Strategies for braiding and ground state preparation in digital quantum hardware

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1 Introduction

1.1 Preface

A quantum computer is a special kind of computer that perform its tasks by leveraging the laws of quantum mechanics. For particular classes of problems, such devices are expected to dramatically outperform conventional (classical) computers [1], such as modern-day laptops or even supercomputers. The greatest possibilities are offered by *digital* quantum computers, which possess the flexibility to perform arbitrary quantum computation. The word 'digital' implies that such a computer operates on standardized memory registers and digitizes the desired computation into elementary operations called gates*. The ability of a digital quantum computer to perform an arbitrary computation is known as universality. Although highly promising due to their universality, application-ready digital quantum computers are hard to implement. In particular, most such realizations need an exceptional degree of precision and noise isolation [2]. To this date, significant questions remain open: for instance, how to reliably implement a digital quantum computer? Furthermore, once such a computer is available, how to utilize it effectively?

One possible way to realize digital quantum computation is by employing a technique called braiding [3–5]. The key elements in such an implementation are anyons — particle-like energy excitations supported by some quantum materials. In a quantum computation, information is to be encoded and modified via mutual interchanges, or braiding, of anyons. The term 'braiding' refers to a similar exchange of strands in the familiar process of making a braid. Anyons and their braiding could be promising building blocks for a digital quantum computer. However, existing theoretical blueprints for anyons so far were elusive to implement [6–10]. This thesis addresses the issue by putting forward alternative proposals for anyon braiding and detection.

For a functioning quantum computer, a promising early application is

*This method can be contrasted with analog quantum computation, which does not require digitization but instead is run on a special-purpose device built to perform a given type of computation.

simulating physical systems [11–14]. One type of potential simulation is preparing a ground state — the lowest energy state of a quantum system. Information obtained from simulated ground states can be used in research or engineering since many chemicals and materials occupy low-energy configurations. On a classical computer, representing a quantum ground state is generally unfeasible [15–17], due to the many-body quantum correlations present in the state. On the other hand, a quantum computer can reproduce these correlations efficiently due to the quantum nature of its hardware. The very procedure of preparing the ground state, however, is less trivial and is currently a subject of active research [18]. In this thesis, we layout several new strategies towards achieving this goal.

1.2 Topological matter and braiding

Solid-state quantum materials have the ability to enter special *topological* phases [19, 20], characterized by elementary excitations with unusual properties. In 2D materials, the topological nature of the phase can manifest itself in so-called *anyonic* statistics of its excitations [3, 21]. In contrast to bosons or fermions, an interchange (braiding) of two *anyons* can modify the system wavefunction by a nontrivial complex phase or even a unitary operation. Perhaps more strikingly, this modification depends only on the topology of the braided worldlines of the anyons. If a species of anyons realizes non-commuting unitary operations via braiding, such anyon statistics is called non-abelian.

Non-abelian statistics is at the foundation of promised quantum computing applications of anyons. Because braiding is discrete, the computation realized by such interchange procedures is a digital one. For some species of anyons, the set of these quantum operations is even universal – they allow to access the Jones polynomials of the knot theory, which are proven to efficiently encode an arbitrary quantum computation [22]. Finally, due to its topological nature, anyon braiding is expected to be robust to external noise and perturbations.

The above properties make topological matter a viable platform for quantum computation. However, the existing proposals for such an application have not yet been realized in a lab. It is therefore of special interest to propose alternative realization platforms for non-abelian statistics. For the same reason, there is an active interest in the novel experimental signatures of such anyonic excitations. We focus on these questions as posed for two types of topological systems: topological superconductors and Fractional Quantum Hall materials.

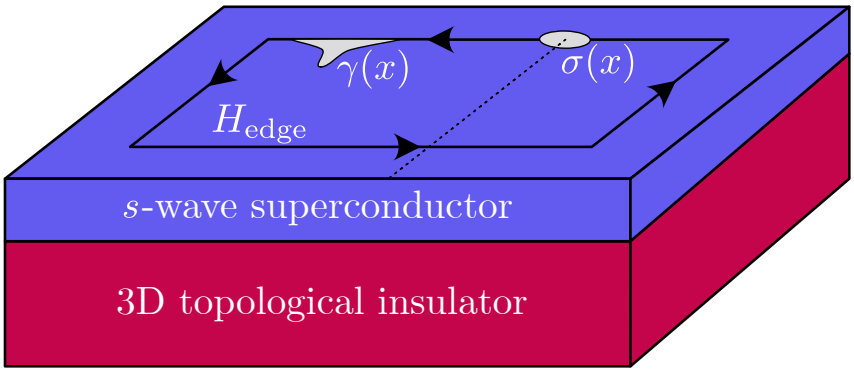

Figure 1.1: Topological superconductivity. A topological superconductor can be effectively realized in a superconductor-topological insulator heterostructure [25]. It is characterized by the presence of a gapless edge mode whose physics is governed by the Hamiltonian H_{edge} (1.2.2). The elementary excitations of the edge mode are Majorana fermions $\gamma(x)$ and edge vortices $\sigma(x)$. (the dotted line shows the associated branch cut)

