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Travelling waves on trees and square lattices

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Travelling Waves on Trees and Square Lattices

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*To the loving memory of my father Tado,
Who first showed me the magic of numbers.*

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