Stimulated raman adiabatic passage in optomechanics

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Mechanical ground state

In section 1 of this chapter we consider the requirements for our optomechanical system to measure the average phonon number of a thermal state of a mechanical mode close to its ground state by observing the Stokes and Anti-Stokes sidebands asymmetry.

In section 2 an experiment is proposed to investigate membrane heating due to the intracavity light fields at low temperatures.

7.1 Requirements

In most quantum optomechanical experiments it is desirable to initialize the mechanical oscillators in their quantum ground state. Therefore, it is essential to know the average phonon occupation $\bar{n}$ of the mechanical modes when they are prepared in a thermal state close to the ground state. One of the known methods of $\bar{n}$ determination is via the sideband asymmetry [9]. This method is based on the different boson enhancement factors of the scattering probabilities $\gamma_{n\to n+1}$ and $\gamma_{n\to n-1}$ of the transition rate between phonon levels n and $n \pm 1$ as mentioned in Chapter 1. The different prefactors lead to the different prefactors of the Stokes and anti-Stokes scattering rates:

$$\Gamma_{\mathrm{S}} = (\bar{n}+1) g_0^2 n_{\mathrm{cav}} \frac{\kappa}{\omega_m^2}, \tag{7.1}$$

$$\Gamma_{\mathrm{AS}} = \bar{n} g_0^2 n_{\mathrm{cav}} \frac{\kappa}{\omega_m^2}, \tag{7.2}$$

where g_0 is the single photon optomechanical coupling rate, n_{cav} is the intracavity photon number, κ is the optical decay rate and ω_m is the frequency of the mechanical mode.

In any measurement the field probe should be weak in order to not disturb the system too much. It's known that high intracavity light fields cause heating of the optomechanical devices. Therefore, we are finding the minimal probe intensity to

measure the thermal state of a mechanical mode close to its ground state via the sideband asymmetry. We are considering a defect mode of our phononic crystal membranes in a cavity with parameters of the cryogenic cavity discussed in Chapter 5. The settings are [9]:
- the membrane is optomechanically coupled with $g_0/2\pi = 1$ Hz (this is what we observe for our 10 cm cavity); the membrane temperature is 1 K, cavity linewidth $\kappa/2\pi = 50$ kHz;
- a strong pump is sent on the red-sideband to cool a high-Q mechanical mode ($\omega_m/2\pi = 1.3$ MHz, $Q = 10^9$) to a thermal state with $\bar{n}_f = 0.1$;
- a probe is sent on cavity resonance to measure the thermal state of the mechanical mode;
- the probe sidebands are detected via heterodyning on a shot-noise limited balanced photodetector with ideal mode overlap;
- all optomechanically scattered photons reach the photodetector: $\kappa_{\text{ext}} = \kappa$.

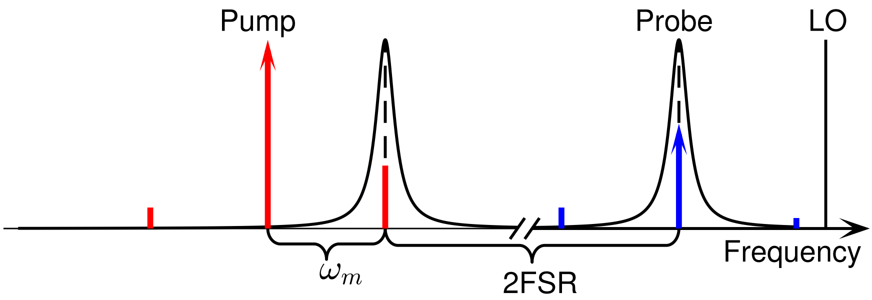

Figure 7.1: Sideband cooling experiment with resonant probe. The Stokes and Anti-Stokes sidebands from the resonant probe are detected via heterodyning with local oscillator (LO), the ratio of the sidebands provides information of the thermal state of the mechanical mode.

To do the measurement both sidebands must be observable above the shot-noise with signal to noise ratio of at least $r_{\text{signal/noise}} = 0.1$ (which is measurable [65, 64]) for the power spectral density.