1.2.1 Topological superconductivity

A good example of a topological superconductor in 2 dimensions is a p-wave superconductor [3, 21, 23, 24]. As a model, one may consider the following ('$p+ip$') Bogoliubov-de Gennes Hamiltonian:

$$H_{p+ip} = \sum_p \left[(\frac{p^2}{2m} - \mu)(c_p^\dagger c_p - b_p^\dagger b_p) + (p_x + ip_y)c_p^\dagger b_p + (p_x - ip_y)b_p^\dagger c_p \right], \tag{1.2.1}$$

where c (b) operators describe the electron (hole) degrees of freedom*. For $\mu > 0$, the Hamiltonian (1.2.1) enters a topological phase, while the phase at $\mu < 0$ is referred to as trivial. Both of these phases are characterized by an energy gap for the bulk excitations. But unlike in the trivial phase, in the topological phase a finite sample of superconductor (1.2.1) would host an additional gapless mode at its edge (Fig. 1.1). This edge mode is chiral, being effectively described by the Hamiltonian H_{edge}:

$$H_{\text{edge}} = \int_x i\gamma(x)\partial_x\gamma(x)dx, \tag{1.2.2}$$

*The absence of a spin degree of freedom in this toy model is justified for a p-wave superconductor, which allows for a superconducting pairing in the spin-polarized channel.

where $\gamma(x)$ is a Majorana fermion: an operator satisfying $\gamma^\dagger(x) = \gamma(x)$ that describes a neutral quasiparticle i.e. one that does not carry charge.

Experimentally, fabricating a topological p-wave superconductor material (1.2.1) turns out to be extremely difficult [26]. Instead, a two-dimensional topological superconductor can effectively be realized [25] with a layer of an s-wave superconductor on a topological insulator substrate (Fig. 1.1). Such a heterostructure has a similar effective Hamiltonian to the $p + ip$ superconductor* (1.2.1) and features the gapless edge mode (1.2.2). Recent experimental works have attempted this realization of topological superconductivity [27, 28], and observed a signature consistent with the Majorana edge mode (1.2.2). However, alternative explanations of the observed data have also been proposed [9, 10, 29], and to date no consensus has been reached on whether the observed signal is from a Majorana mode. Optimistically, in the near-future one expects the body of such evidence to grow further [7], and two-dimensional topological superconductivity to be finally established and harnessed in a lab setting.

1.2.2 Majorana anyons: bulk and boundary

In addition to the Majorana edge mode, topological superconductors also provide a platform for anyonic braiding statistics. The anyonic excitation that has this statistics is an Abrikosov vortex of supercurrent [30], which in a topological superconductor hosts a zero-energy Majorana bound state (or zero-mode). While playing a crucial role in the vortex exchange properties, Majorana zero modes γ_α themselves obey statistics of a fermionic type:

$$\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha = 2\delta_{\alpha,\beta}. \tag{1.2.3}$$

The anyonic statistics of the vortices is rooted in the Aharonov-Bohm [31] effect imposed by the vortex. In particular, a 2π phase winding of the superconducting order parameter Δ can be translated into a π phase shift for the fermionic variables. This can be represented by a branch-cut boundary condition for the fermions (Fig. 1.2a), with implications for the vortex statistics [32]. Consider the example sketched in Fig. 1.2, where two vortices σ_1 and σ_2 are interchanged. Due to the inevitable crossing of a

*Although unlike H_{p+ip}, it satisfies the time-reversal symmetry.

