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## **On shape and elasticity: bio-sheets, curved crystals, and odd droplets**

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On Shape and Elasticity:  
Bio-sheets, Curved Crystals, and Odd  
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The cover shows artistic depictions of various geometric elements related to the research on two-dimensional curved media presented in this work. Front cover: helical tubulin sheet on a background of curved patterns with different values of curvature. Back cover: the background of curved patterns is interrupted by a curly ribbon-like sheet. The lower background is replaced by a perfect triangular lattice, which is only possible due to the lack of Gaussian curvature of the flat cover. On top of the lattice, three-dimensional discretized surfaces model the faceting and flattening of emulsion droplets, whose crystalline interface instead is forced to have defects. Cover design implemented by digital artist Sofía Navarro Abarca.

To my parents, who gave us wings to fly.



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