoli, & Kamionkowski, 2006; Clarke, Copeland, & Moss, 2020). This contribution can be calculated analytically to be :

$$N_{\rm eff,GW} \sim \frac{rA_s}{n_T}(A^{n_T} - B^{n_T}) \tag{3.7}$$

where A and B are two real numbers and $A, B \gg 1$. This contribution is clearly extremely small for red spectra ($n_T \leq 0$) but may be important in inflationary theories where blue spectra ($n_T > 0$) can be produced (see e.g. (Mukohyama, Namba, Peloso, & Shiu, 2014; Namba, Peloso, Shiraishi, Sorbo, & Unal, 2016; Stewart & Brandenberger, 2008; Hebecker, Jaeckel, Rompineve, & Witkowski, 2016; Peloso, Sorbo, & Unal, 2016; Giarè & Melchiorri, 2021; Giarè & Renzi, 2020)). Consequently the only interaction between PGWs and neutrinos considered in this work is the one arising from neutrino anisotropic stress after neutrino decoupling at $T \lesssim 1$ MeV (Kojima, Kajino, & Mathews, 2010). These constraints are virtually unmodified when we further extend our baseline model, allowing the whole neutrino sector to vary *i.e.* $N_{\rm eff} + \sum m_\nu$. The 2D contours for both our forecasts are reported in Figure 3.2. A strong correlation now arises between α and the other inflationary parameters conversely to what we found with the Planck data. This is due to the power of CMB-S4 of resolving the B-mode spectrum, consequently breaking the degeneracy between r and n_s. Nevertheless, the situation is unchanged for the scalar running. The strong bound we find on the scalar running is in fact due to imposing the α-model a priori. Even a StageIV experiment would not have the required accuracy to measure the tiny scalar running predicted by α-attractor inflation. When r, n_s and α_S are independently varied (*i.e.* neglecting the consistency relation in equation (3.4)) the running is fixed only with an error $\sigma(\alpha_S) = 0.0029$ at 68% C.L., an order of magnitude higher than when the α-model is imposed *a priori* and in good agreement with the expected sensitivity for the CMB-S4 experiment (K. N. Abazajian et al., 2016).

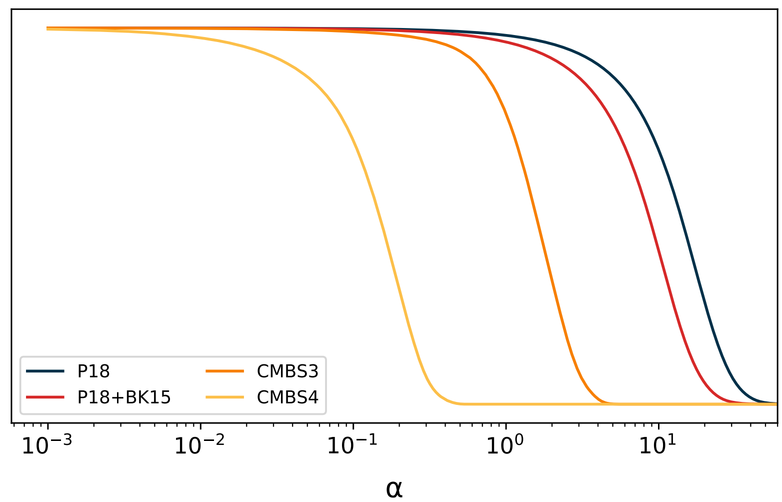

Figure 3.3: 1D posterior for the parameter α for several experimental configurations. These posterior distributions are obtained with the method described in Appendix 3.5. The CMB Stage-III (CMBS3) constraint is obtained assuming a target sensitivity of $r_{\rm lim} = 0.01$ corresponding to $\alpha_{\rm lim} \approx 3$. This sensitivity would be achievable by a stage-III experiment such as SPT-3G (Benson et al., 2014) or BICEP3 (Grayson et al., 2016).

We conclude noting that, as well as for current data, one can forecast the corresponding upper limit on α from equation (3.5) in the pessimist scenario which CMBS4 will not be able to detect a tensor-to-scalar ratio above the target sensitivity. Assuming $r_{\rm lim} = 0.001$ (K. Abazajian et al., 2022), one finds $\alpha_{\rm lim} \approx 0.26$ which would exclude Starobinsky inflation at ten standard deviations. With respect to Planck data, CMBS4 will provide an improvement on the measure of $\alpha_{\rm lim}$ of two orders of magnitude even in the pessimistic case of not detecting any B-mode polarization signal. The constraints on r/α will be instead improved only by a factor of four leading to $r/\alpha = 0.00386 \pm 0.00035$ when neutrinos parameters are fixed to their ΛCDM values. We show in Figure 3.3 a comparison of the upper bounds on α achievable by the experimental configurations considered in this work.