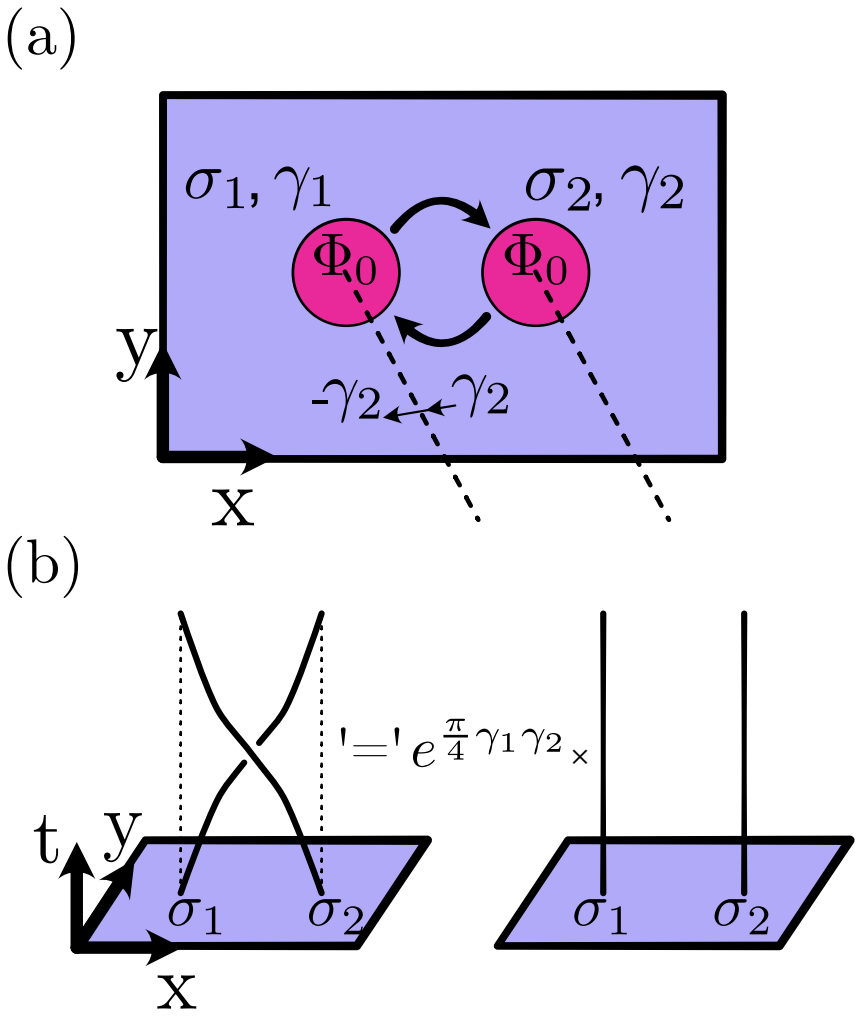

Figure 1.2: Braiding of vortices in a 2D topological superconductor. (a) In a topological superconductor, Abrikosov vortices $\sigma_{1,2}$ host Majorana fermion modes $\gamma_{1,2}$ at zero energy. The magnetic flux quantum $\Phi_0 = h/2e$ carried by each vortex induces a branch-cut boundary condition (dotted lines). Any fermion operator flips the sign once crossing the branch cut. Such crossing is guaranteed to happen whenever the two vortices are exchanged. (b) In space and time, the vortex exchange process implies braiding of their worldlines (on the left). Compared to no exchange (on the right), such a braiding operation transforms the Majorana degrees of freedom $\gamma_{1,2}$ with a unitary: $U_{12} = e^{\frac{\pi}{4}\gamma_1\gamma_2}$.

branch cut, Majorana operator γ_2 has to flip a sign. In algebraic form, this exchange operation produces a unitary U_{12} such that $U_{12}^{-1}\gamma_2 U_{12} = -\gamma_1$, $U_{12}^{-1}\gamma_1 U_{12} = \gamma_2$. By virtue of the Majorana operator algebra (1.2.3), these relations imply the following form of the unitary:

$$U_{12} = e^{\frac{\pi}{4}\gamma_1\gamma_2} = \frac{1}{\sqrt{2}}(\mathbf{I} + \gamma_1\gamma_2). \tag{1.2.4}$$

More generally, a similar exchange of vortices σ_α and σ_β would be represented by a unitary operation $U_{\alpha\beta} = e^{\frac{\pi}{4}\gamma_\alpha\gamma_\beta}$. As different $U_{\alpha\beta}$ do not generally commute, the vortices σ_α have the exchange statistics of non-abelian anyons.

An intense recent interest has been drawn to the relation between the edge modes and anyonic excitations. Since Majoranas play a role in the anyonic statistics of the bulk excitations, some have proposed to use edge Majoranas to produce the anyonic statistics [33]. Edge mode theory also permits vortex-like excitations, which can be characterized by branch cuts (illustrated in Fig. 1.1). Some of the work presented in this thesis is motivated by the question: can one reproduce the bulk anyonic statistics using edge vortices?

1.2.3 Fractional Quantum Hall Effect. Laughlin quasiparticles

In some two-dimensional materials subjected to strong magnetic fields at low temperatures, the Hall conductivity [34] is measured to be quantized:

$$\sigma_{xy} = q\frac{e^2}{h}, \tag{1.2.5}$$

which is known as the Quantum Hall Effect. In the Integer Quantum Hall Effect (IQHE) [35], the constant q in (1.2.5) is always an integer, while in the Fractional Quantum Hall Effect (FQHE) [36] it can also take fractional values. Similarly to topological superconductivity, the key to Quantum Hall physics is in the gapless edge modes [37], which in this case carry the quantized Hall currents. In the IQHE such gapless modes are populated by ordinary electrons, and the quantization (1.2.5) with an integer q follows from the conductance quantization theorem for 1D channels [38]. Meanwhile, the non-integer q of the FQHE materials seemingly contradicts this theorem. This striking effect is explained by the presence of many-body excitations with fractional charge, rather than electrons which carry unit charge, in the gapless edge mode. In the Laughlin model [39], which