Let's start with the required cooling light intensity to reach $\bar{n}_f = 0.1$. Using

$$\bar{n}_f = \frac{\Gamma_{\text{opt}}\bar{n}_{\text{min}} + \Gamma_{\text{m}}\bar{n}_{\text{th}}}{\Gamma_{\text{opt}} + \Gamma_{\text{m}}}, \tag{7.3}$$

where $\bar{n}_{\text{min}} \approx \kappa^2/16\omega_m^2$, we can estimate the required $\Gamma_{\text{opt}} \approx \frac{k_B T}{\hbar Q}\frac{1}{n_f}$, $\Gamma_{\text{opt}}/2\pi = 220$ Hz. The rates of Stokes and anti-Stokes scattering for the resonant probe are

$$\Gamma_{\text{S}} = (\bar{n}_f + 1)g_0^2 n_{\text{cav.probe}}\frac{\kappa}{\omega_m^2}, \tag{7.4}$$

$$\Gamma_{\text{AS}} = \bar{n}_f g_0^2 n_{\text{cav.probe}}\frac{\kappa}{\omega_m^2}, \tag{7.5}$$

where $n_{\text{cav.probe}} = \frac{4P_{\text{probe}}}{\kappa\hbar\omega_L}$. The weaker anti-Stokes sideband has the Lorentzian lineshape with linewidth Γ_{opt} due to scattering with the mechanical mode cooled by

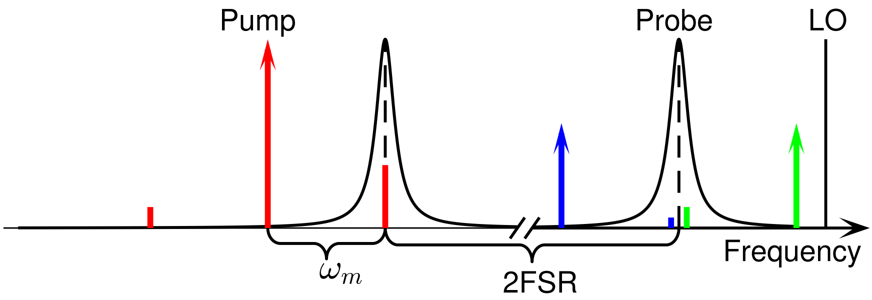

Figure 7.2: Sideband cooling experiment with two off-resonant probes. The probes are detuned by $\Delta = \pm(\omega_m + \delta)$ such that the Stokes and anti-Stokes sidebands are separated by more than Γ_{opt} to avoid interference between them. $\Gamma_{\text{opt}} \ll \delta \ll \kappa$.

the pump light fields. The intensity of the anti-Stokes sideband is $I_{\text{AS}} = \Gamma_{\text{AS}}\hbar\omega_L = \bar{n}_f \frac{4g_0^2}{\omega_m^2} P_{\text{probe}}$. Such a signal will have the following spectrum:

$$I_{\text{AS}}(\omega) = \frac{1}{2\pi} \frac{\Gamma_{\text{opt}}}{(\omega - \omega_{\text{AS}})^2 + \Gamma_{\text{opt}}^2/4} I_{\text{AS}}, \tag{7.6}$$

which means $I_{\text{AS}} = \int_{-\infty}^{\infty} I_{\text{AS}}(\omega)\, d\omega$.

Interference of this sideband with a local oscillator with intensity I_{LO} and frequency ω_{LO} on a balanced photodetector will result in the difference signal

$$2\sqrt{I_{\text{LO}} I_{\text{AS}}} \sin{(\omega_{\text{LO}} - \omega_{\text{AS}})t}. \tag{7.7}$$

The frequency of the local oscillator is chosen such that in the range of frequencies around $|\omega_{\text{LO}} - \omega_{\text{AS}}|$ the balanced photodetector is shotnoise limited.

The power spectral density of this signal will be

$$I_{xx}(\omega) = 2 I_{\text{LO}} I_{\text{AS}}(\omega) \tag{7.8}$$

with the maximum value

$$I_{xx,\text{signal}}(\omega_{\text{AS}}) = I_{\text{AS}} \frac{2}{\pi \Gamma_{\text{opt}}}. \tag{7.9}$$

This should be $r_{\text{signal/noise}}$ of the shot noise power spectral density $I_{xx,\text{SN}} = 2\hbar\omega_L I_{\text{LO}}$:

$$I_{xx,\text{signal}}(\omega_{\text{AS}}) = r_{\text{signal/noise}} I_{xx,\text{SN}} \tag{7.10}$$

resulting in

$$\Gamma_{\text{AS}} = \frac{r_{\text{signal/noise}}\pi}{2} \Gamma_{\text{opt}}. \tag{7.11}$$

We see that I_{LO} drops out. Using the expressions for Γ_{opt} and Γ_{AS} we obtain

$$P_{\text{probe}} = \frac{r_{\text{signal/noise}}\pi}{8} \frac{k_B T \omega_L}{Q} (\frac{\omega_m}{g_0})^2 \frac{1}{\bar{n}_f^2}. \tag{7.12}$$

For the given physical properties this will result in $P_{\rm probe} \approx 0.2$ mW.