3.4 Conclusions

We have carried out a Bayesian analysis with current CMB and LSS data to constrain inflationary observables (the scalar spectral index n_s, its running α_S and the tensor-to-scalar ratio r). With the current constraining power on n_s and imposing the α-attractor model a priori in our analysis, the possible values of the ratio r/α are narrowed in a band of around 0.004. However current data do not have enough sensitivity to break the degeneracy between r and α and consequently to constrain any deviation from the Starobinsk inflationary model due to the fact the predicted tensor-to-scalar ratio is much smaller than the current upper limit of LSS and CMB data. Consequently, we focused our attention on forecasting the constraints achievable by a future CMBS4 experiment assuming a tensor-to-scalar ratio corresponding to the value obtained from Planck data imposing the α-attractor model a priori, see section 3.2.

The forecast is performed from a Bayesian statistical approach, where α is let free to be sampled from a logarithmic prior distribution. Future CMB-S4 experiments will then be able to constrain α as long as the value of r is above the target sensitivity expected from such experiment *i.e.* $r > 0.001$ (K. N. Abazajian et al., 2016; K. Abazajian et al., 2022). Conversely, in the pessimist scenario that even in future CMB-S4 data will not measure a tensor amplitude above the target sensitivity, the situation will be exactly as for current data and only an upper limit on the value of α could be placed. We forecasted the corresponding limit on α to be $\alpha_{\text{lim}} \approx 0.26$, an improvement of two orders of magnitude with respect to Planck data alone.

In conclusion, a future CMB-S4 experiment will have enough sensitivity to significantly constrain single-field slow-roll inflationary models. In the case of an optimistic detection of a non-vanishing tensor amplitude, it would be able to shed light on both the energy scale and the shape of the inflationary potential, while in the pessimistic scenario of a non-detection of tensor modes it would still be able to place a tight upper limit on the value of α and exclude Starobinsky inflation at 10σ . We underline that, when the running of the spectral index α_S is free to vary, it is always different from zero as expected from the inflationary consistency relation of the α attractor model. However, we show that the value expected for the scalar running given the current constraints on the scalar index is so small that it will not be detectable by a future CMB-S4 experiment (with an expected sensitivity of $\Delta\alpha_S \sim 0.003$), but it may be reachable when information from future weak lensing and galaxy clustering measurements will be included (Euclid Collaboration et al., 2020; Font-Ribera et al., 2014; LSST Science Collaboration, Abell, et al., 2009). The combination of future weak-lensing surveys and CMBS4 would possibly reach a target sensitivity on $\Delta\alpha_S \sim 0.001$, a factor of three better than CMBS4 alone[1]. This is enough to constrain α_S at a level compatible with the value expected from α-attractor models. Note that a measure of α_S would constitute a smoking-gun for inflation as well as a measure of a non-zero tensor amplitude. Therefore, future LSS surveys and CMB experiments will either give us a measure of both r and α_S in the most optimistic scenario or they will be able to significantly reduce the available parameter space for single-field slow-roll inflation in the most pessimistic one.

[1]This is derived assuming an improvement of a factor $\sigma(\alpha_S)_{\text{Planck}}/\sigma(\alpha_S)_{\text{CMBS4}} \sim 3$ of the forecasted constraints on α_S with respect to the combination of weak-lensing and Planck data from Tab.21 of (Amendola, Appleby, et al., 2018)

3.5 Appendix: from a two sigma bound to an upper limit

In this section we briefly describe the procedure used to convert the two sigma bound on r/α into an upper limit on α assuming an experimental threshold $r_{\rm lim}$. Let us start noting that an upper limit on the tensor-to-scalar ratio, $r < r_{\rm lim}$ at 95% C.L. can be represented by an half-normal distribution with standard deviation, σ, given by the following equation:

$$\int_{-r_{\rm lim}}^{r_{\rm lim}} \mathcal{N}(x\,|\,0,\sigma)dx = 0.95 \tag{3.8}$$

Solving for σ and applying an inverse transform sampling technique, we can extract samples from the half normal distribution. Then, for each samples of the half Gaussian of r/α, we can use equation (3.5) to calculate a sample of the distribution of α. This procedure allows to reconstruct the posterior of α starting from the bound on r/α and it is equivalent to perform a full MCMC analysis with an experimental configuration that can reveal tensor modes with amplitude $r > r_{\rm lim}$ at 95% C.L. . As shown in Fig.(3.3), the results on α agrees almost perfectly with the approximate results obtained considering a delta distribution for r_0 and $r_{\rm lim}$. Thus, we conclude that the uncertainties in the measure of r/α can be negligible in deriving an upper limit for the values of α.