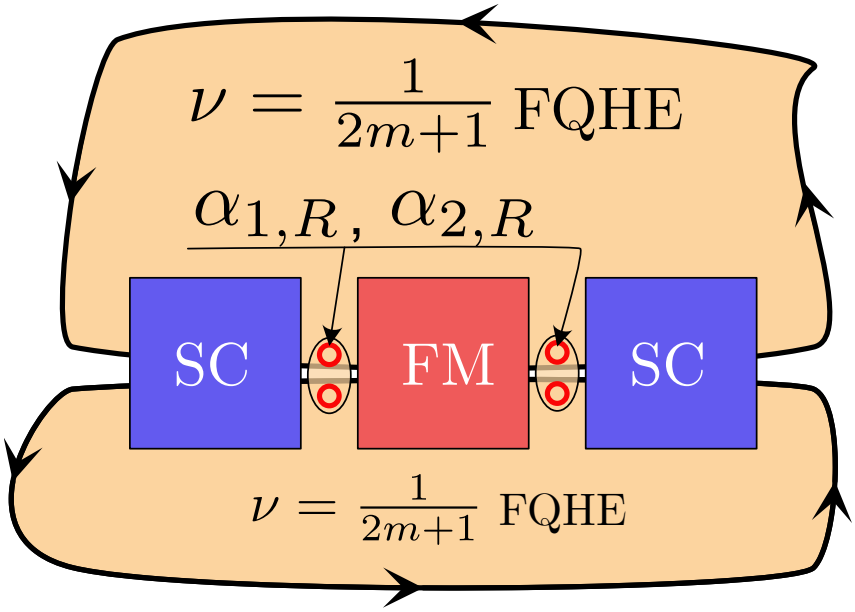

Figure 1.3: Combining two samples of Laughlin FQHE materials allows one to enact parafermionic zero-modes. For that, one has to deposit a sequence of ferromagnets (FM) and superconductors (SC) on top of the interfacing sample edges. Each interface between a ferromagnet and a superconductor hosts two zero-modes: one per edge mode. The respective operators are characterized by parafermionic statistics.

describes the subclass of FQHEs with inverse odd $q = \frac{1}{2n+1}$, there is a single species of such fractional excitations. These Laughlin quasiparticles $\psi(x)$ carry charge qe and have anyonic exchange statistics:

$$\psi(x)\psi(y) = \psi(y)\psi(x)e^{iq\pi}, \quad x > y. \tag{1.2.6}$$

Since the exchange unitary is represented only by an overall phase $e^{iq\pi}$, the Laughlin anyons are abelian.

With material in an FQHE phase of the Laughlin type, one can also achieve non-abelian braiding statistics. For this, one needs to orchestrate so-called parafermionic zero-modes [40]. This can be realized if a pair of counterpropagating FQHE edge modes are gapped out by a sequence of ferromagnets interlaced with superconductors (Fig. 1.3). On sites $j \in \mathbf{Z}$ between the superconducting and ferromagnetic domains, a parafermionic

zero-mode $\alpha_{j,L/R}$ emerges. L and R here stand for left-propagating and right-propagating, depending on the edge where $\alpha_{j,L/R}$ is localized. The parafermion operators $\alpha_{j,L/R}$ themselves have statistics similar to Laughlin quasiparticles, e.g.

$$\alpha_{j,R}\alpha_{k,R} = \alpha_{k,R}\alpha_{j,R}e^{iq\pi}, \quad j > k, \tag{1.2.7}$$

which is abelian. However, a gradual exchange of two parafermion-hosting sites can be used to produce a nonabelian operation [40]. Such a relation between abelian and non-abelian statistics of parafermions can be closely paralleled with Majorana braiding in a topological superconductor. Indeed, Majorana fermions in Abrikosov vortices do not carry nonabelian statistics, while the braiding of those vortices is nonabelian. In contrast to Majorana-hosting vortices, however, the non-abelian braiding of parafermionic sites allows for an even larger set of unitary operations [40]. In this thesis, we will use this fact as a motivation for our investigation of parafermions but not focus on their braiding per se. Instead, we are interested in constructing new and useful ways to characterize parafermionic zero mode statistics (1.2.7) in experiment.

1.3 Preparing ground states with a quantum computer

In this section, we transition from the basic physics of digital quantum hardware and move on to its potential utilization. One of the most promising applications of quantum computers is preparing a simulation of a many-body ground state. A version of this task is likely [18] to be among the first problems that are solved on a quantum computer with a speed-up relative to its classical counterparts. The ground state preparation problem is relevant to quantum physics as well as classical physics and computer science. As a natural example, it arises when studying the low-temperature properties of many-body systems. This includes [18] multi-electron systems in solid state physics, chemistry, and spin systems. In computer science, a cost function optimization is also a common task, with application to machine learning and logistics [41]. Such tasks can also be mapped onto a ground state finding problem for a particular Hamiltonian [16].

