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Strategies for braiding and ground state preparation in digital quantum hardware

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Strategies for braiding and ground state preparation in digital quantum hardware

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Cover: An interlacing of colored threads which illustrates braiding and the notion of a quantum ground state. Regions with regular patterns represent the onset of order at low energies. The orange-to-blue gradient on the threads shows the transition from hot to cold as one moves closer to the ground state.

To my family and to Marina.

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