Explicit computation of the height of a Gross-Schoen Cycle

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Summary

Explicit Computation of the Height of a Gross-Schoen Cycle

In this thesis, we study the Beilinson-Bloch height of the Gross-Schoen cycle on a curve over a field.

Let X be a genus g smooth curve over a field k (a number field or a function field). To an element $e \in \mathrm{Div}^1(X)_{\mathbb{Q}}$, we can associate a Gross-Schoen cycle Δ_e in $\mathrm{CH}^2(X^3)$. The cycle is an alternating sum of small diagonals on X^3. When k is a global field, the height (studied by A. Beilinson, S. Bloch, B. Gross and C. Schoen) of Δ_e can be used to measure the non-triviality of Δ_e.

In Chapter 1, we review Arakelov theory and Zhang's work on the heights of Gross-Schoen cycles. The main result of this chapter is Theorem 1.5.16, in which we show that the height for genus $g \geq 3$ curves over $\mathbb{Q}$ is unbounded. The proof relies on the Northcott property for Gross-Schoen cycles proved by S. Zhang.

In Chapter 2, we recall some moduli properties of genus 3 curves and Klein's formula for smooth plane quartic curves.

In Chapter 3, we focus on Arakelov geometry of genus 3 curves. We explain how to explicitly compute the admissible invariants of genus 3 pm-graphs. The main result of this chapter is a sufficient condition for the heights of a family of genus 3 curves to go to infinity.

In Chapter 4, we numerically compute the height of a canonical Gross-Schoen cycle of a particular plane quartic curve over $\mathbb{Q}$.