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Explicit computation of the height of a Gross-Schoen Cycle

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Explicit Computation of the Height of a Gross-Schoen Cycle

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