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## Explicit computation of the height of a Gross-Schoen Cycle Wang, R.

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## Summary

## Explicit Computation of the Height of a GrossSchoen Cycle

In this thesis, we study the Beilinson-Bloch height of the Gross-Schoen cycle on a curve over a field.

Let $X$ be a genus $g$ smooth curve over a field $k$ (a number field or a function field). To an element $e \in \operatorname{Div}^{1}(X)_{\mathbb{Q}}$, we can associate a Gross-Schoen cycle $\Delta_{e}$ in $\mathrm{CH}^{2}\left(X^{3}\right)$. The cycle is an alternating sum of small diagonals on $X^{3}$. When $k$ is a global field, the height (studied by A. Beilinson, S. Bloch, B. Gross and C. Schoen) of $\Delta_{e}$ can be used to measure the non-triviality of $\Delta_{e}$.

In Chapter 1, we review Arakelov theory and Zhang's work on the heights of GrossSchoen cycles. The main result of this chapter is Theorem 1.5.16, in which we show that the height for genus $g \geq 3$ curves over $\mathbb{Q}$ is unbounded. The proof relies on the Northcott property for Gross-Schoen cycles proved by S. Zhang.

In Chapter 2, we recall some moduli properties of genus 3 curves and Klein's formula for smooth plane quartic curves.

In Chapter 3, we focus on Arakelov geometry of genus 3 curves. We explain how to explicitly compute the admissible invariants of genus 3 pm -graphs. The main result of this chapter is a sufficient condition for the heights of a family of genus 3 curves to go to infinity.

In Chapter 4, we numerically compute the height of a canonical Gross-Schoen cycle of a particular plane quartic curve over $\mathbb{Q}$.