The hope for the success of quantum computers in ground state simulation lies in the exponentially greater expressiveness of a quantum computer compared to a classical one. However, one does not expect such an exponential speed-up in the context of every problem. In particular,

NP-complete ground state search problems, e.g. for classical spin models [42], are not expected to be polynomially solvable on a quantum computer. For quantum many-body Hamiltonians, accessing the ground state even in some restricted Hamiltonian families is already QMA-complete [43] (quantum analog of NP-complete). On the other hand, any *polynomially* complex quantum computation can be mapped [44] onto the problem of distilling the exact ground state of a particular Hamiltonian from a given good approximation thereof. These mathematical results highlight the relevance of ground state preparation to general quantum speed-ups and the significance of approximating target ground states efficiently.

To prepare a ground state on a quantum computer, one needs to build and utilize an appropriate quantum algorithm. On a broad level, there already exist multiple paradigms constructing such algorithms. These include adiabatic quantum computing [45], quantum phase estimation [46], and variational quantum algorithms [47]. An ongoing effort is devoted to optimizing such approaches for practical use. This optimization is especially critical if an algorithm is to be employed in the near-term [11], before existing quantum hardware is sufficiently protected from the errors. For such applications, the exact time complexity of an algorithm is as crucial as its asymptotic scaling. In addition to optimizing existing ideas, creating original methods for ground state preparation is also of interest. A novel approach to this problem may yield a sizeable improvement in time complexity, especially in the context of specific niche applications.

1.3.1 Variational quantum algorithms

A variational quantum algorithm (Fig. 1.4) is a quantum-classical hybrid algorithm which aims to approximate the ground state of a given Hamiltonian by utilizing the variational principle [47]. Since quantum circuits are in general exponentially hard to represent classically, a variational quantum algorithm employs a quantum circuit as a powerful variational ansatz. To enable the optimization procedure, the energy of the ansatz state is being measured at the end of the circuit by means of sampling from the target Hamiltonian. The quantum circuit is then tuned to ensure the minimization of said energy. Variational quantum algorithm is well-suited for use on near-term quantum devices [47], being not highly sensitive to noise and not requiring an implementation of deep quantum circuits. The success of this method depends on the efficiency of the energy measurement [48–50], choice of optimization procedure [51–53], and the expressiveness

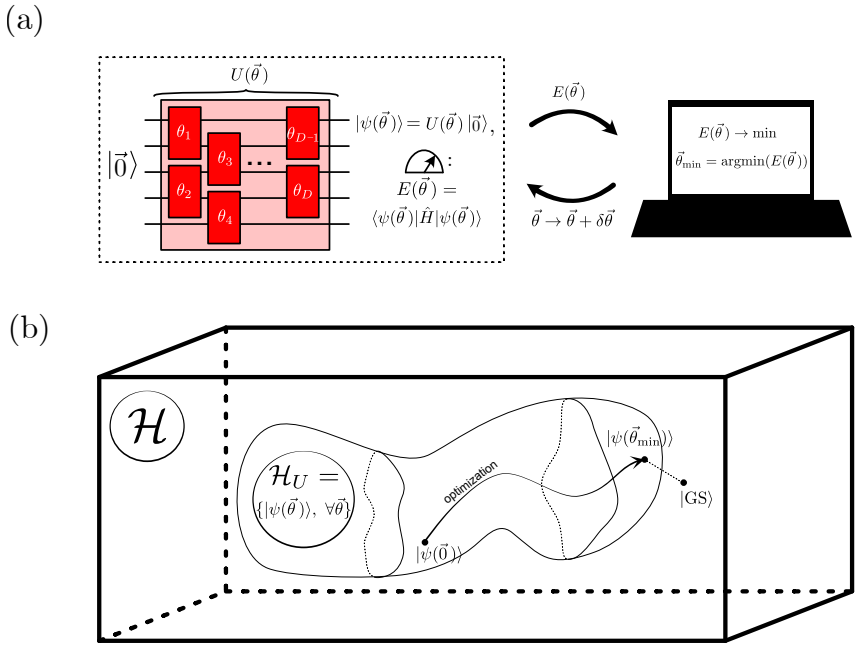

Figure 1.4: Variational quantum algorithms. (a) The hybrid quantum-classical scheme that was introduced in [47]. The algorithm employs a tunable ansatz circuit $U(\vec{\theta})$, and aims to find the value of parameters $\vec{\theta}_{\min}$ which minimizes the variational energy $E(\vec{\theta})$. Quantum hardware need not be continuously in a coherent state since the information is processed classically. This is the key advantage of such an algorithm for near-term implementation. (b) Schematic illustration of the ansatz expressivity. In the total Hilbert space of the system, $\mathcal{H}$, only the small subset $\mathcal{H}_U$ is spanned by the variational states. It is a manifold that is usually much lower in dimensionality (polynomial versus exponential), and generally, the ground state $|GS\rangle$ lies outside of it. The state $|\psi(\vec{\theta}_{\min})\rangle$ produced by the variational algorithm only approximates $|GS\rangle$. It is the task of ansatz construction that the expected separation between $|GS\rangle$ and $|\psi(\vec{\theta}_{\min})\rangle$ is ensured to be minimal. For that, $\mathcal{H}_U$ has to span the physically relevant part of the Hilbert space.

of the ansatz circuits [54–56]. Since the very potential for exponential speed-up is rooted in the capacity of the quantum circuit, such ansatz design is particularly crucial.