This probe power will result in the intracavity photon number 3×10^4 times higher than the intracavity photon number due to the cooling pump light and will provide the dominant heat load on the membrane. Therefore, we conclude that observation of the sideband asymmetry of a resonant probe is not practical for our system parameters.

The intracavity photon number due to the probe can be minimized by introducing a pair of equal intensity light tones with detunings $\Delta = \pm(\omega_m + \delta)$ instead of a resonant probe [85], where $\Gamma_{\rm opt} \ll \delta \ll \kappa$. Taking into account the expression for the anti-Stokes scattering rate with an arbitrary detuning Δ

$$\Gamma_{\rm AS} = \bar{n}_f g_0^2 n_{\rm cav.probe} \frac{\kappa}{\kappa^2/4 + (\Delta + \omega_m)^2}, \tag{7.13}$$

the intracavity photon number $n_{\rm cav.probe}$ for the case of the two off-resonant probes will decrease $\frac{4\omega_m^2}{\kappa^2}/2 = 1300$ times. Even in this case the intracavity photon number due to the probe will be approximately 20 times higher than the intracavity photon number due to the cooling light fields. Therefore, the temperature of the membrane might increase when the probe light is introduced resulting in an increase of $\bar{n}_f$.

To make the membrane heating effect of the probe light small compared to the sideband cooling the single photon detection scheme discussed in the previous chapter can be used. In this case the reflected light is sent through the filtering system tuned to the right cavity resonance shown in Fig. 7.2 towards the single photon detector. Instead of the two off-resonant probes only one probe is used at a time. First, the probe with intensity $1/10$ of the cooling light fields and $\Delta = +\omega_m$ detuning is sent to the cavity and the Stokes rate $\Gamma_{\rm S,1}$ is measured, then the same intensity probe and $\Delta = -\omega_m$ is sent and the anti-Stokes rate $\Gamma_{\rm AS,1}$ is measured. The intracavity photon number due to these probe light fields will be $1/10$ of the intracavity photon number due to the cooling light fields. The expected scattering rates will be:

$$\Gamma_{\rm S,1} = 150 \text{ Hz}, \tag{7.14}$$

$$\Gamma_{\rm AS,1} = 13 \text{ Hz}. \tag{7.15}$$

The measured dark count rates of our single photon detectors are approximately 0.01 Hz making the expected rates possible to measure.

7.2 Membrane thermometry via Anti-Stokes sideband

The main uncertainty of the sideband asymmetry experiment and the quantum STIRAP experiment is how much the membrane is heated due to the strong driving light fields. It is important for two reasons: first, the time of one phonon to enter the mechanical mode from the environment is proportional to the temperature of the environment. Second, it was demonstrated [86] that the quality factor of a SiN membrane increases factor of ~ 4 upon cooling the membrane from 1 K to the dilution fridge temperature.

Here we will show that by measuring the anti-Stokes scattering rate it is possible to determine the dependence of the membrane temperature $T_{\rm env}$ on the intracavity photon number. Let's consider an experiment where only the cooling light fields

