Explicit computation of the height of a Gross-Schoen Cycle

Wang, R.

Stellingen

behorende bij het proefschrift getiteld

Explicit Computation of the Height of a Gross-Schoen Cycle

I. Zhang's admissible invariants on pm-graphs satisfy the contraction lemma and are additive on pm-graphs of genus $g > 1$.

II. The normalized Beilinson-Bloch height of canonical Gross-Schoen cycles on genus $g > 2$ curves over $\mathbb{Q}$ is unbounded.

III. A dual graph satisfying Condition $(\mathfrak{H})$ means that it is a wedge sum of irreducible pm-graphs with at most 1 cycle. If a family of genus 3 curves $\{C_n\}_{n\in\mathbb{N}}$ over $\mathbb{Q}$ has the following properties:

- the dual graphs of $\{C_n\}_{n\in\mathbb{N}}$ at finite places satisfy Condition $(\mathfrak{H})$,
- the points $\{c_n\}_{n\in\mathbb{N}}$ on $\overline{\mathcal{M}}_3(\mathbb{C})$ corresponding to $\{C_n \otimes \mathbb{C}\}_{n\in\mathbb{N}}$ converge to a point which corresponds to a stable curve having dual graph satisfying Condition $(\mathfrak{H})$,

then their normalized heights of canonical Gross-Schoen cycles goes to infinity.

IV. For the smooth curve

$$C : -X^3Y + X^2Y^2 - XY^2Z + Y^3Z + X^2Z^2 + XZ^3 = 0$$

over $\mathbb{Q}$, the height of its canonical Gross-Schoen cycle (denoted by Δ_C) is $\langle \Delta_C, \Delta_C \rangle \approx 0.60$.

V. The canonical Gross-Schoen cycle of a hyperelliptic curve X over a field is rationally equivalent to $0 \in \mathrm{CH}^2(X^3)$.

VI. Let X be a smooth non-hyperelliptic curve of genus $g \geq 2$ defined over the number field k with a stable model $\mathcal{X}$ over O_k. Let $\Delta \in \mathrm{CH}^2(X^3)_{\mathbb{Q}}$ be a canonical Gross-Schoen cycle on X^3. Then the equality

$$\langle \Delta, \Delta \rangle = \frac{6(2g+1)}{g-1} \left(\deg \det f_* \overline{\omega}_{\mathcal{X}/O_k} - \sum_{v \in M(k)} \lambda(X_v) \log Nv \right)$$

holds, where $\overline{\omega}_{\mathcal{X}/O_k}$ is the Arakelov dualising sheaf, $M(k)$ is the set of places of k and $\lambda(\cdot)$ denotes the lambda invariant defined by Zhang.

VII. The two plane curves defined by

$$X^3Z + X^2YZ + X^2Z^2 - XY^3 + XYZ^2 + Y^2Z^2 + YZ^3 = 0,$$
$$X^3Z + X^2Z^2 + XY^3 - XY^2Z + Y^2Z^2 - YZ^3 = 0$$

over $\mathbb{Q}$ are semistable.

VIII. For a smooth curve X of genus 2, the nodal curve X' obtained by gluing two non-conjugate points on X is not a plane quartic curve.

IX. It is hard to compute the contributions from infinite places precisely, although approximations are easy to carry out. It is hard to know if we can compute contributions from the finite places, although we get precise values if they are computable. Gain always comes with loss, and vice versa.

Ruihua Wang,
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