To be efficient, a variational ansatz circuit needs to be tailored to the problem at hand. In the quantum chemistry context, a common way to do

this [47, 54] is through the unitary coupled-cluster (UCC) method. The UCC state has the form:

$$|\psi_{\mathrm{UCC}}\rangle = U\,|\psi_{HF}\rangle = e^{T_1+T_2+T_3+\cdots}\,|\psi_{\mathrm{HF}}\rangle\,, \tag{1.3.1}$$

where $|\psi_{\mathrm{HF}}\rangle$ is a Hartree-Fock approximation to the fermionic ground state, and anti-Hermitian generators T_n consist of $2n$-fermion operators, each adding n excitations to the state $|\psi_{\mathrm{HF}}\rangle$. For example:

$$T_1 = \sum_{\alpha,\beta} T_1^{\alpha\beta} c_\alpha^\dagger c_\beta, \quad T_2 = \sum_{\mu,\nu,\lambda,\rho} T_2^{\mu\nu\lambda\rho} c_\mu^\dagger c_\nu^\dagger c_\lambda c_\rho \tag{1.3.2}$$

With the appropriate coefficients in T_n, UCC is able to lower the energy beyond that of $|\psi_{\mathrm{HF}}\rangle$ by introducing the right type of correlations. As using the generators with all possible orders n is computationally prohibitive, one often truncates the UCC to single and double excitation operators T_1, T_2 only. This is also referred to as UCCSD (SD stands for 'singles and doubles'). The efficiency of this approach is provable on the perturbative level, where it is ensured by the linked-cluster theorem [57].

To utilize UCCSD in a variational quantum algorithm, one needs to implement the unitary in Eq. (1.3.1) as a digital quantum circuit. In most cases this is not possible directly, and instead requires a Trotter-Suzuki approximation [58, 59]. For example, a UCCSD unitary can be approximated ('Trotterized') into K Trotter steps as follows:

$$e^{T_1+T_2} \simeq \prod_{k=1}^{K} \left[\prod_{\alpha,\beta} e^{\frac{1}{K}\left(T_1^{\alpha\beta} c_\alpha^\dagger c_\beta + \mathrm{h.c.}\right)} \prod_{\mu,\nu,\lambda,\rho} e^{\frac{1}{K}\left(T_2^{\mu\nu\lambda\rho} c_\mu^\dagger c_\nu^\dagger c_\lambda c_\rho + \mathrm{h.c.}\right)} \right]. \tag{1.3.3}$$

Given this form of the ansatz, realizing (1.3.3) as a quantum circuit is a routine procedure. Indeed, fermionic operators can be mapped onto Pauli matrices, and exponentials of Pauli strings can be represented with a quantum circuit using the standard procedures of [60]. By tuning the coefficients $T^{\alpha\beta}$, one can then employ the ansatz circuit (1.3.3) in a variational procedure.

Along with UCC, there exist alternative approaches to ansatz construction. One example is the Hamiltonian Variational Ansatz [56], which is based on the principle of adiabatic state preparation. Since adiabatic evolution is capable of producing complex ground states, its crudely Trotterized version is expected to have similar capabilities. The accuracy of such preparation is further enhanced by the variational procedure, making Hamiltonian Variational Ansatz a sound approach. Another method is

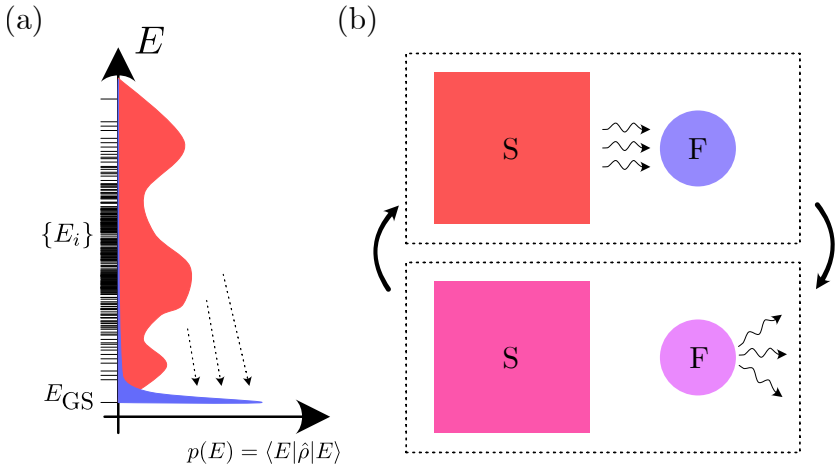

Figure 1.5: Quantum cooling. (a) In quantum mechanics, cooling implies a modification of the density matrix such that it shifts to lower energies (red to blue profile). With perfect cooling, the final state would be dominated by the ground state (blue profile). (b) To cool a physical system (S), it is standard to mediate the process by a controlled system (fridge, F). The process is two-fold. Firstly, the system and fridge are brought to interact. The energy from the system is then spontaneously transferred to the fridge (illustrated by a change of color). Secondly, the fridge has to emit surplus energy into the environment. The latter process is to be driven by external control, and would not happen under equilibrium thermodynamics. In algorithmic quantum cooling, one can achieve this by manually driving the fridge to its ground state.

