Tautological differential forms on moduli spaces of curves

Lugt, S. van der

Stellingen

behorende bij het proefschrift

Tautological differential forms on moduli spaces of curves

van Stefan van der Lugt

1. Let $r \geq 0$, and $\chi \in \mathbb{Z}$. There are, up to isomorphism, only finitely many contracted r-marked graphs of characteristic χ. These graphs have at most $2(r - \chi)$ unmarked vertices.

2. Let $d \in \mathbb{Z}$ be an integer. If d is negative, the set $\mathrm{CG}(r, r-d)$ of isomorphism classes of contracted r-marked graphs of characteristic $r - d$ is empty for all $r \geq 0$. If $d \geq 0$, then there exists a polynomial $f_d \in \mathbb{Q}[x]$ of degree $2d$ such that

$$|\mathrm{CG}(r, r-d)| = f_d(r) \quad \text{for all } r \geq 0.$$

   The leading coefficient of f_d is $1/(2^d \cdot d!)$.

3. Let $r \geq 0$ and $g \geq 2$ be integers. Let $\mathcal{C}_g \to \mathcal{M}_g$ be the universal family of genus g curves, and let $\mathcal{C}_g^r$ denote the r-fold fiber product of $\mathcal{C}_g$ over $\mathcal{M}_g$. Then the ring $\mathcal{R}^*(\mathcal{C}_g^r)$ of tautological forms on $\mathcal{C}_g^r$ is a finite-dimensional $\mathbb{R}$-vector space.

4. For all $r \geq 0$ the dimension of the real vector space $\mathcal{R}^2(\mathcal{C}_g^r)$ of tautological differential 2-forms on $\mathcal{C}_g^r$ equals $\frac{1}{2}r(r+1)+1$ if $g = 2$ and $\frac{1}{2}r(r+1)+2$ if $g \geq 3$.

5. Objects on moduli spaces are objects that occur universally among the families these moduli spaces classify.

6. Let E be a semi-stable elliptic curve over a number field K. There exists a function $B(N) = N \log N + O(N)$ such that the following holds: for each K-torsor C of E, and each point $P \in C(\bar{K})$ with field of definition $L \supseteq K$, the following inequality holds:

$$\log \left| N_{K/\mathbb{Q}}(\Delta_{L/K}) \right| \leq B([L : K]).$$

7. When using a computer to carry out extensive computations, one should not only take into account the efficiency of the used algorithms, but also the efficiency of the underlying software libraries.

8. Let P be any point on earth. The average daytime length at P can be less than 12 hours, but also more than 0.5 years. Currently there exists a point P on the northern hemisphere for which the average daytime length is longer than 0.5 years. Such a point does not exist on the southern hemisphere.

   (joint with Raymond van Bommel)

9. In order to understand a subject, one should teach it. In order to teach a subject, one should understand it.