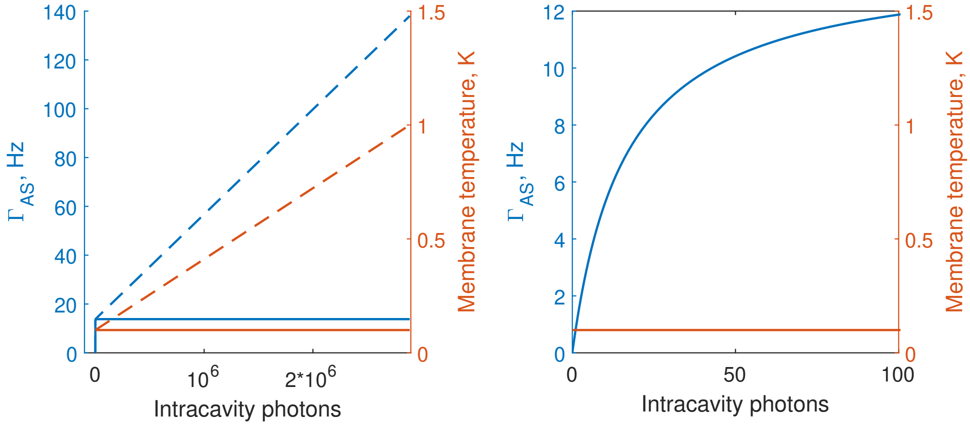

Figure 7.3: Anti-Stokes scattering rate used for thermometry. The left plot shows the scattering rate and the membrane temperature in the absence of membrane heating due to the pump light (solid lines) and in the presence of the heating (dashed lines). The membrane temperature is assumed to be proportional to the intracavity photon number. The right plot shows the scattering rate and the membrane temperature for low intracavity photon numbers. 2.9×10^6 intracavity photons result in $g/2\pi = 1.7$ kHz and $\bar{n}_f = 0.1$. For large intracavity photon numbers the scattering rate is proportional to the membrane temperature and can be used for direct thermometry.

with detuning $\Delta = -\omega_m$ are present. In the sideband-resolved regime the anti-Stokes scattering rate will be

$$\Gamma_{\mathrm{AS}} = \frac{k_B T_{\mathrm{env}}}{\hbar Q(1 + \frac{\omega_m \kappa}{4Q g_0^2 n_{\mathrm{cav}}})}. \tag{7.16}$$

The dependence of Γ_{AS} on the intracavity photon number is shown in Fig. 7.3 by the blue solid line. For $n_{\mathrm{cav}} \gtrsim 1000$ Eq. 7.16 becomes independent of n_{cav}:

$$\Gamma_{\mathrm{AS}} \approx \frac{k_B T_{\mathrm{env}}}{\hbar Q}. \tag{7.17}$$

In this case the anti-Stokes scattering rate is directly proportional to the physical temperature of the membrane T_{env}. Here we assume that $Q = \mathrm{const}$. If the quality factor changes with temperature, than it can be measured independently via a mechanical ringdown. In the presence of membrane heating due to the intracavity light the anti-Stokes scattering rate will increase with the increase in the pump light intensity as shown by dashed lines in Fig. 7.3. Here we assumed direct proportionality between the temperature increase and the intracavity photon number with the membrane temperature being equal to 1 K for intracavity photon number necessary to reach $\bar{n}_f = 0.1$.

By choosing a membrane mode with not too high quality factor such an experiment is feasible even with a heterodyne detection scheme.

7.3 Conclusions

We analyzed the possibility to find the average phonon ocupation $\bar{n}$ of a mechanical mode by measuring the sideband asymmetry for our system using a balanced photodetector. It was supposed that the membrane is thermalized to a bath with $T = 1$ K and a strong enough pump light fields are used for the sideband cooling to reach $\bar{n} = 0.1$. It was found that if a single probe resonant with the cavity is used with light intensity high enough to do the sideband asymmetry measurement then the intracavity photon number due to the probe will be 3×10^4 times higher than the intracavity photon number due to the cooling light fields which already might cause heating of the membrane. Such a probe will definitely disturb the system by heating the membrane and the measured $\bar{n}$ will be higher than $\bar{n}$ without the probe.

If two probes are used simultaneously at $\omega_{\rm cav} \pm \omega_m$ then the intracavity photon number due to the probe will be 23 times higher than the intracavity photon number due to the cooling light fields which is much better but still may heat the membrane.

A way out can be to use a single photon detector instead of the balanced photodetector to increase the sideband measurement sensitivity. The pump photons must be filtered out by the filtering cavities as discussed in Chapter 6. The expected rate of detection of the Stokes and anti-Stokes photons are much higher than the dark count rate of our detectors making a non-disturbing measurement of $\bar{n}$ possible.

We also proposed a method to measure how much the membrane is heated due to the intracavity light fields. A cooling light field is sent to the cryogenic cavity and its light intensity is varied. If the measured anti-Stokes scattering rate is independent of the cooling light intensity for a certain intensity range then the membrane is not heated. Otherwise, it's possible to make an absolute measurement of the membrane temperature as a function of the intracavity photon number. This would require the ability to trace the cavity resonant with a probe intensity < 1 nW. We demonstrated locking with light intensity of 2 nW with some room to further decrease the required probe intensity. Alternatively, the modulation of the cryogenic cavity resonance frequency by the cryostat pulse tube should be removed and the cooling laser should be locked to a reference cavity.

These considerations show that the introduction of the filtering cavities together with SNSPD detection create an opportunity for new exciting experiments.