ADAPT [55], which suggests to add tunable elements to the quantum circuit one at a time, while optimizing the parameters of such an ever-growing ansatz. The added unitary is to be picked adaptively, by estimating and optimizing the energy decrease that is expected from this addition. ADAPT is digital by construction and automatically adapts itself to the problem of interest, thus forming another viable approach to ansatz design. The methods presented above have both strengths and limitations — each is founded on specific analytical consideration, with some concessions made for compatibility with near-term quantum hardware. As such, there is an ongoing effort to design new methods of ansatz construction. The ultimate goal of these novel methods is to improve the stringent trade-off between the capacity of the circuit and its ease of implementation.

1.3.2 Dissipative quantum algorithms

Dissipative approach to quantum computing [61] implies performing a computation through an engineered evolution of an open quantum system, rather than an isolated one. This paradigm is less developed relative to the more established scheme of unitary-based computing, albeit it is theoretically sound. In particular, it was proven that any polynomially-complex algorithm performed with a standard quantum circuit can be mapped onto dissipative quantum hardware with only polynomial overhead [61]. Any evolution of an open quantum system, conversely, can be directly realized on digital quantum hardware by using ancillary registers. Intriguingly, with the latter procedure, one may apply the dissipative paradigm directly to digital quantum computing — potentially leading to new and powerful algorithms.

In the context of applying digital quantum hardware to ground state preparation, the dissipative paradigm is of direct relevance. Indeed, the low-energy states in nature are normally produced via a dissipative process of cooling (Fig. 1.5). Cooling is also employed in quantum engineering, for instance when initializing a null computational basis state of a quantum computer [62–64]. In the context of digital quantum computation, cooling was proposed early on [12] as a method of ground state simulation. In this case, even a single ancillary qubit can be used to emulate a fridge. Indeed, to drive the qubit to its ground state is straightforward — and being able to prepare the low energy state of the fridge is key to controlled cooling (Fig. 1.5b). Despite this being an interesting possibility, this idea remained largely undeveloped after the work of Lloyd [12]. Instead, most of research in algorithmic quantum cooling is focused on analog quantum computation [65, 66].

Another way to include dissipation into a state preparation protocol is to introduce weak measurements [67, 68]. Compared to conventional (projective) quantum measurements, weak measurements extract less information about the system and modify its state in a less drastic fashion. These unusual measurements require an auxilliary 'detector' system, which is coupled to the subject system only for a brief period of time. Application of such weak measurements to quantum state preparation is actively studied under the framework of quantum control theory [69–73] . In the traditional approaches, the system is controlled quantum-coherently, and the information obtained from the weak measurements can be fed back to modify the direction of this unitary evolution. A feedback of this type is classified as closed-loop quantum control. Compared to pre-defined unitary evolution, including such feedback allows to complete the state

preparation task with fewer resources or on a shorter timescale. Closed-loop quantum control can even be taken to the extreme, by removing the quantum-coherent part of the evolution altogether [69, 74, 75]. In this case, the weak measurements are used to inform the subsequent evolution, which is also driven by weak measurements. This approach, which goes under the name "control-free control", is now under active investigation both in experiment and theory [69].

1.4 This thesis

Chapter 2

Non-abelian statistics in topological superconductors can be realized by exchanging vortices that host Majorana zero-modes. However, using the bulk Abrikosov vortices for this purpose has proven to be impractical. Instead, many experimental groups turned to using Majorana zero-modes at the ends of superconducting nanowires, an effort that has also proven to be challenging. In this chapter, we propose to use a topological superconductor to realize the non-abelian braiding of an itinerant edge vortex with a bulk vortex. A voltage-driven Josephson junction can be employed to deterministically produce an edge vortex available for a braiding procedure. After a braiding operation, the vortices are to be fused back at another Josephson junction utilizing another voltage bias. We predict that the charge produced after the fusion is sensitive to the braiding operation. In particular, a single electron charge is produced after the braiding, and no charge if no braiding has occurred.

Chapter 3

We put forward a scheme to realize and detect another phenomenon of non-abelian statistics - anyon fusion. For this, we suggest employing the chiral edge vortex architecture introduced in the previous chapter. In a topological superconductor, by fusing Majorana-hosting vortices one produces a mixture of zero and one Dirac fermion. We design an experiment to capture this property, using four edge modes and four Josephson junction terminals. We propose to create two pairs of such vortices (1 and 2, 3 and 4) at the input terminals, and fuse them at the output terminals in a different configuration (1 with 3, 2 with 4). We predict, that the vortex fusion produces an equal weight superposition of two electrons and no electrons in the two output channels. This reproduces the fusion rule with

a direct effect on the output observables: charge transfer and fermion parity. In particular, we show that (a) the average fermion parity in each of the two leads is exactly vanishing upon fusion and (b) charge transfer is directly correlated, with the difference of output currents exhibiting zero noise while their sum remaining noiseful.

