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Tautological differential forms on moduli spaces of curves

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Tautological differential forms on moduli spaces of curves

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