Chapter 4

Parafermionic zero-modes allow to realize non-abelian braiding based on a Laughlin type Fractional Quantum Hall material. Unfortunately, they were not yet shown to be realized in an experiment. It is therefore of high interest to characterize parafermions with appropriate observables. To that end, we investigate the possibilities offered by the Clauser-Horne-Shimony-Holt (CHSH) inequality. It is a version of Bell inequality, which signifies the extent of quantum correlations between two separated physical systems. Firstly, we generalize this and other notions of Bell nonlocality to the context of non-hermitian and potentially non-commuting observables. These are characteristic for systems of parafermions, due to the anyonic nature of these quasiparticles. Secondly, for such generalized observables, we draw up several bounds and relations for the intra-system and inter-system correlations. We show how these can be probed with parafermions, and predict that our correlation bounds are saturated much tighter when the two subsystems host non-commuting, rather than commuting observables. Paradoxically, the non-commutation of observables in these separated anyonic systems could be interpreted as superluminal signaling. This is only a simulation of such signaling, however. We show that the relativistic causality is automatically restored when such paradoxical correlations are to be probed in a physical experiment.

Chapter 5

Successfully employing a variational quantum algorithm for ground state preparation requires the use of appropriate ansatz circuits. The main requirements for such an ansatz are high expressivity and ease of use with digital quantum hardware. One such prospective ansatz, inspired by computational quantum chemistry, is a Trotterized version of Unitary Coupled Cluster. The ansatz is analytically justified by the linked cluster theorem, which proves its efficiency in the perturbation theory. Unfortunately, this approach relies on Trotterization which is not exact. In this chapter, we put forward an approach to ansatz creation that follows the linked-cluster theorem while not relying on Trotterization. The basis of the construction

is Quantum Combinatorial Ansatz (QCA). QCA is given by a sequence of tunable Pauli string-generated rotations, spanning the entire N-qubit Hilbert space in a minimum number of such elementary rotations. We give a rule for a systematic reduction of QCA to practical size, based on many-body perturbation theory. This ansatz construction turns out to satisfy the linked-cluster theorem, therefore proving its efficiency in the perturbative limit. Finally, we numerically test a few variants of QCA-based ansatz constructions by applying them to Ising spin chains. We find that these allow for a good asymptotic convergence to the ground state in the paramagnetic and the ferromagnetic phases of this model. As expected from perturbative analytics, in the weakly coupled limit of the model, the variant of an ansatz construction that satisfies the linked-cluster theorem shows optimal performance.

Chapter 6

Using dissipative approaches in digital quantum hardware is an area of research that is currently under active development. One natural application is ground state preparation. In this chapter, we use the principle of cooling to design a ground state preparation algorithm. To that end, we propose to simulate the interaction between the target system and a single qubit, transferring the energy to the qubit. The qubit, therefore, plays the role of a fridge. To simulate the cooling process, this has to be combined with the internal system and fridge evolutions. The final ingredient of the protocol is the measurement and reset of the fridge qubit, which introduces the dissipative element necessary to reduce the system energy. We develop the protocol by first pinning down the case of a single-qubit system and then extending the protocol to a general system of N qubits. The role of energy conservation is established, given by an extension of the Fermi Golden Rule to the case of finite-time evolution. We study the issues of Trotter error and Heisenberg energy uncertainty and propose two scalable approaches to tackle these. The BangBang approach, characterized by high Trotter error and large Heisenberg uncertainty, yields low implementation complexity and is thus suitable for near-term implementation. The complementary LogSweep approach yields an asymptotically vanishing energy uncertainty and Trotter error. However, this asymptotic accuracy is at the cost of extensive circuit complexity. Numerically, we apply LogSweep to transverse-field Ising chains in their paramagnetic and critical states. The time complexity of such ground preparation turns out to scale algebraically with a desired precision.

Chapter 7

This chapter is focused on another dissipative approach to quantum state preparation: steering by generalized measurements. To reduce the time in which the target state is prepared, we propose to actively choose the applied measurements on-the-go. This active choice is to be based on the information obtained from the previous measurements. The possible policies for active-decision steering are influenced by the presence of entanglement in the target state. For strongly correlated target states, creating a policy for active-decision steering is most challenging, due to the vastness of the many-body Hilbert space. Therefore, for efficient decision-making special Hilbert space "navigation techniques" are needed. Two approximate ways of representing such navigation are developed: via (i) a cost function landscape and (ii) a semiclassical Quantum State Machine. From such simplified Hilbert space representations, one derives the respective active-decision policies. We numerically apply these policies to two paradigmatic targets: AKLT state and W-state. In each case, the introduction of active decision achieves up to factor 10 speed-up of the target state preparation.